Discussion

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THE EQUITY PREMIUM IN THE UNITED STATES has been puzzling and challenging both to theorists and empiricists in the profession of finance and economics. Estimating the equity premium from the time series of excess equity returns is as challenging as explaining it with economic models. The paper by Pástor and Stambaugh deals with the estimation rather than with the economic explanation of the equity premium. The most interesting aspect of the paper is its employment of economists' ideas alongside statisticians' procedures. The first idea is that the equity premium should be linked to volatility. The second is that a rise in price should reduce the equity premium. The third is that the shifts in the equity premium over regimes should not be very large. The paper constructs a prior distribution to express those thoughts and conducts Bayesian inference using MCMC methods and a model incorporating structure-breaks. The paper's approach is attractive because it incorporates economic theory and intuition into the prior distribution. It is this aspect that distinguishes economists from statisticians.

The estimate of the equity premium delivered in Pástor and Stambaugh's paper is sensible. To appreciate the equity premium estimated by Pástor and Stambaugh, in Figure 1, I plot the 30-year moving average of equity returns, short-term interest rates, and excess returns. The data, kindly provided by Pástor and Stambaugh, are the same as those used in their paper except the data before 1834 are also used. The equity premium estimated by Pástor and Stambaugh fluctuates within the range of four to six percent. This result does not seem to disagree with the long-run real return of seven percent that Siegel (1998, 1999) has argued for and the long-run average real interest rate of around two percent. It is also consistent with the long-run average excess return of 4.74 percent and the associated standard error of 1.11 percent. In addition, the estimated equity premium rises before World War II and declines thereafter. The rise before World War II coincides with a gradual increase in the equity return and a downward trend in the short-term interest rate. The decline after World War II coincides with a large increase in the equity return until the early 1960s and a steep climb of the interest rate during the period from 1960 to 1990. Finally, the estimated equity premium drops sharply during the 1990s. This result is consistent with the views of many economists including Siegel (1999) and Shiller (2000) who pay attention to the sky-high price-earning ratio near the end of the 1990s.

* Columbia University, Graduate School of Business. I thank Darrel Duffie, Ravi Jagannathan, and Michael Johannes for helpful comments on my discussion.

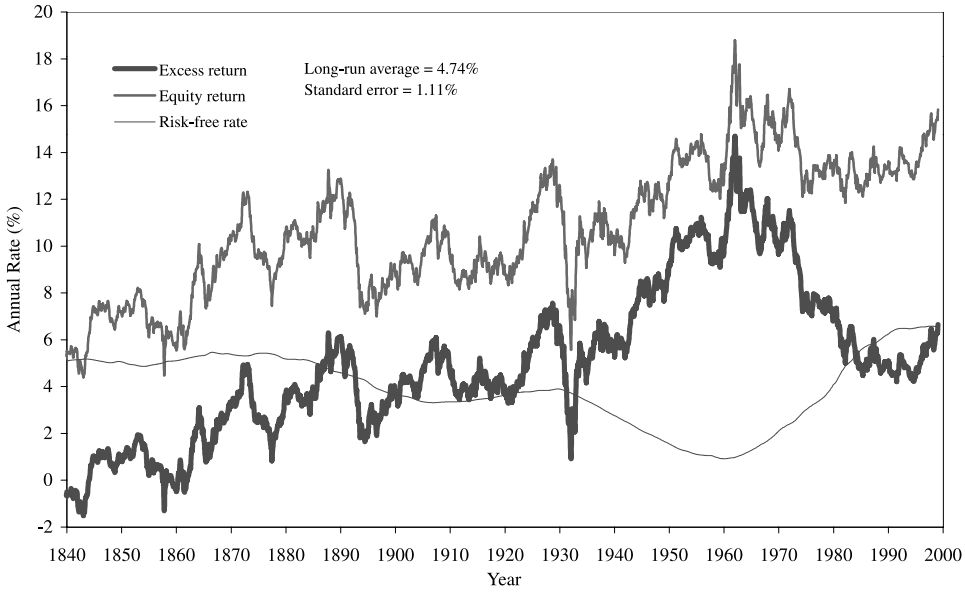

Figure 1. 30-year moving average of equity returns and interest rates.

I. Comments

Since the main contribution of Pástor and Stambaugh's paper is to incorporate economic theory and intuition into the prior distribution for this problem, I only comment on the three prior beliefs they introduced.

A. The Positive Link Between the Premium and Volatility

The prior belief in the positive link between the equity premium and volatility comes from classic economic theory. In stable regime i, the prior distribution of volatility σ_i is restricted by the premium μ_i as in the following equation:

$$\mu_i = \gamma\psi_i\sigma_i^2, \quad i = 1,\cdots,K+1 \tag{1}$$

where γ is a parameter independent of i, and ψ_i is a parameter depending on i. The prior distributions of γ and ψ_i are gamma distributions. There is a restriction on μ_i and σ_i because the prior mean of ψ_i is 1 and the prior variance of ψ_i is 0.2. The volatility $\sigma_{j,j+1}$ in transition regimes and the probability (p_{kk}, $k = 1,\cdots,2K$) of breaks are independent of the equity premium in the prior distribution. This prior belief affects both the range and shape of the curve of the estimated equity premium, as demonstrated in Figure 4 of Pástor and Stambaugh's paper.

In the above mathematical setup of the prior distribution, the premium only depends on the volatility in stable regimes. It implies that shifts in the equity premium are driven by shifts in the volatility between stable regimes but are not affected by the changes in the probability and magnitude of regime shifts or the volatility in transition regimes. This structure ignores investors' desire to hedge the risk of regime change. At any point in time, the possibility of experiencing a regime switch is an important part of the risk investors have to bear, so they will discount security prices because of this risk. When the risk of regime change and the associated premium varies, the equity premium in a regime, therefore, should also depend on the probability (p_{kk}, $k = 1,\cdots,2K$) and magnitude ($\mu_{j+1} - \mu_j$) of regime shifts and the volatility ($\sigma_{j,j+1}$) in transition regimes, as well as the volatility (σ_i) in stable regimes. Even when the risk of regime change and the associated premium is constant over time, it is still not reflected in the model specified in equation (1) because the premium is assumed to be proportional to the volatility. In the case where regime-change risk is constant over time, the equity premium should be a linear function of volatility with an intercept, that is, $\mu_i = \gamma_o + \gamma\sigma_i^2$, where γ_o represents the premium on the risk of regime change.

The premium on the risk of regime change outlined above corresponds to the jump-risk premium in the literature of continuous-time finance. If the jump risk can be diversified away in a portfolio consisting of a large number of stocks, there will be no premium on this risk and the structure in equation (1) will be correct. Since Pástor and Stambaugh assume a regime-switching model for the aggregate equity index, the jump risk cannot be diversified away, and there must be a premium on such risk. The evidence of jump-risk premium has been shown in the early literature by Jarrow and Rosenfeld (1984) and more recently by Pan (2000). A prior distribution that incorporates an equilibrium model of the jump-risk premium would be interesting.

B. The Negative Relation Between the Premium and Price

The prior belief in the negative relation between the equity premium and stock price comes from a basic accounting rule. When a security's price moves up, the expected return is lower if the expected cash flow from the security and the risk-free rate do not change. The equity premium then should decrease before it increases. For this reason, Pástor and Stambaugh model the equity premium during transition regime j as

$$\frac{\mu_j + \mu_{j+1}}{2} + b_j(\mu_{j+1} - \mu_j), \tag{2}$$

where μ_j and μ_{j+1} are, respectively, the equity premiums before and after the transition regime j. Since the negative relation between the equity pre-

mium and the stock price is captured by a negative value of the parameter b_j, they choose the prior distribution such that

$$b_j \sim \mathrm{N}\left(-15.13, \quad \frac{1}{3} \times 15.13\right). \tag{3}$$

Almost all the mass of the prior probability distribution of b_j is below zero.

This prior belief mainly effects the direction of the changes in the estimated equity premium, especially after 1930, as demonstrated in Figure 5 of Pástor and Stambaugh's paper. If there are transition regimes in which the prior belief expressed by (2) and (3) is imposed, a large movement of the excess return always produces an opposite movement of the estimated equity premium. If there are no such transition regimes, the curve of the estimated equity premium resembles the moving average of the excess returns plotted in Figure 1.

The assumed distribution of excess returns during transition regimes does not model the effects of changes in the risk-free rate or changes in either the expectation or risk of equity cash flows. A shift in the premium due to such changes is not necessarily accompanied by a price move in the opposite direction. This is a limitation of the above prior specification. If the equity premium increases because the risk-free rate drops, it does not have to decrease before it increases. Neither should the equity premium decrease before it increases if it becomes higher because earnings grow faster and riskier while the equity price remains the same. An economist who has the prior distribution specified by equations (2) and (3) views the price movement as the dominant force that changes the equity premium.

It is well known that the rate of return on short-term U.S. government securities has changed substantially in the past. In Figure 1, the moving-average of the risk-free rate decreases before 1960 and sharply increases during 1960 to 1990. Changes in the risk-free rate also affect equity returns because the lower (higher) cost of bonds increases (decreases) the return to equity holders. In fact, the declining trend of the risk-free rate before 1960 is associated with the upward trend of the equity return. The rising trend of the risk-free rate during 1960 to 1990 is associated with the downward trend of the equity return.

It is also well known that the structure of equity cash flows has changed during the past century and fluctuates over the business cycles. Siegel (1998) reports that the median earnings yield is 7.35 percent during 1871 to 1945 and 6.96 percent during 1946 to 1996. The real earnings growth is 0.72 percent per year during 1871 to 1945 and 3.25 percent during 1946 to 1996. The median dividend yield is 5.16 percent during 1871 to 1945 and 3.75 percent during 1946 to 1996. Earnings growth is rather high in the late 1990s, but it cannot keep up with the sharp increase in the equity price. These changes in the structure of equity cash flows might be associated with the structural breaks of the equity premium.

Large movements of equity prices are often associated with the changes in fundamental economic variables such as interest rates, corporate earnings, and dividends. Therefore, to a large extent, Pástor and Stambaugh's prior distribution captures the changes in the equity premium, especially the drop of equity premium in the late 1990s, in a rather reasonable way. However, it might be more fruitful to incorporate valuation models into the prior distribution rather than the simple accounting rule. Such a prior distribution will combine the information obtained from the interest rates and price-earnings ratios. Given the large quantity of literature on the valuation of the aggregate equity market based on fundamental economic variables, it is natural that these variables affect economists' prior beliefs in the equity premium.

C. The Range of Shifts in the Equity Premium

Pástor and Stambaugh believe that large shifts in the equity premium are unlikely. To express such a belief, the prior distribution of the equity premiums in stable regimes, denoted by the vector μ, is specified as

$$p(\mu|\bar{\mu}) \propto \exp\left[-\frac{1}{2\sigma_\mu^2}(\mu - \bar{\mu}\iota)'(\mu - \bar{\mu}\iota)\right], \quad \mu > 0 \tag{4}$$

where $\bar{\mu}$ is a positive hyperparameter following a noninformative prior distribution, and ι is the vector of 1s. Pástor and Stambaugh use the variance σ_μ^2 to control the strength of the belief. As expected, the main effect of this prior belief is to dampen the fluctuations of the estimated equity premium over regimes. Comparing the solid curves in Figure 2 and Figure 4 of Pástor and Stambaugh's paper, the plots of the estimated equity premium are very similar, except that the one in Figure 2 has a much narrower range than the one in Figure 4. Without the prior belief expressed by equation (4), the estimated equity premium ranges from 4 to 10 percent rather than from 4 to 6 percent. Clearly, the prior belief expressed by equation (4) is needed to shrink the estimate to the desired range.

All of us should wonder why large shifts in the equity premium are unlikely. Although Pástor and Stambaugh pose this belief as a wise man's wisdom, it is useful for us to think of some economic reasons. If the equity premium is the premium on risk, it will have large shifts either because equity risk jumps or because investors' risk aversion changes drastically. Since the volatility of the U.S. equity index fluctuates within some range, the link between the premium and the volatility should constrain the shifts in the equity premium. The degree of risk aversion is usually implied in the valuation model. Therefore, the prior belief in the link between the premium and volatility and the prior belief in the valuation model should already impose restrictions on the shifts in the equity premium. If the prior beliefs in the two economic models work well, we should not need the informative prior distribution expressed by equation (4).

II. Summary

To deliver a reasonable estimate of the equity premium, Pástor and Stambaugh incorporate a pricing model that links the premium to volatility in stable regimes, an accounting rule that relates the premium to price changes, and the view that changes in the equity premium are unlikely to be extreme. Their work points to an important and promising direction for research on the equity premium. I suggest two extensions. First, the pricing model should be extended to link the premium to both the risk of regime change and the volatility in stable regimes. Second, a valuation model that uses fundamental economic variables should replace the accounting rule. I think a prior distribution incorporating both the pricing model and the valuation model should deliver a reasonable estimate of the equity premium. The view that shifts in the equity premium are unlikely to be extreme should be a consequence of the belief in the two economic models.

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