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# Energetic Tail Evolution of Auroral Electron Spectra

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**Abstract.** Based on a Fokker-Planck formalism a mechanism is presented, generating consistent with spacecraft observations a high energy tail electron population from the interaction of a broad-band Alfvén wave spectrum under auroral plasma conditions. The structure of the resulting suprathermal tails of the electron velocity space distributions is shown to be well represented by the family of kappa distributions, hence providing a physical interpretation of the origin of auroral energetic electron observations. © 2001 Elsevier Science Ltd. All rights reserved

## 1 Introduction

As indicated by numerous spacecraft observations, most astrophysical plasmas are found to have velocity distribution functions exhibiting non-Maxwellian suprathermal tails (Mendis and Rosenberg, 1994). The advantage in using a generalized form of a Maxwellian to fit energetic particle properties was pointed out by (Vasyliunas, 1968) in conjunction with magnetosheath electron observations or by Leubner (1982, 1999b) to study the Jovian multi-banded whistler emission. The success in shaping velocity space observations by a kappa-distribution is documented in numerous investigations of space plasma wave-particle interaction processes and instabilities (Xue et al., 1996; Chaston et al., 1997). Plasma sheet ion and electron spectra are well represented at high energies by a power law in particle speed (Christon et al., 1991) and excellent agreement between the Dynamic Explorer DE 2 data and a combined electron-proton-hydrogen atom aurora theory was found from a kappa-distribution approach (Decker et al., 1995). Recently detailed studies of auroral current density and energy fluxes were successfully carried out fitting low orbiting Freja electron flux spectra by a power law at high energies (Janhunen and Olsson, 1998; Olsson and Janhunen, 1998).

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The generation of velocity space distributions exhibiting pronounced energetic tails can be interpreted to be a consequence of several different acceleration mechanisms, as DC parallel electric fields or a field-aligned potential drop, produced when magnetic field lines are being reconnected (Menietti and Smith, 1993; Hoffman, 1993). Non-thermal features of velocity distributions may result also from an acceleration mechanism by wave-particle interaction due to the presence of broadband lower hybrid or Alfvén wave turbulence. In the latter context, the problem of wave induced particle energization is formulated within a Fokker-Planck approach, where the velocity space diffusion is induced by Landau interaction (Dendy et al., 1995; Leubner and de Assis, 1998; Leubner, 1999a). From an initially Maxwellian electron equilibrium distribution the time evolution of the distribution function due to the interaction of an Alfvén wave-spectrum is simulated numerically, where major attention is drawn to the specific shape of the diffusion operator, generating energetic electron tails in velocity space. In addition, it is shown that collisional drag provides a limitation for the particle energization possibly resulting in a saturated stage.

## 2 Theory

The general equation for the collective, time dependent development of the velocity space distribution in response to a wave spectrum affecting the particles via Cherenkov interaction, as well as in response to particle collisions and external electric fields reads

$$\frac{\partial f}{\partial t} = \left[\frac{\partial f}{\partial t}\right]_{waves} + \left[\frac{\partial f}{\partial t}\right]_{collisions} + \left[\frac{\partial f}{\partial t}\right]_{E-field} \quad (1)$$

Here $f(v, t = 0)$ shall be represented by a starting one-dimensional Maxwellian equilibrium distribution function

$$f(v,0) = \frac{N}{v_{th}\sqrt{\pi}} \exp\left[\frac{v^2}{v_{th}^2}\right] \quad (2)$$

where $v_{th} = \sqrt{2k_BT/m}$ is the thermal speed and $N$ denotes the particle density. Neglecting external electric field contributions, we model the wave-particle interaction by a diffusive process and consider the time evolution of the parallel velocity distribution function $f(v)$ with respect to the magnetic field $\mathbf{B}_0$, being regulated by the Fokker-Planck equation as

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v}\nu(v)\left[vf + v_{th}^2\frac{\partial f}{\partial v}\right] + \frac{\partial}{\partial v}D(v)\frac{\partial f}{\partial v} \quad (3)$$

Here $\nu(v)$ and $D(v)$ denote the velocity dependent collision and diffusion operators, respectively. The first term on the right hand side of equation (3) restores the distribution function to a Maxwellian. The consideration of a uniform magnetic field $\mathbf{B}_0$ allows with regard to axial symmetry a reduction of the three dimensional problem to two dimensions. The reduction to a reasonable one-dimensional approach can be achieved by assuming low collisionality such that changes in the pitch angle are negligible and we note that the main dynamics of Alfvén wave-particle energy exchange due to Landau interactions is regulated in parallel direction.

The quasi-linear. one dimensional diffusion operator due to Cherenkov wave-particle interaction (Davidson, 1972) may be written as

$$D(v) = (\frac{e}{m}\cos\theta)^2 \int \frac{d\Omega dK}{2\pi} S_E(K,\Omega)\pi\delta(\Omega - Kv\cos\theta) \quad (4)$$

where $S_E(K,\Omega)$ is the spectral energy density of the wave turbulence and the Dirac term describes the wave-particle resonance. The $\cos\theta$ dependence is a consequence of the wave parallel electric field component where $\theta$ is the angle between the wave vector and the magnetic field $\mathbf{B}_0$.

Next, we adopt a broadband spectrum $R(k)$ of Alfvén waves, trapped inside an envelop of extension $L_{\parallel,\perp}$ parallel and perpendicular to the magnetic field $\mathbf{B}_0$, characterized by their wavenumbers $k$ and frequencies $\omega(k)$. Performing the resonant interaction between the wave turbulence and the plasma electrons in the wave packet frame and using a Gaussian shape for the wave envelope, the diffusion coefficient for parallel electron acceleration can be written as

$$D(v) = \frac{L_\perp A^2\cos\theta}{\sqrt{8\pi}av}\sum_k |R(k)|^2 g(a,\gamma)\exp(-a\gamma^2) \quad (5)$$

where $A$ is the amplitude of the wave packet and $a$ denotes a parameter determining over which part of the wave packet the diffusion acts. Here $g(a,\gamma)$ enters from the consideration of spatially confined turbulence where $\gamma = ikL_\perp[\omega(k) - kv\cos\theta]/ak\ v\cos\theta$.

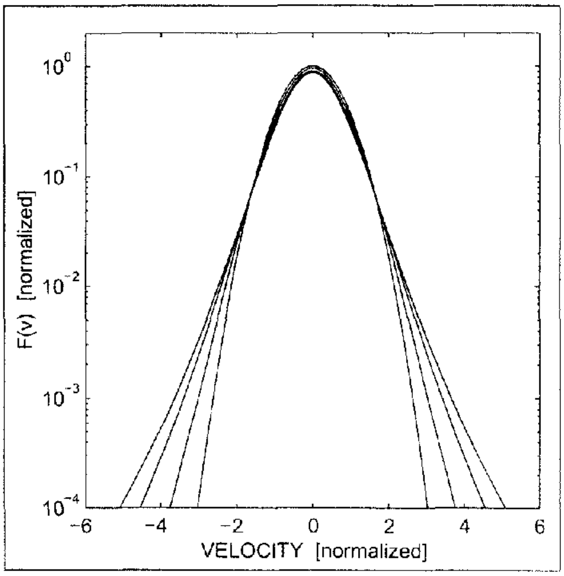


**Fig. 1.** A family of kappa-distributions for $\kappa = 3, 5, 10$ and $\infty$. The outermost curve exhibits with $\kappa = 3$ pronounced non-thermal tails, the innermost curve represents with $\kappa = \infty$ an isotropic Maxwellian. Velocities are normalized to the thermal speed.

Equation (5) permits in combination with the Fokker-Planck approach from equation (3) the simulation of wave-particle energy exchange due to a broadband Alfvén wave packet with $\cos\theta \simeq 1$, obeying the dispersion relation for dispersive Alfvén waves

$$\omega^2 = k_\parallel^2 v_A^2 \frac{1 + k_\perp^2 r_{gi}^2(3/4 + T_e/T_i)}{1 + k_\perp^2 c^2/\omega_{pe}^2} \quad (6)$$

where $\omega_{pe}$ is the electron plasma frequency. Usually the parallel wavelength is much larger than the ion gyroradius $r_{gi} = m_i v_{th} c/(eB_0)$ wherefore the conditions $k_\parallel \ll k_\perp$, $k_\perp^2 r_{gi}^2 \sim 1$ hold. The dispersive Alfvén wave described by the dispersion relation (6) contains the ion thermal corrections and additionally takes care of the inertia of the electron background plasma. For Alfvén waves the main damping mechanism is Landau damping by electrons since the electron thermal speed is comparable to the Alfvén velocity for high temperature space plasmas. Thus, Alfvén waves absorbed in this context can resonantly interact to create an extended tail in the particle velocity space distribution.

Finally, let us consider the generally observed high energy tails in the particle distributions from a one dimensional analytical form of the family of kappa-distributions as (Leubner, 1982)

$$F(v) = \frac{N}{v_{th}\sqrt{\pi}}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-1/2)}(1 + \frac{v^2}{\kappa v_{th}^2})^{-(\kappa+1)} \quad (7)$$

hence, representing a power law in particle speed, see Fig. 1.

The parameter $\kappa$ shapes predominantly the suprathermal tails of the distribution, $\Gamma$ denotes the standard Gamma function and as $\kappa \to \infty$, $F(v)$ approaches a Maxwellian. The advantage in using the function (7) instead to model the energetic particles by a power law at high energies was outlined in numerous studies as (Vasyliunas, 1968; Christon et al., 1991; Ma and Summers, 1998), trying to fit accurately spacecraft observations, wherefore we search for appropriate generation mechanisms of kappa-like suprathermal tails in velocity space.

## 3 Results and Conclusions

The numerical simulation of the time evolution of the electron distribution due to a diffusive process of wave-particle interaction and collisions is treated within the Crank-Nicholson implicit scheme. As initial condition a Maxwellian equilibrium velocity distribution is introduced. For magnetospheric-auroral plasmas the basic parameters are chosen as $N = 25\ cm^{-3}$, $T_e = 1 \times 10^5 K$ and $B_0 = 2200\gamma$, noting that a simultaneous change of the parameters produce similar results for different physical systems. A generation mechanism representing the family of kappa-distributions accurately enough in the context of wave-particle interaction due to a spectrum of Alfvén waves provides a possible theoretical explanation for the formation of persistent kappa-like distribution functions in magnetospheric-auroral plasmas.

As an example Fig. 2 illustrates the velocity dependence of a normalized diffusion operator, evaluated from equation (5) with respect to the dispersion relation (6) for a broadband Alfvén wave spectrum, where $L_\perp \ll L_\parallel$ is assumed.

Introducing this diffusion properties into the Fokker-Planck formalism, Fig. 3 shows the time evolution of the distribution function under auroral conditions. Starting from a Maxwellian, equidistant snapshots in time are presented and may be compared with the family of kappa-distributions in Fig. 1. The resulting structures demonstrate impressively that kappa-like suprathermal tails in the electron distribution, exhibiting a smooth onset in velocity space are generated by a broadband Alfvén wave-particle energy exchange process. In Fig. 4 a non-zero collision frequency $\nu$ is adjusted such, that the electron energization due to Alfvén wave turbulence is saturated at a balancing level between energy input and collisional thermalization. Hence, although not of general importance with respect to magnetospheric plasma environments, the possibility to simulate the time evolution of wave-induced particle acceleration towards a saturated stage by applying a simple collision term into the Fokker-Planck approach is demonstrated.

It turns out that the specific shape of the diffusion operator in velocity space, as calculated from the wave power spectrum and dispersion relation, enters as dominant model parameter. Consistent with the family of

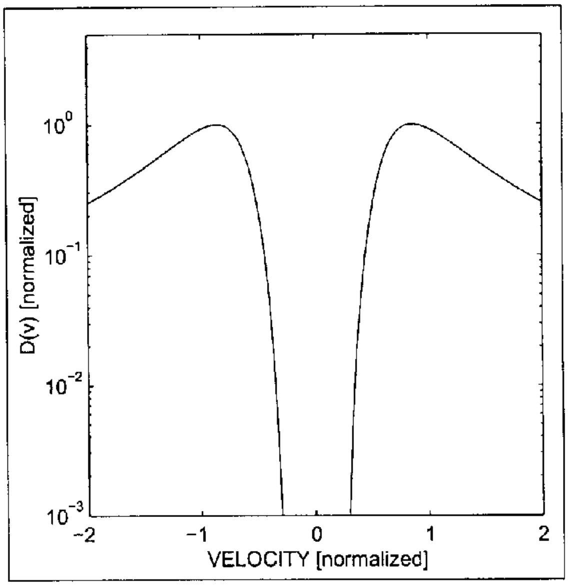


**Fig. 2.** Velocity dependence of the diffusion coefficient for a broadband Alfvén wave spectrum. Velocities are normalized to the Alfvén speed.

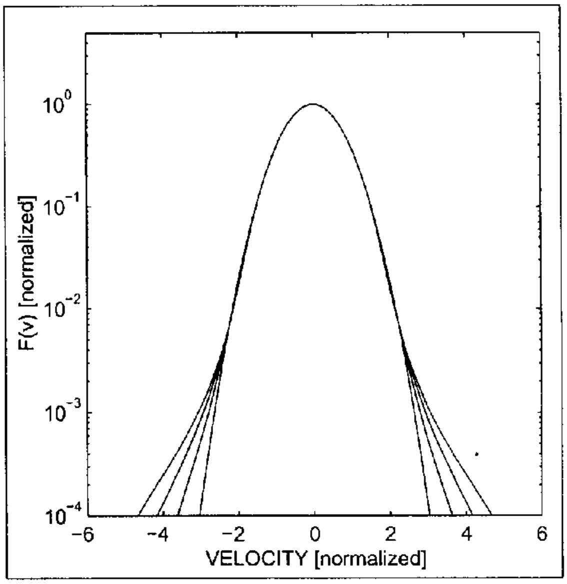


**Fig. 3.** Time sequence of the formation of a suprathermal electron population due to an Alfvén wave spectrum. Non-thermal features reproduce well those of the kappa-distribution family. Velocities are normalized to the thermal speed.

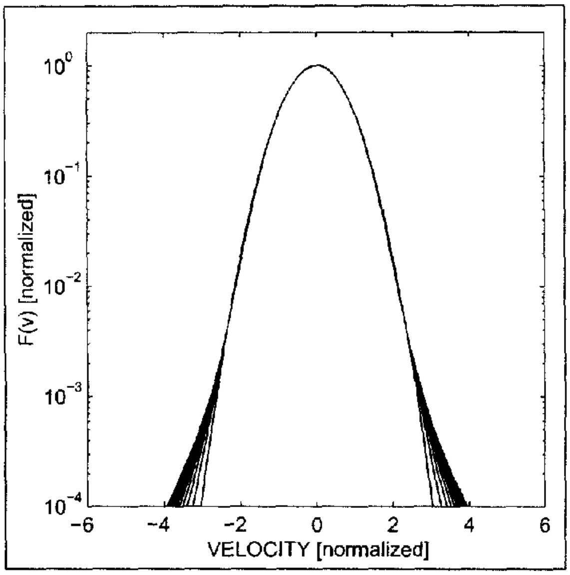


**Fig. 4.** Time evolution of a Maxwellian into a saturated stage due to Alfvén wave-particle energy exchange and collisional drag. Velocities are normalized to the thermal speed.

kappa-distributions a significant fraction of suprathermal electron populations can be generated by broadband Alfvén wave spectra, representing accurately spacecraft observations. Thus, a justification for the use of the analytical family of kappa distributions for auroral space plasma studies is provided.

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