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CONDITIONS FOR THE EXISTENCE OF CHAOTIC OSCILLATIONS IN NUCLEAR REACTORS*

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It is shown on the basis of point kinetics equations with delayed neutrons that if the impulse feedback function is negative, nonmonotonic, and possesses several maxima and the coefficient of amplification of feedback is sufficiently large, then chaotic self-excited oscillations of the following type arise in nuclear reactors. Neutron bursts with random intensity occur in random time interals in the reactor, and the neutron density between the bursts oscillates at a low level. The mechanism for the appearance of chaos is described and one-dimensional mappings which approximately determine the chaotic dynamics are constructed. Three types of reactors (boiling water, with gaseous core, pulsed) where such chaotic oscillations can arise are indicated. The results obtained point the way to determining other types of reactors with stochastic behavior. 4 figures, 10 references.

In [1–3], it is shown on the basis of a point kinetics model with no delayed neutrons that chaotic oscillations of the following type can arise in reactors. Neutron bursts with random intensity arise in a reactor in random time intervals, and the neutron density in the intervals between these bursts oscillates at a low level. A condition for the appearance of such oscillations is that the feedback transfer function – the response of reactivity to a step change in neutron density – must be negative and nonmonotonic [2, 3].

In the present paper, it is shown that similar chaotic oscillations can also arise in reactor models with delayed neutrons. However, the condition for their existence is that it is not the transfer but the impulse function – the response of reactivity to a pulsed change in neutron density – that must be negative and nonmonotonic. Since dynamic processes in reactors are largely determined by delayed neutrons, dynamic chaos in specific types of reactors should be judged according to the properties of the impulse function.

We shall describe the change in neutron density by the kinetics equations

$$l\dot{n} = (1+n)\delta k + \sum_{i=1}^{k} \lambda_i c_i - \beta n; \tag{1}$$

$$\dot{c}_i = -\lambda_i c_i + \beta_i n; \quad i = 1, \ldots, k, \tag{2}$$

where $n(t)$ is the relative deviation of the neutron density from a stationary value, l is the lifetime of prompt neutrons, δk is the reactivity, $c_i(t)/l$ is the relative deviation of the concentration of nuclei-emitters of delayed neutrons of the ith group from the

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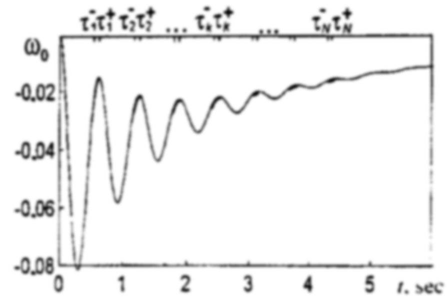

Fig. 1. Impulse function $\omega_0(t)$ of the form of (6).

stationary value; k is the number of groups; λ_i is the decay constant of these nuclei; β_i is the relative yield of delayed neutrons as a result of a fission event; $\beta = \sum_{i=1}^{k} \beta_i$; and t is the time.

We describe linear feedback on reactivity by the equations

$$\dot{\mathbf{u}} = P\mathbf{u} + \mathbf{d}n; \quad \delta k = \mathbf{b}^T \mathbf{u}, \tag{3}$$

where $\mathbf{u}(t)$, $\mathbf{d}$, $\mathbf{b} = \theta \mathbf{b}_0 \in R^m$, and P is a constant $m \times m$ Hurwitz matrix.

The transfer function $h(t)$ and the impulse function $\omega(t)$ for the feedback in (3) are the response $\delta k(t)$ to a step $n(t) = 1(t)$ or impulsive $n(t) = \delta(t)$ perturbation n with $\mathbf{u}(0) = 0$:

$$h(t) = \mathbf{b}^T[\exp(Pt) - I]P^{-1}\mathbf{d}; \qquad \omega(t) = dh/dt = \mathbf{b}^T \exp(Pt)\mathbf{d}. \tag{4}$$

It is well known [4, 5] that all solutions of the system of equations (1)–(3) are bounded, if

$$\omega(t) < 0 \quad \text{for} \quad t > 0. \tag{5}$$

We shall assume that the condition of (5) holds and that both states of equilibrium of system (1)–(3) are unstable. Under these restrictions, steady motions in system (1)–(3) are self-excited oscillations.

It is shown in this paper that the condition for these oscillations to be stochastic is that $\omega(t)$ must be nonmonotonic and the following condition, which is more stringent than the condition of (5), must hold:

$$\omega_0(t) = \omega(t) - \omega(0)\exp(s_0 t) < 0 \quad \text{for} \quad t > 0, \tag{6}$$

where $s_0 < 0$ is the eigenvalue of the matrix P closest to the imaginary axis. It is real, since otherwise condition (5) would break down for large t. An example of the function $\omega_0(t)$ leading to chaos is shown in Fig. 1. The function $\bar{\omega}_0(t)$

$$\bar{\omega}_0(t) = \omega(t) + \omega(0)[h(t)/h(\infty) - 1] < 0 \quad \text{for} \quad t > 0 \tag{7}$$

proves to be close to Eq. (6).

During self-excited oscillations a reactor can become subcritical ($\delta k(t) < 0$) or supercritical ($\delta k(t) > 0$). Chaotic oscillations are possible with ($\delta k(t) > \beta$ for some t) and without ($\delta k(t) < \beta$ for all $t > 0$) the reactor reaching instantaneous kinetics. The first types of oscillations are characteristic for fast feedbacks, i.e., feedbacks for which the characteristic variation time of $\omega(t)$ and $\delta k(t)$ is comparable to or much less than the characteristic decay time of delayed-neutron emitters ($|\mathrm{Re}\, s_i| \cong \lambda_j$ or $|\mathrm{Re}\, s_i| > \max_j \lambda_j$; s_i are the eigenvalues of the matrix P). The second types of oscillations occur for small feedbacks ($|\mathrm{Re}\, s_i| < \min_j \lambda_j$). A mechanism of chaos

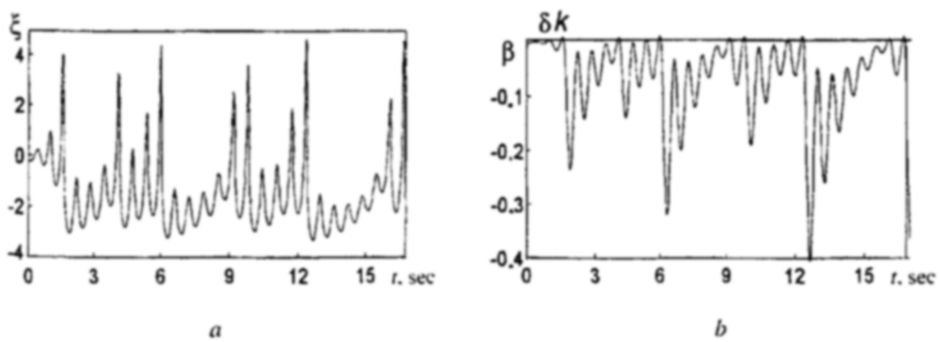

Fig. 2. Chaotic oscillations of the quantity $\xi(t) = \ln[1 + n(t)]$ (*a*) and the reactivity $\delta k(t)$ (*b*).

with instantaneous kinetics is described below. The description of the mechanism of chaos without instantaneous kinetics will be similar, except that the phrase "the reactivity reached β and a fission pulse on prompt neutrons has occurred" must be replaced by the words "the reactivity has reached zero and the neutron pulse on delayed neutrons has occurred."

Let the reactivity exceed β at a certain moment in time. In this case, the reactor becomes supercritical on prompt neutrons, and a neutron burst (fission pulse) starts. The burst continues as long as $\delta k(t) > \beta$, and it stops as a result of the negative feedback of (3) for $\delta k(t) < \beta$. A sharp drop in neutron density and a transition to slow kinetics occur. After the pulse, the reactivity in the region $\delta k(t) < \beta$ behaves similarly to an impulse function $\omega_0(t)$ of the form of (6), repeating all inflections and extrema. The oscillations of $\delta k(t)$ give rise to oscillations of the neutron density at a quite low level, determined by the burst and by the delayed neutrons. They continue for a time τ until δk once again reaches β and the next neutron pulse occurs. The oscillations of $\xi(t) = \ln[1 + n(t)]$ and $\delta k(t)$, corresponding to the function $\omega_0(t)$ in Fig. 1, are shown in Fig. 2.

If the function $\omega_0(t)$ is nonmonotonic, then the behavior of $\delta k(t)$ after bursts in the region $\delta k < \beta$ is also nonmonotonic. The same time intervals correspond to the sections of growth and decay of the functions $\delta k(t)$ and $\omega_0(t)$, irrespective of the pulse. The reactivity can then reach the value β only on limited sections of increasing function $\delta k(t)$, lying to the left of one of its maxima. Similar sections and maxima of the function $\omega_0(t)$ and the corresponding time intervals $\Delta\tau_i = [\tau_i^-, \tau_i^+]$ (these intervals and sections are shown in Fig. 1) correspond to these sections and maxima of $\delta k(t)$. In this connection, for a nonmonotonic function $\omega_0(t)$, the time τ between bursts cannot be arbitrary, it must lie within the bounded set of intervals $\delta\tau_i$. We shall say that if $\tau \in \Delta\tau_i$, then the next neutron burst is initiated by the *i*th maximum of the impulse function. In Fig. 2*b*, the reactivity successively reaches β on the maxima of $\delta k(t)$ with the numbers 4, 2, 1, 5, 1, 3, 1, 6, and 1.

The time τ depends on the intensity of the neutron burst, which we shall characterize by the scalar quantity γ. This quantity is proportional either to the maximum neutron density or the energy released in the reactor over the time of the pulse. The larger the value of γ, the larger τ is. If $\omega_0(t)$ is nonmonotonic, then as γ increases, $\tau(\gamma)$ increases monotonically within the admissable intervals $\Delta\tau_i$ and for certain values of γ_i, it jumps abruptly from $\Delta\tau_i$ to $\Delta\tau_{i+1}$, i.e., the function $\tau(\gamma)$ is discontinuous at $\gamma = \gamma_i$. When the condition of (6) is satisfied, the values of τ are not bounded and as γ increases, they run through all intervals $\Delta\tau_i$. However, if the condition of (6) breaks down and $\omega_0(t^*) = 0$, then for arbitrary γ the time τ does not exceed t^*. Then all maxima of $\omega_0(t)$ in the region $t > t^*$ are blocked, i.e., they cannot give rise to neutron bursts. For this reason, when condition (6) breaks down, the chaos mechanism being described also breaks down.

The intensity of the next burst depends on the derivatives $\dot{\delta k}$, $\ddot{\delta k}$, ... with which the reactivity δk reaches β and not on the maximum of the impulse function with which the burst is initiated. If $\delta k(t)$ at $t = \tau$ touches the line $\delta k = \beta$, then $\dot{\delta k} = 0$ and there is no impulse. The larger the value of the derivative with which $\delta k(t)$ crosses the value β, the larger the value of γ^* of the next impulse is. Impulses initiated by the same *i*th maximum of $\omega_0(t)$ with $t \in \Delta\tau_i = [\tau_i^-, \tau_i^+]$ can take on very diverse values. Conversely, impulses caused by different maxima can be identical. We denote by $\Gamma_i = \{\gamma : \gamma = \gamma^*(\tau);\ \tau \in \Delta\tau_i\}$ a series of impulses engendered by the condition $\delta k(t) = \beta$ being satisfied on the *i*th maximum of $\omega_0(t)$. The next impulse can be judged according to the derivatives of the impulse function, which approximately correspond to the derivative of δk at the moment τ. The impulse is larger for those $\tau \in \Delta\tau_i$ for which $\dot{\omega}_0(\tau)$ is larger. For $\omega_0(t)$ in Fig. 1, the impulse grows with τ increasing from $\tau = \tau_i^-$ to $\tau = \tau_i^+$. In the general case, we have

$$\gamma^* = \gamma^*(\tau) = f[\dot{\delta k}(\tau), \ddot{\delta k}(\tau), ...] \cong \tilde{f}[\dot{\omega}_0(\tau), \ddot{\omega}_0(\tau), ...], \tag{8}$$

i.e., the next burst depends on the time between the bursts. Substituting the function $\tau(\gamma)$ into Eq. (8), we obtain the one-dimensional mapping

$$\gamma^* = \gamma^*(\gamma) = \varphi(\gamma), \tag{9}$$

which determines the next burst according to the preceding burst.

We shall now determine the conditions under which the oscillations arising will be stochastic. It is well known that dynamical chaos is realized in a situation where all motions of the system, being globally bounded, are at the same time unstable. In our case, the condition of (5) ensures boundedness. Instability of motions will arise in situations when the series of bursts Γ_i proliferates, i.e., each series Γ_i engenders in the next cycle several different series Γ_j and not just one series. This means that after impulses of one (any) series Γ_i, engendered by the ith maximum of the function $\omega_0(t)$, the time τ up to the next burst will lie in several intervals $\Delta\tau_j = [\tau_j^-, \tau_j^+]$, so that the next impulses can be engendered by several maxima of the function $\omega_0(t)$, i.e., they can belong to several series of impulses Γ_j. In this case, there arise chaotic oscillations during which, together with the next impulse γ and the time τ between bursts, the series Γ_i to which the impulse belongs, the interval $\Delta\tau_i$ to which τ belongs, and the corresponding interval $\Delta\tau_i$ of the maximum of the function $\omega_0(t)$ that gives rise to the next impulse are all random.

We note that the intensity of neutron bursts which is required for the series Γ_i of impulses to proliferate and the chaos mechanism being described to appear will be attained in the case of fast feedbacks for sufficiently large values of $\dot{\omega}(0)/\omega(0)$ and the coefficient of amplification of the feedback $\theta(\delta k = \theta b_0^T\mathbf{u})$. An increase of θ at first gives rise to instability of the point $M_0 = 0$ and then to chaos according to one of the known scenarios.

The proliferation of the series of impulses can be represented as a graph with vertices $\Gamma_1, \Gamma_2, ..., \Gamma_N$ and edges γ_{ij}, directed from Γ_i to Γ_j only if the series of impulses from Γ_i engenders pulses from Γ_j. Motion along the edges of a graph corresponds to a semiinfinite sequence $\Gamma_{i1}, \Gamma_{i2}, ..., \Gamma_{im}, ...$ (a path on the graph). If the set of paths on the graph does not reduce to a finite number of closed contours or a periodic repetition, then a continuum of different paths exists. Each unstable trajectory corresponds to its own path $\{\Gamma_i\}$ and a corresponding sequence of intervals $\{\Delta\tau_i\} = \Delta\tau_{i1}, \Delta\tau_{i2}, ..., \Delta\tau_{im}, ...$ Moreover, for each path $\{\Gamma_i\}$ there exists an unstable trajectory which engenders the path. If the continuous set of all such unstable (saddle) motions is attractive, then it forms a chaotic (strange or quasihyperbolic) attractor within which stochastic self-excited oscillations occur.

A strict substantiation of what has been said above can be obtained by analyzing the multidimensional Poincaré mapping, using the general methods, developed in [6, 7], for analyzing such mappings. In the present paper, we shall confine ourselves to constructing and analyzing one-dimensional mappings of the type of (9), whose stochasticity serves as an approximate criterion for chaos in the system under study and whose properties determine the properties of the chaotic oscillations.

Let us consider a ray $L_0(\gamma)$ emanating from the point $\mathbf{u}_0$ in the direction of the vector $\mathbf{g}$ and lying in the plane $\delta k = \mathbf{b}^T\mathbf{u} = \beta$ ($\mathbf{b}^T\mathbf{u}_0 = \beta$, $\mathbf{b}^T\mathbf{g} = 0$):

$$L_0(\gamma) = \{n, c_i\mathbf{u}: n = n_0 = \text{const}; c_i = \beta_i\gamma; \mathbf{u} = \mathbf{u}_0 + \gamma\mathbf{g}\};$$

$$\mathbf{u}_0 = \beta(P^{-1}\mathbf{d})/(\mathbf{b}^TP^{-1}\mathbf{d}); \;\; \mathbf{g} = \mathbf{d} - (\mathbf{b}^T\mathbf{d})(P^{-1}\mathbf{d})/(\mathbf{b}^TP^{-1}\mathbf{d}), \tag{10}$$

where γ is the coordinate along the ray. It corresponds to a neutron burst, since after the impulse the trajectory lies on the surface $\delta k = \beta$ at a point whose projection on L_0 is proportional to the impulse.

The trajectories of system (1)–(3), which start on the ray $L_0(\gamma)$ and enter the region $\delta k < \beta$, will once again after a time $\tau(\gamma)$ end up on the surface $\delta k = \beta$ at the points $T_1(L_0(\gamma))$. Then they emerge into the region $\delta k > \beta$ and return onto the surface $\delta k = \beta$ at the points $T_2[T_1(L_0(\gamma))] = T(L_0(\gamma))$. The ray L_0 thereby transforms first into the curves $T_1(L_0(\gamma))$ and then into the curves $T(L_0(\gamma))$, lying, just like the ray L_0, in the plane $\delta k = \beta$. We obtain a mapping of the form (9) by projecting $T(L_0)$ onto L_0.

If the condition (6) is satisfied and $\omega_0(t)$ is nonmonotonic, then the functions $\tau(\gamma)$ and $\gamma^*(\gamma)$ are discontinuous, and the mappings $T_1(L_0)$ and $T(L_0)$ consist of several curves. The characteristic forms of the function $\tau(\gamma)$, and the images $T_1(L_0)$ and $T(L_0)$ in projection on the $\dot{\delta k}$, $\ddot{\delta k}$ plane, and the mapping $\gamma^*(\gamma)$ are shown in Figs. 3 and 4*a*. The mapping $\gamma^*(\gamma)$ will have several intervals of continuity $\Delta\gamma_i = (\gamma_{i-1}, \gamma_i)$ according to the number of maxima of $\omega_0(t)$. Reaching the value $\Delta\gamma_i$ signifies that the

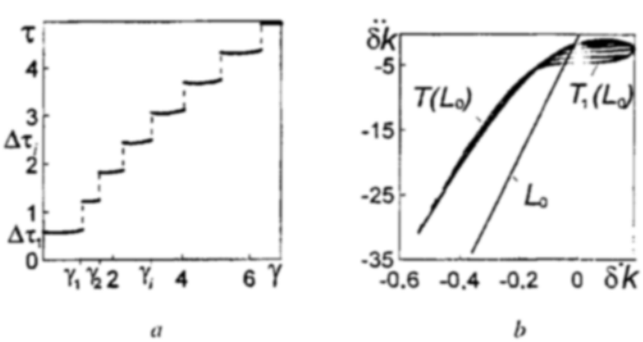

Fig. 3. $\tau(\gamma)$ on the ray L_0 (*a*) and the images $T_1(L_0)$ and $T(L_0)$ of the ray L_0 (*b*).

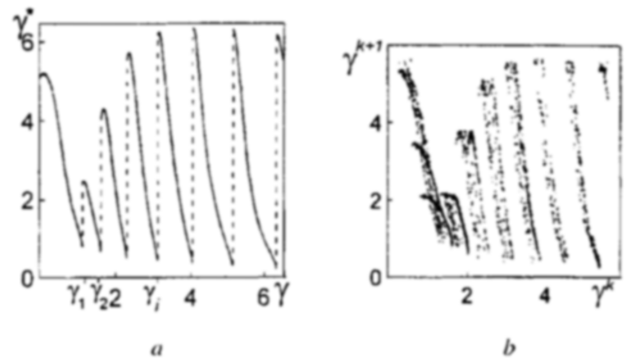

Fig. 4. One-dimensional mapping of the ray L_0 into L_0 (*a*) and the pairs (γ^{k+1}, γ^k) of successive points of intersection of the surface $\delta k = \beta$ by the chaotic trajectory in the projection on the ray L_0 (*b*).

next neutron impulse will belong to the *i*th series Γ_i. The graph described above is constructed according to $\gamma^*(\gamma)$. The vertices of the graph are the intervals $\Delta\gamma_i$. The edges $\overline{\gamma_{ij}}$ connect $\Delta\gamma_i$ with $\Delta\gamma_j$, if the mapping $\Gamma_i = \gamma^*(\Delta\gamma_i)$ intersects $\Delta\gamma_j$.

A change in the coefficient of intensification of feedback θ changes the scale in the mapping $\gamma^*(\gamma)$ along the ordinate. For sufficiently large θ, there arises a situation where $\gamma^*(\gamma)$ transforms into itself an interval $\Delta\gamma = [\gamma_0, \gamma_N]$, it possesses on this interval several intervals of continuity $\Delta\gamma_i$, and is a stretching mapping at each point $\Delta\gamma$, with the exception, possibly, of small segments. Such mappings are stochastic, and this also makes the self-excited oscillations in system (1)–(3) stochastic.

The main qualitative features of the function $\tau(\gamma)$ and the mapping $T_1(L_0)$ are preserved if the feedback along *n* is broken in Eq. (3), setting $n = -1$. In this case, for trajectories starting at L_0, we have $\mathbf{u}(t) = P^{-1}\mathbf{d} + \exp(Pt)[\mathbf{u}_0 - P^{-1}\mathbf{d} + \gamma\mathbf{g}]$, and we obtain $\gamma(\tau)$ from the condition $\mathbf{b}^T\mathbf{u} = \beta$: $\gamma(\tau) = [\beta - \mathbf{b}^T P^{-1}\mathbf{d} - \mathbf{b}^T\exp(P\tau)(\mathbf{u}_0 - P^{-1}\mathbf{d})]/[\mathbf{b}^T\exp(P\tau)\mathbf{g}]$.

Using relations (4), (7), and (10) for $\omega(t)$, $h(t)$, $\overline{\omega}_0(t)$, $\mathbf{u}_0$, and $\mathbf{g}$, we obtain the function $\gamma(\tau)$ and the projection $T_1(L_0)$ onto the $\dot{\delta k}$, $\ddot{\delta k}$ plane in the form

$$\gamma(\tau) = [\beta/h(\infty)+1]h(\tau)/\overline{\omega}_0(\tau);\ h(\infty) = -\mathbf{b}^T P^{-1}\mathbf{d}; \tag{11}$$

$$\dot{\delta k}(\tau) = \mathbf{b}^T\dot{\mathbf{u}}(\tau) = -\omega(\tau)[\beta/h(\infty)+1] + \gamma(\tau)\dot{\overline{\omega}}_0(\tau); \tag{12}$$

$$\ddot{\delta k}(\tau) = \mathbf{b}^T\ddot{\mathbf{u}}(\tau) = -\dot{\omega}(\tau)[\beta/h(\infty)+1] + \gamma(\tau)\ddot{\overline{\omega}}_0(\tau). \tag{13}$$

We obtain the function $\tau(\gamma)$ as the inverse to function (11). For a nonmonotonic function $\overline{\omega}_0(\tau)$, $\gamma(\tau)$ will be nonmonotonic and $\tau(\gamma)$ will be discontinuous. The minimum values of τ corresponding to γ should be taken for $\tau(\gamma)$.

We obtain a model analytic mapping $\gamma^*(\gamma)$ from the condition (8), using derivatives (12) and (13). In the simplest case, we obtain

$$\gamma^*(\gamma) = \bar{\theta}\dot{\delta k}[\tau(\gamma)] = \bar{\theta}\{-\omega[\tau(\gamma)](\beta/h(\infty)+1)+\gamma\dot{\bar{\omega}}_0[\tau(\gamma)]\}; \quad \bar{\theta} = \text{const}. \tag{14}$$

The expressions for $\gamma^*(\gamma)$ employ only the explicit form of the functions $\omega(\tau)$, $h(\tau)$, $\bar{\omega}_0(\tau)$, $\dot{\bar{\omega}}_0(\tau)$. When the condition (7) is satisfied, $\bar{\omega}_0(t)$ is nonmonotonic, and $\bar{\theta}$ is sufficiently large, the mappings (11) and (14) are stochastic and explain qualitatively correctly the appearance of chaos in the reactor models in system (1)–(3).

We shall now determine the structure of Eqs. (3) that is required for chaos. A substitution of variables puts the system (3) into the form

$$\dot{x} = s_0(x-n); \quad \dot{\mathbf{y}} = B\mathbf{y} + \bar{\mathbf{d}}x; \quad \delta k = ax + \theta\bar{\mathbf{b}}_0^T\mathbf{y}, \tag{15}$$

where $x(t) \in R^1$; $\mathbf{y}(t)$, $\bar{\mathbf{b}}_0$, $\bar{\mathbf{d}} \in R^{m-1}$; a, θ = const; $s_0 < 0$ is the leading (closest to the imaginary axis) real eigenvalue of the matrix P. In the case (15), the function $\omega_0(t)$ of the form (6) is the response $\theta\bar{\mathbf{b}}_0^T\mathbf{y}$ to an impulsive change $n(t) = \delta(t)$ with $x(0) = \mathbf{y}(0) = 0$. The function $\omega_0(t)$ is nonmonotonic and negative if the transfer function $h_0(t) = \theta\bar{\mathbf{b}}_0^T[\exp(Bt) - I]B^{-1}\bar{\mathbf{d}}$ from x to $\theta\bar{\mathbf{b}}_0^T\mathbf{y}$ has the same properties. For these properties and sufficiently small $|a|$, an increase of θ inevitably leads to the above-described mechanism of chaos in system (1), (2), and (15).

If the spectrum of the matrix B consists of several eigenvalues $\alpha_i \pm i\omega_i$, $\alpha_i < 0$, $\omega_i > 0$, then system (15) becomes

$$\dot{x} = s_0(x-n); \quad \delta k = ax + \theta\sum_{i=1}^{(m-1)/2}(b_i y_i + c_i z_i); \tag{16}$$

$$\dot{y}_i = \alpha_i y_i - \omega_i z_i - \alpha_i x; \quad \dot{z}_i = \omega_i y_i + \alpha_i z_i - \omega_i x; \quad i = 1, \ldots, (m-1)/2. \tag{17}$$

The condition $h_0(t) < 0$ and $\omega_0(t) < 0$ for system (16) and (17) holds, for example, if $\theta > 0$; $b_i < 0$, $c_i = 0$, $i = 1, \ldots, (m-1)/2$. The functions $h_0(t)$ and $\omega_0(t)$ are nonmonotonic for sufficiently small $0 < -\alpha_i/\omega_i < 0.3$.

We note that the curves in Figs. 1–4 correspond to system (1) and (2) in the case of feedback (16) and (17) with one oscillatory link (17) ($m = 3$) and the parameters $s_0 = -0.25$; $\alpha_1 = -1$; $\omega_1 = 10$; $a = c_1 = 0$; $\theta b_1 = -0.195$; $l = 10^{-4}$ sec; $\lambda_1 = 0.012$ sec^{-1}; $\lambda_2 = 0.03$ sec^{-1}; $\lambda_3 = 0.111$ sec^{-1}; $\lambda_4 = 0.301$ sec^{-1}; $\lambda_5 = 1.14$ sec^{-1}; $\lambda_6 = 3.01$ sec^{-1}; $\beta_1/\beta = 0.033$; $\beta_2/\beta = 0.219$; $\beta_3/\beta = 0.196$; $\beta_4/\beta = 0.395$; $\beta_5/\beta = 0.115$; $\beta_6/\beta = 0.042$; $\beta = 0.0065$.

The functions $\tau(\gamma)$ and $\gamma^*(\gamma)$ and the images $T_1(L_0)$ and $T(L_0)$ for the ray (10) ($n_0 = 0$) were constructed numerically using the trajectories of the system. The chaotic trajectory and the successive points (n^k, c_i^k, x^k, y_1^k, z_1^k) of intersection of the trajectory and the surface $\delta k = \beta$, $k = 1, \ldots, 6000$ from above (after the impulse) were obtained numerically. These points are projected onto the ray L_0 ($\gamma^k = 4x^k + 10z_1^k + 0.192$), and the projections γ^k obtained are plotted in the (γ^{k+1}, γ^k) plane in Fig. 4*b*. The points cluster near the branches of the mapping $\gamma^*(\gamma)$ in Fig. 4*a*; this indicates that the one-dimensional approximation for chaotic motions is admissable.

We note that in [8] Eqs. (16) and (17) with $m = 3$ were used to describe feedbacks in models of a boiling water reactor. For specific parameters of the reactor, it is shown in [8] that an increase of θ results in chaotic oscillations. However, their mechanism is different from the one considered in the present paper. It is a well-known mechanism associated with the presence of a homoclinic loop of a saddle-focus equilibrium state $M_0 = 0$ [6, 7]. It occurs in the absence of maxima in the function $\omega_0(t)$ and when the value $-\alpha_1/\omega_1$ is sufficiently large ($-\alpha_1/\omega_1 > 0.4$).

The equations for the feedbacks in reactor models with a gaseous core can also be put into the same form (16), (17) with $m = 3$ [9]. As shown in [9], the acoustic self-excited oscillations arising in such reactors can be stochastic. Depending on the paramters of the reactor, the chaos mechanism found in the present work and the chaotic oscillations similar to those considered in [8] can arise.

For an appropriate choice of feedbacks on reactivity, the type of chaotic oscillations examined here can be obtained in pulsed reactors. A different mechanism of chaos in such reactors has also been found and studied in [10]. In accordance with this mechanism, generation of impulses occurring in equal time intervals and stochasticity of their energy occur not only as a result of the action of internal feedbacks on reactivity (as in the present paper), but primarily as a result of the external modulation of reactivity with a prescribed period.

In conclusion, we note that the results obtained in this work point the way to determine other types of reactors where stochastic self-excited oscillations can arise. For small amplitudes, they can serve as a steady operating regime of a reactor. The results obtained make it possible to determine the region of stochasticity and to choose reactor parameters so as to obtain a stochastic regime, if it is desirable, and conversely to avoid such a regime, if the characteristics of chaotic oscillations are unsatisfactory from the standpoint of reactor operation.

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