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Evolving structural design solutions using an implicit redundant Genetic Algorithm

A.M. Raich and J. Ghaboussi

Abstract Performing synthesis during conceptual design provides substantial cost savings by selecting an efficient design topology and geometry, in addition to selecting the structural member properties. A new evolutionary-based representation, which combines redundancy and implicit fitness constraints, is introduced to represent and search for design solutions in an unstructured, multi-objective structural frame problem. The implicit redundant representation genetic algorithm, in tandem with the unstructured problem domain definition, allows the evaluation of diverse frame topologies and geometries. The IRR GA allows the representation of a variable number of location independent parameters, which overcomes the fixed parameter limitations of standard GAs. The novel frame designs evolved by the IRR GA synthesis design method compare favourably with traditional frame design solutions calculated by trial and error.

Key words genetic algorithm, redundancy, variable parameters, topology optimization, geometry optimization

1 Introduction

The challenge of efficient structural design is clear when viewed through spectacles focused on optimization: Provide the least cost design over the life-cycle of the structure. Optimization lowers the total cost of a structure by reducing the volume of material and the fabrication and construction time. The exploration for cost-efficient designs, however, is prematurely limited if the design topology and geometry are not allowed to change during optimization. The cost-savings resulting from shape optimization of the structural members is limited to the final design stage. More substantial cost savings can result from design changes made during the preceding conceptual design stage when synthesis of design topology and geometry occurs.

Supporting the synthesis of design alternatives during conceptual design requires evolving the topology and geometry of frame structures, in addition to optimizing the member properties. To achieve this goal, the design problem is modelled as an unstructured problem. Design alternatives are represented using location independent, variable parameters provided by the implicit redundant representation genetic algorithm (IRR GA). The IRR GA also provides the method for searching the highly complex and deceptive space of design alternatives.

Optimizing the topology and geometry of frame structures presents a new challenge to researchers. The topology of the structure defines the number of joints, the joint support locations, and the number of members connected to each joint. The structural geometry defines the location of joints within the problem domain and the length of the members. Optimizing the geometry of the structure requires designating the x, y, or z coordinates of the joints as design variables. Topology optimization requires the ability to add and remove members and joints from the structure, either heuristically or implicitly during the search for design solutions. Adding the representational flexibility to model topology and geometry alterations makes the frame problem nearly unapproachable using any existing design or optimization method, including the ground structure approach. Frame solutions with diverse topologies and geometries provide similar levels of satisfaction of multiple design objectives and may have similar fitness values.

2 Overview of genetic algorithms in structural optimization

This paper presents an evolutionary-based, random directed search methodology called IRR GA that extends the GA first proposed by Holland (1975) and the simple

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GA (SGA) further developed by Goldberg (1989). SGAs have been applied in a wide range of disciplines on an even more diverse set of optimization, classification, control, design, and adaptive evolution problems. SGAs have four properties that make them fundamentally different from traditional optimization methods (Goldberg 1989).

1. GAs work with a coding of the parameters, not the parameters themselves.
2. GAs use a population of solutions, not a single solution.
3. GAs use payoff information, not additional information or derivatives.
4. GAs use probabilistic transition rules, not deterministic transition rules.

Researchers have applied genetic algorithms (GA) to structural truss optimization problems, including the optimization of trusses with fixed topology and fixed geometry (Yang and Soh 1997); the optimization of trusses with fixed topology and variable geometry (Wu and Chow 1995); and the optimization of trusses with variable topology and geometry (Rajan 1995; Rajeev and Krishnamoorthy 1997). The GA truss topology and geometry optimization methods cited, including those based on the ground structure approach (Hajela and Lee 1995; Rajan 1995), are not directly transferable to frame optimization problems. Several researchers have applied GAs for shape optimization of frame structures with fixed geometry and topology (Camp *et al.* 1998) and to provide limited geometry optimization of frame structures (Jenkins 1997). The synthesis method developed using the IRR GA and the unstructured problem domain formulation discussed by this paper provides concurrent topology and geometry optimization of frame structures without requiring the definition of a ground structure or heuristic rules for adding or deleting members.

3 Implicit redundant representation Genetic Algorithm

In order to provide an evolutionary-based method capable of synthesizing design alternatives, a more flexible GA representation is required. The IRR provides a mechanism that allows essential and redundant sections of a string to interact dynamically by using a string length that is longer than the length required to encode only the parameter values (Raich and Ghaboussi 1997). Each population individual represents a complete solution and all individuals have the same string length.

3.1 Introduction of the IRR genotype representation

The specific location of each encoded parameter value or gene instance is not designated explicitly by the IRR. Instead, each gene instance is allowed to drift within the length of the string as shown in Fig. 1a. Each gene instance in an IRR string consists of two parts: a preselected Gene Locator (GL) pattern identifying the location of the gene instance in the string and a specified number of useful bits of the gene instance that encode the parameter values. To decode the parameter values from the IRR string, the string is parsed until a GL pattern is found indicating a gene instance.

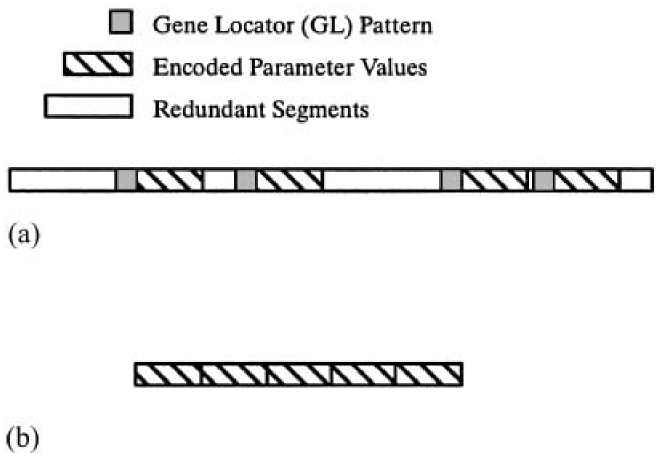


Fig. 1 Comparison of generic IRR GA and SGA genotype representation. (a) IRR GA, (b) SGA

The parameter values are encoded using binary or real numbers similar to SGAs. The number of bits, n, encoding each parameter within a gene instance sets explicitly the range of the parameter values, such as a two-bit binary number that represents the integer numbers $\{0, 1, 2, 3\}$. Typically, a mapping adjusts the ranges for continuous parameters or discrete parameters based on the encoded number of bits. The genotype/phenotype IRR GA representation is distinct from the SGA representation. In SGAs, each parameter value is represented as a n-bit binary number, but the encoded binary values are concatenated together without any redundant bits as shown in Fig. 1b. The fixed string length in SGAs is determined by adding the lengths of the individual encoded values.

The portions of the IRR GA string that are not part of a gene instance contain redundant material. Incorporating redundant, or noncoding segments, has been researched previously (Levenick 1991; Wu and Lindsay 1996; Raich and Ghaboussi 1997). Each redundant segment consists of a variable number of bits that separate the gene instances in the string. The IRR GA provides several benefits to the evolutionary process: redundant segments protect existing gene instances from the disruption of crossover and mutation and in future generations new gene instances may be designated within previously redundant segments by the actions of crossover or mutation.

In addition to providing redundancy, the IRR does not require the designer to specify the number of parameter values to be represented. Problems in which the optimal number of parameters is not known are common in design. The SGA works with a fixed number of parameters, which limits its effectiveness on design problems. The IRR GA allows the number of parameters encoded to change dynamically from generation to generation and between individuals in the population. No external constraints are required to process over or underspecified strings, since the IRR GA strings are the same length. The benefit of providing dynamic redundancy allows the IRR GA to search for solutions effectively in unstructured problems by encoding a variable number of location independent design parameters and allowing self-organization of the linkage of the encoded design parameters. Varying the number of parameters encoded allows the IRR GA to alter the structural topology and geometry. A severe reduction in the number of elements represented in the IRR GA genotype often occurs during evolution. This type of dynamic behaviour cannot be captured by SGAs.

3.2 IRR GA computation algorithm

The iterative computational algorithm for the IRR GA is shown in Fig. 2 and is the same algorithm used for the SGA (Goldberg 1989). The designer selects the population size based on diversity and computational issues and randomly initializes the bit values of the population individuals. The population size, N, controls the number of fitness evaluations (i.e. structural analysis) performed during each generation. The remaining components of the genetic algorithm (GA) selected by the designer are: fitness evaluation function, selection scheme, crossover method and rate, and mutation rate. The solution represented by each individual is decoded from the IRR GA genotype and evaluated using the defined fitness function. An individual with a high fitness will have an increased chance of being propagated into the next generation; those individuals with low fitness may not be selected at all. The genetic operations of selection, crossover, and mutation are performed to determine the individuals in the new population. The IRR GA uses the same selection, crossover, and mutation operators as the SGA. Typically, the entire population of N individuals is replaced by the children generated by crossover and mutation during each generation. The iterative process of fitness evaluation and genetic manipulation is continued until a stopping criterion related to the maximum number of generations or a desired population convergence is satisfied.

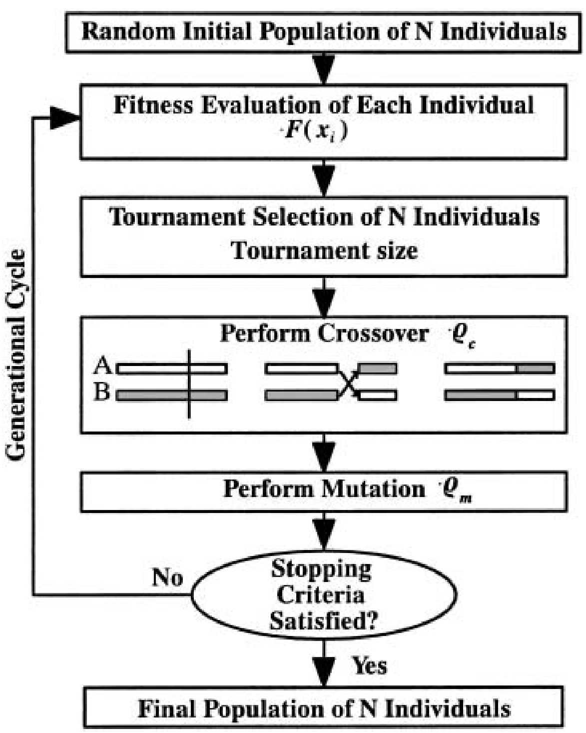


Fig. 2 Statement of computational algorithm for the IRR GA

4 Unstructured problem formulation

Design synthesis has two principles driving it: providing partial optimality of design (in some sense the best design) and ensuring feasibility of design. Synthesis alternatives are found in the search space bounded by the infinite and ill-defined unconstrained space of all possible design alternatives and the small and well-defined space of constrained mathematical programming design alternatives. The synthesis of structural design alternatives is supported in this research by defining an unstructured problem domain. The unstructured design problem formulation removes the explicit constraints put on the number of design variables modeled and, therefore, design solutions can be generated and compared that have diverse topologies and geometries. The IRR GA is highly-suited to model unstructured problem domains because of the flexibility of parameter representation and global search properties provided. Defining the required design grammar for the design problem is simplified since the grammar is explicit in the genotype/phenotype relationship provided by the IRR GA representation itself.

The level of unstructuredness provided by the statement of the problem domain can be controlled by constraining the design variables within the problem domain. A tradeoff occurs in the process of determining the level of unstructuredness that is beneficial to the overall process of synthesis. Removing constraints allows for a more diverse set of synthesis alternatives to be explored by expanding the search space. Increasing the space of design alternatives, however, has a high computational cost attached. Limiting the size of the search space by constraining specific design variables lowers the computational cost, but at the cost of possibly excluding beneficial design alternatives.

The problem objective function, constraints, and parameter value ranges define a fitness landscape within the search space. A structured problem with a fixed number of variables provides a single, static fitness landscape within the search space, even if the landscape is multimodal and deceptive. In unstructured problem domains, there is no assurance that the fitness landscape remains static. Instead, each distinct topology and geometry considered will have its own fitness landscape defined in a distinct dimensional search space. If the topology or geometry changes, then a new fitness landscape will be defined. The search for synthesis design solutions in an unstructured problem domain is performed over a nonstationary, multimodal, meshed fitness landscape. Unstructured problems, therefore, can be categorized as highly deceptive problems (Goldberg 1989).

5 Design problem statement

The design problem selected is the synthesis of a plane frame structure with a maximum total structure width of 60'–0" and a maximum structure height of 36'–0" (three floor levels). The corresponding structured and unstructured frame problem domains are shown in Figs. 3a and b,

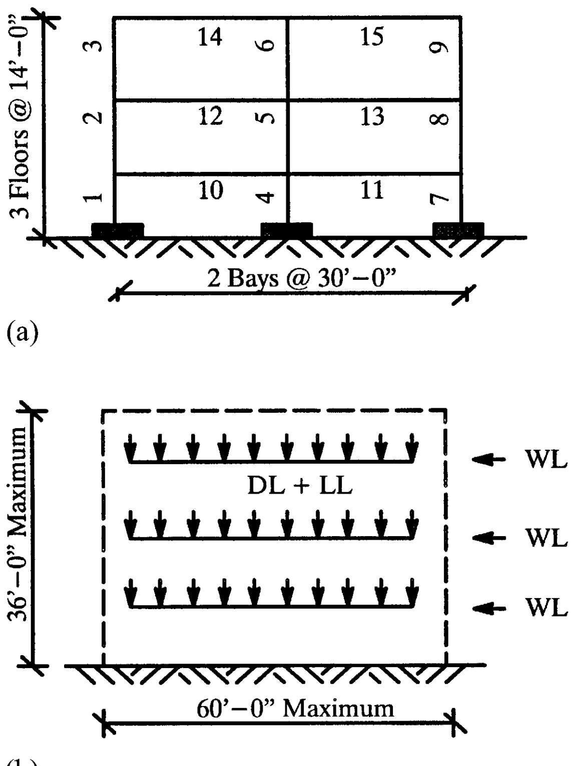


Fig. 3 Model of structured and unstructured problem formulation for frame design. (a) Structured problem domain, (b) unstructured problem domain

respectively, to provide a comparison of the two approaches. The structured frame problem domain defines the location and connectivity of all 15 members in the structure using a fixed topology and geometry. The baywidths and floor heights for the two-bay, three-storey structure are provided and the loading configurations are fixed. The only remaining design variables are the individual member properties. In structured problems, the synthesis of design alternatives is limited to shape optimization since the topology and geometry remains fixed. The unstructured frame problem domain, in comparison, is defined by: dimensional bounds placed on the maximum structure width and height and the statement of the location of plane(s) of possible applied loading and possible support placement. The fixed design parameters are the magnitudes of the dead load, live load and wind loading and the designation of pinned support nodes. The loading applied to the frame structure is a function of the number of storeys and bays defined and varies for each proposed frame synthesis alternative. All other required design information, including the number and location of structural nodes and members, member properties, support information, member connectivity, number of storeys, and number and size of bays, is specified by the design variables.

5.1 Definition of the IRR GA frame synthesis gene instance

Assembling a frame design solution within the unstructured problem domain shown in Fig. 3b requires knowledge about the number of members, the member areas, and the member locations in the structure as defined by the nodal coordinates. All the required design information to model a frame member is encoded in a single IRR gene instance identified by the GL pattern [1 1 1] in the order shown in Fig. 4a: the x-coordinate of node 1 ($X1$); the y-coordinate of node 1 ($Y1$); the x-coordinate of node 2 ($X2$); the y-coordinate of node 2 ($Y2$); the depth of the nonhorizontal member (Depth 1); the depth of any horizontal member connected to the right of node 1 (Depth 2); and the depth of any horizontal member connected to the right of node 2 (Depth 3). The design information defines the nonhorizontal member coordinates, nodal incidences, and member depths as shown in Fig. 4b.

The design variable value ranges are set by the number of binary bits that encode each variable. The nodal x-coordinates, $X1$ and $X2$, are encoded as six-bit binary numbers that are mapped by the following function: $(X1 - 31.0) \times 12.0$, which provides a x-coordinate value range of $(-372.0, 384.0)$. The y-coordinates, $Y1$ and $Y2$, are encoded as two-bit binary numbers. Each of the four encoded binary values corresponds to a floor level of 0, 1, 2, or 3. The three member depths are three-bit binary numbers that encode eight discrete member depths $\{5, 10, 15, 20, 25, 30, 40, 50\}$. The member area and section modulus are calculated using the decoded mem-

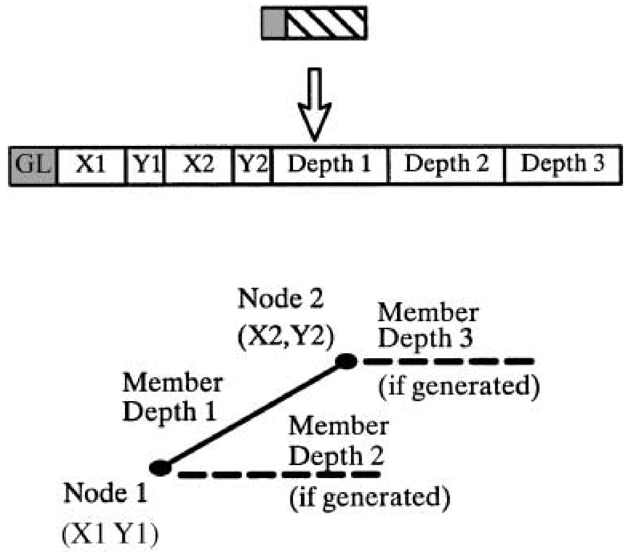


Fig. 4 IRR GA gene instance and the corresponding phenotype definition of a single frame member for the unstructured frame problem

ber depth. The structural frame members are steel tube sections with fixed width and thickness and a variable depth.

The frame member information encoded in each gene instance does not depend on the location of the gene instance in the string. In addition, there are no explicit constraints applied to the fitness function that influence the number of members encoded in the string length. The total number of frame members (gene instances) encoded in each IRR GA genotype is implicitly constrained by the fitness and penalty functions and will vary among the individuals in the current population. Allowing the number of gene instances to vary among the individuals in the population provides the mechanism for evaluating structures with varying topology and geometry.

5.2
Frame assembly from individual members

Assembling a complete frame structure consists of defining: the nonhorizontal member locations using the nodal coordinates decoded from the IRR genotype and generating the horizontal members based on the locations of the nodal coordinates defined by the nonhorizontal members. The two horizontal member depths decoded from the gene instance for each nonhorizontal member are used when a horizontal member is generated. The horizontal members are generated between each pair of adjacent nodes defined on the same floor level after the nonhorizontal member information has been decoded. A horizontal member could be connected to either the starting or ending node of the decoded nonhorizontal member. The depth of horizontal member is provided by the value of the horizontal depth (Depth 2 or Depth 3) decoded for the designated starting node of the horizontal member as shown in Fig. 4.

Three repair strategies were applied to the assembled frame structures as required: assigning a minimal fitness to frames with fewer than two supports to prevent analyzing unstable structures; replacing nodes closer than 5'–0" with a single node to prevent the automatic generation of very short members; and removing single nodes in the structure that carry no loading.

5.3
Frame synthesis fitness and penalty functions

Typically, a frame design problem has a single objective: provide minimum weight while satisfying flexural strength and deflection requirements. Optimizing the single objective in an unstructured frame problem domain, however, results in the evolution of minimal structures (two member frames) that carry no loading. Therefore, a second objective is required: maximize the total floor space provided by the frame. The nonpenalized GA fitness functions that optimize the volume (minimum weight), F_V and floor area, F_F, can be stated for the frame synthesis design problem,

$$F_V = \left[\frac{C_V - \sum_{i=1}^{m} \varrho A_i \ell_i}{C_V} \right]^{d_V} ,$$

$$F_F = \left[\frac{\sum_{j=1}^{m_h} h_j(x_j)}{L_H} \right]^{d_F} , \tag{1}$$

where m is the total number of members; m_h is the number of horizontal members; C_V is a selected scalar value larger than the maximum expected volume; L_H is the maximum total floor space provided by the dimensional bounds; and a_V and a_F are selected exponential power terms.

A stress penalty function, P_S, reduces the fitness of frame design solutions that violate the code-specified maximum stress criteria:

$$P_S = \left[\frac{C_S - \prod_{j=1}^{m} \mathrm{Int}(M_j, M_{j\mathrm{all}}, P_j, P_{j\mathrm{all}})}{C_S} \right]^{a_S} , \tag{2}$$

where Int() is the interaction ratio defined by the LRFD code, M_j is the design moment in member j; $M_{j\mathrm{all}}$ is the allowable moment in member j; P_j is the design axial force in member j; $P_{j\mathrm{all}}$ is the allowable axial force in member j; a_S is a selected exponential power term; and C_S is a selected scalar value that is larger than the maximum stress interaction penalty.

The frame design solutions are required to meet serviceability limits. The horizontal deflection of the structure must satisfy the NEHRP allowable inter-storey drift limits and the vertical deflection of the structural members is limited to a deflection of less than $\ell/360$ along the member. The deflection penalty functions, P_{HD} and P_{VD}, reduce the fitness of design solutions that have excessive horizontal and vertical deflections (with a subscript of H for horizontal and V for vertical deflection):

$$P_D = \left[\frac{C_D - \prod_{\ell=1}^{n} \left(1.0 + \frac{\Delta_\ell}{\Delta_{\max}}\right)}{C_D} \right]^{a_D} , \tag{3}$$

where n is the number of nodes considered for horizontal or vertical deflection; Δ_ℓ is the horizontal or vertical deflection of node ℓ exceeding the set limit; $\Delta_{\max}$ is the maximum limit on horizontal or vertical deflection for the nodes; a_D is a selected exponential power term, and C_D is a selected scalar value that is larger than the maximum expected horizontal or vertical deflection penalty.

Aesthetics are introduced into the frame synthesis search process by promoting the symmetric placement of structural members and nodes, while still allowing the placement of nonsymmetrical members and nodes. Penalties for nonsymmetrical members and nodes in the structure are calculated using a two foot tolerance. Nodal and member symmetry penalty functions, P_{SN} and P_{SM}, can be stated that penalize the design solution:

$$P_{SN} = \left[\frac{1.0}{\sum_{k=1}^{n} 0.2 \times NumSym} \right]^{a_{SN}} ,$$

$$P_{SM} = \left[\frac{1.0}{\sum_{k=1}^{m} Sym(k,j)} \right]^{a_{SM}} , \tag{4}$$

where $NumSym$ is the number of nonsymmetrical nodes; a_{SN} and a_{SM} are selected exponential power terms; and $Sym(k,j)$ is 0 if members k and j are symmetric or 1 if members k and j are nonsymmetric.

Applying the LRFD load combinations to potentially nonsymmetric frame structures requires a total of four load cases: two load cases for Dead Load + Live Load on alternating spans and two load cases for Dead Load + Wind Load from each direction. Three of the penalty functions, stress (P_S), horizontal deflection (P_{HD}), and vertical deflection (P_{VD}), must be evaluated for each of the four specified load cases applied to the structure to determine the total penalty function. A separate structural analysis is required for each individual in the IRR GA population. The unstructured formulation of the plane frame design problem creates a difficulty in applying the gravity and wind loading to the structure. The loading cannot be applied to a fixed set of members or nodes, since the same members and nodes are not always present due to variable geometry and topology. Instead, gravity load is applied uniformly along the horizontal members defined at each floor level, which do not necessarily relate to equal spans. The wind load is applied to the exterior nodes defined at each floor level. If a specific floor level is not defined, the wind load is transferred to the adjacent floors.

A product composite penalty term, P_{TOT} that magnifies the differences existing between the individual penalty terms defined in (2) to (4) was defined:

$$P_{\mathrm{TOT}} = \sum_{k=1}^{\ell} P_S^k \times \sum_{k=1}^{h} P_{HD}^k \times \sum_{k=1}^{j} P_{VD}^k \times P_{SN} \times P_{SM} , \tag{5}$$

where ℓ is the number of loading cases analyzed; h is the number of load cases analyzed for horizontal deflection; and j is the number of load cases analyzed for vertical deflection. Table 1 presents a summary of the values of the scalar terms used for the fitness and penalty functions defined in (1) to (4).

The product composite fitness function is composed of the two fitness terms defined in (1) and the ten penalty terms defined in (5):

$$\max F(\mathbf{x}) = F_V \times F_F \times P_{\mathrm{TOT}} . \tag{6}$$

Table 1 Values of scalar variables for calculating the fitness and penalty functions

Scalar term	Scalar value
C_V	600.0
C_S	2000.0
C_{VD}	2000.0
C_{HD}	2000.0
a_V	1.0
a_F	1.0
a_{VD}	4.0
a_{HD}	4.0
a_{SN}	0.1
a_{SM}	0.1
L_H	2268.0

5.4 Selection of IRR GA string length

The provision of an appropriate level of redundancy within the string length is an important design consideration (Raich and Ghaboussi 1997). Initially, the level of redundancy is determined by the probability of an occurrence of the selected GL pattern within the string length. For an IRR GA string length of 600 bits and a total encoded gene instance length of 22 bits, an average of 16 members (gene instances) are randomly initialized in each individual (genotype). During the GA evolutionary trials performed, the number of gene instances encoded decreased. The initially overspecified string protected the population from premature convergence by reducing the average stress and deflection penalties during early generations, which lowered the severity of the penalties.

5.5 Discussion of the genetic control operators

The search space for the frame design problem includes multiple, equally optimal solutions. To prevent the population from converging to a single optimum, fitness sharing distributed the population among multiple solutions so that only a few individuals were maintained in the vicinity of each solution in the search space (Goldberg and Richardson 1987). The sharing function applied was the same as defined by Goldberg (1989) with a similarity measure, σ_S, of 0.05 to control the size of the niche. A Euclidean distance measure related the similarity between the satisfaction of the individual objective and penalty terms for all individuals in the population. The fitness of each individual was reduced based on the number of similar individuals in the current population as defined by the niche count, m_i. Tournament selection used the modified fitness values to select the next generation population. The individual with the highest fitness in the randomly chosen tournament group of size n was selected to be the winner. The size of the tournament subgroup controlled the level of selection pressure applied. The tournament competition continued until a new population of individuals were selected. To prevent the loss of the fittest individual in the current population due to low selection pressure or the disruption of crossover or mutation, an elitist strategy was used. The fittest individual in the current population was copied to the next generation bypassing any genetic manipulation.

A multiple crossover operator increased the number of string segments recombined and reduced the size of the string segments exchanged. A random, normal distribution selected the number of crossover sites using a mean of 10 crossovers and a standard deviation of two. Two individuals were randomly paired from the set of tournament selected individuals; the string was cut virtually at multiple, random locations; and the portions of the strings between the cuts were exchanged. The crossover rate varied randomly between 0.2 to 0.7. Mutations prevent the loss of diversity in the population by introducing new genetic information or reintroducing previously lost information (Goldberg 1989). Single bit mutation was applied probabilistically to each population individual by flipping the encoded bit values from zero to one, or vice versa, using a mutation rate of either 0.0025 or 0.0033.

6 Frame synthesis design results

Experimental trials were performed using the IRR GA to model the unstructured plane frame problem domain defined in Fig. 3b. Three frame synthesis design solutions evolved by the IRR GA using a population size of 200, a string length of 800 bits, and a tournament size of 10 after 1500 generations are presented in Fig. 5. The IRR GA trials were randomly initialized using different random seeds. The product composite fitness function defined by (6) was used. Each of the IRR GA trials converged to frame design solutions having three storeys and the maximum floor space allowed.

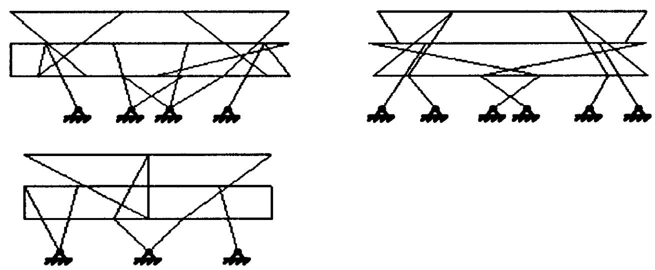


Fig. 5 IRR GA frame design solutions represented by the fittest population individual

The beneficial influence of the member symmetry penalty on the evolution of design features and complete frame design solutions is evident. The IRR GA converged the search process towards symmetrical designs. Incorporating symmetry into the design process by penalizing unsymmetrical solutions allowed the evaluation of single members before promoting the addition of symmetrical members to the structure. To support member symmetry, the IRR GA genotype must maintain two separate gene instances within the genotype. The self-organization of the location independent gene instances along the IRR GA genotype protects the pairs of gene instances from the disruption of crossover and mutation. If flexibility of encoding is not provided, representing structures having multiple pairs of symmetrical members is difficult.

The results of additional IRR GA frame synthesis design trials are presented using a less unstructured problem formulation, which provided a slightly smaller search domain. The nodal x-coordinates ($X1$ and $X2$) encoded in the genotype gene instances were encoded using three-bits, which provided a 10'–0" joint spacing along each floor, instead of the 1'–0" spacing used in the previous trials. Multiple, randomly initialized IRR GA trials were performed using the product composite fitness function defined by (6), a population size of 100 or 200, a string length of 600, and a tournament size of 5. In addition, IRR GA trials were performed using a sum composite fitness function that added the fitness and penalty terms in (6) instead of multiplying the terms. The same control parameters were used for the product and sum composite fitness function IRR GA trials. The reduced population size and string length for these trials resulted from constraining the size of the search space by reducing the number of possible x-coordinate locations assigned. The frame synthesis design solutions obtained after 500 generations for the IRR GA trials are shown in Fig. 6.

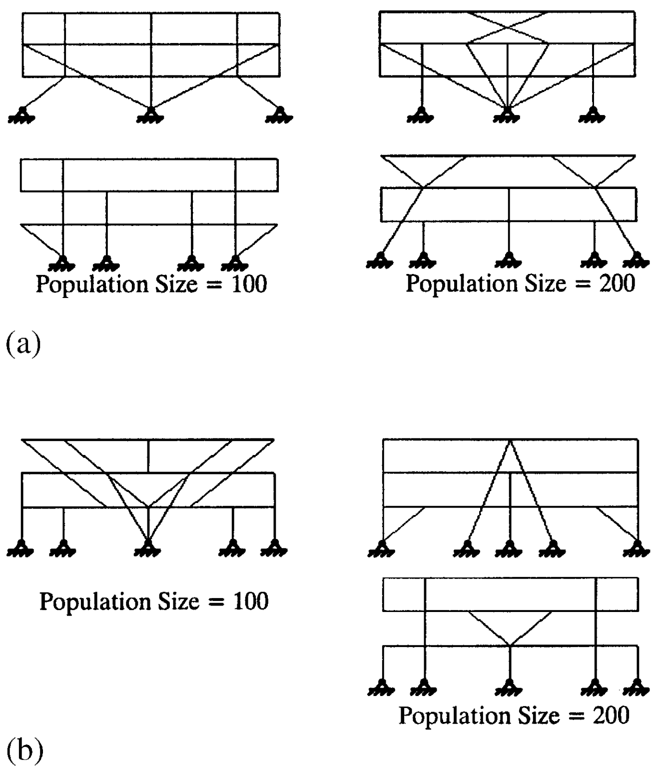


Fig. 6 IRR GA frame design solutions represented by the fittest population individual after 500 generations with a 10'–0" restriction on the x-coordinate spacing. (a) Product composite fitness function, (b) sum composite fitness function

The frame synthesis design solutions shown in Fig. 6 provide complete frame design solutions that incorporate inclined columns to aid in resisting wind loading, tension members carrying gravity loading, and stiff, triangular substructures. A striking feature of these designs was the specification of separate load carrying systems for the individual floors. For the design solution shown in the bottom, right-hand corner of Fig. 6a, the second and third floor loadings are carried to the foundation through an arch structural system. The first floor loading is carried on additional vertical one-storey columns. The evolved design solutions satisfied the symmetry penalties to a greater extent than the trials performed without restricting the x-coordinates of the nodes.

6.1 The evolution of frame synthesis design solutions

The evolution of frame synthesis design solutions can be investigated by examining the features of the fittest individual in the IRR GA population at specific generations. Figure 7 presents an overview of the evolutionary search process for two of the IRR GA frame synthesis design trials in Fig. 6.

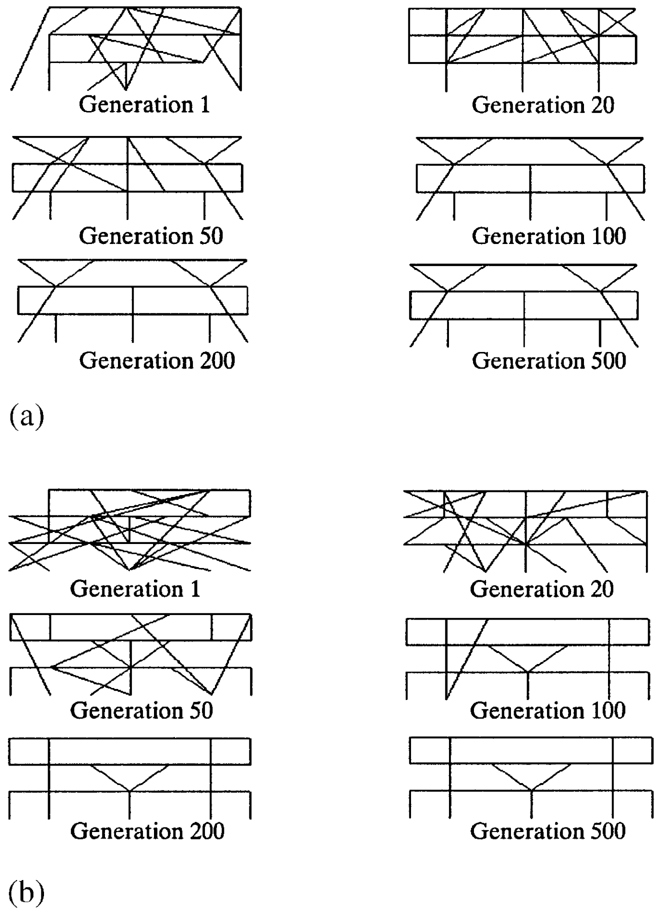


Fig. 7 Example of the evolution of the best IRR GA design solution at each generation with a 10'–0" restriction on the x-coordinate spacing. (a) Product composite fitness function, (b) sum composite fitness function

The IRR GA evolutionary process starts with a population of randomly initialized individuals. After one generation, the frame design solution represented by the IRR GA population individual is not random since one tournament selection has been performed. Although three storeys are defined by the design solution, the floor space provided by each storey does not maximize the domain boundaries. The frame design solutions also have more members than are required to prevent assigning extremely high stress and deflection penalties to a high percentage of the population. The best frame design solution at 20 generations included several design features similar to the features found in the final design solution. After 50 generations, the influence of the floor space objective and the nodal symmetry penalty begins to appear. The floor space provided is extended towards the domain boundaries and the nodal coordinates are placed in nearly symmetrical positions. The synthesis of the topology and geometry of the design solution continued during the first 200 generations. Shape optimization of the member depths was performed after 200 generations on the fixed structural topology and geometry evolved. The disruptive effect of crossover and mutation on the IRR GA genotype, however, makes the shape optimization process difficult. The flexibility provided to encode variable topologies and geometries during synthesis does not provide the best representation for performing size optimization. The best frame design solution at 500 generations is symmetric and optimizes the floor space and volume objectives. The design solution also satisfies the stress, deflection, and symmetry penalties for each of the four applied load cases.

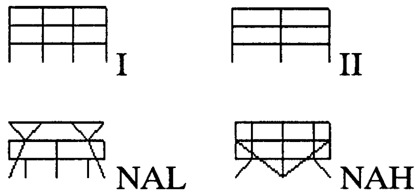
To support the exploration of design alternatives during synthesis, a diverse population is essential. The diversity of the population maintained during evolution can be investigated by comparing the maximum fitness reached by the population at each generation with the average fitness of the population. The average fitness of the population did not converge to the maximum fitness after synthesis was finished. Convergence of the population to the maximum fitness shows that the population individuals were becoming similar. Instead, a high level of population diversity was maintained throughout the entire evolutionary process of synthesis and optimization for the IRR GA trials.

6.2 Comparison of IRR GA and standard frame design solutions

The quality, or optimality, of the IRR GA frame design solutions cannot be determined directly based on a comparison with known optimal frame design solutions. The frame design problem has numerous optimal solutions that each satisfy the constraints and optimize the objectives equally well using different structural configurations of member sizes, topologies, and geometries. Three frame solutions were designed using a trial and error process for standard frame topologies and geometries. The standard frame design solutions have vertical columns and rectangular bays and are shown graphically in Table 2 as frame design solutions I, II, and III. Standard frame design I has three, 12'–0" storeys and three, 20'–0" bays. Standard frame design II has three, 12'–0" storeys and two, 30'-0" bays. Standard frame design III has three, 12'–0" storeys and an 10'–0" wide interior bay and the two, 25'–0" wide exterior bays.

Table 2 presents the structural properties of the three standard frame design solutions. The comparison categories are the total volume of the structure, the maximum horizontal deflection, and the maximum vertical deflection. Standard frame design solution I provided the lowest volume of the three design solutions. All three design solutions, I, II, and III, provided approximately the same level of structural deflection. In addition, the standard design solutions had relatively low average member stress ratios under both gravity and wind loading.

Table 2 Evolutionary frame solutions vs. trial and error frame solutions



Structure	Volume (ft^3)	Δ_{max} Horz. (in.)	Δ_{max} Vert. (in.)
Solutions generated by trial and error			
I	99.666	0.5241	0.25
II	102.666	0.5585	0.25
III	105.499	0.5704	0.25
Solutions generated by IRR GA			
NAL	104.506	0.7216	0.27
NAL (Modified)	99.811	0.4281	0.28
NAL (Ratio)	90.665	0.7772	0.55
NAH	108.422	0.558	1.44
NAH (Modified)	118.336	0.216	0.89

Two IRR GA frame synthesis design solutions were selected from the results presented in Fig. 6 for comparison with the standard frame design solutions. Both IRR GA trials restricted the nodal x-coordinates to 10'–0" spacings. The structural properties of the IRR GA trials (NAL and NAH) are shown in Table 2. The IRR GA frame design solutions provide competitive solutions when compared with the standard frame design solutions generated by trial and error. Both IRR GA trials had volumes that compared favorably with the standard frame design solution volumes. The average horizontal and non-horizontal member stress ratios for the IRR GA synthesis solutions were lower than those maintained by the standard design solutions.

The IRR GA synthesis design method evolved solutions having symmetric topologies and geometries, but not necessarily having symmetric member sizes. The two IRR GA trials presented were re-evaluated after providing symmetric member sizes and limiting excessive nodal deflections. The NAL (Modified) trial provided a very competitive design solution. The NAH (Modified) trial reduced the excessive deflections at the expense of increasing the structural volume. The excessive deflections in the two IRR GA frame solutions were located at the nodes of cantilever members.

In addition, one IRR GA trial was selected and changes were made to the evolved structural member sizes to increase the average stress ratios in the horizontal and nonhorizontal members, resulting in the best frame design solution of all the alternatives examined. The IRR GA trial NAL (Ratio) design solution had the lowest volume (90.665) of all the designs considered. The average stress ratios were increased to the same level as those provided by the standard frame design solutions. The low stress ratios maintained by the IRR GA design solution could be controlled in future IRR GA trials by penalizing the design solutions for member under-stress in addition to the current penalty for member over-stress.

7 Conclusions

A new evolutionary-based representation was introduced to represent and search for synthesis design solutions in unstructured problem domains. The implicit redundant representation genetic algorithm (IRR GA) uses redundancy to encode a variable number of location independent parameters and allows the representation to self-organize. The IRR GA represented and searched for design synthesis alternatives in a highly unstructured, plane frame problem domain. Two levels of unstructured problem formulations were examined to determine the effectiveness of the IRR GA representation on synthesizing frame design solutions. Constraining the allowable spacing of the x-coordinates along each floor, in addition to imposing member and nodal symmetry penalties, aided the synthesis process by reducing the size of the search space.

The IRR GA evolved novel design solutions using an unstructured problem formulation that minimized volume while maximizing the floor space subject to satisfying stress, deflection, and symmetry penalties. The IRR GA did not require the statement of heuristics to add or remove members or the definition of a ground structure topology and geometry for the design domain. The benefits of providing topology and geometry synthesis using an unstructured formulation in conjunction with the IRR GA were shown. The frame design solutions evolved by the IRR GA synthesis design method compared favourably with traditional frame design solutions calculated by trial and error.

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