FLEXIBLE PAVEMENT THERMAL STRESSES WITH VARIABLE TEMPERATURE

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ABSTRACT: A comprehensive analytical treatment of flexible pavement thermal stresses with variable temperature is presented in this paper. General solutions for equations of equilibrium expressed in terms of displacement and variable temperature are derived by Laplace transformation, Hankel transformation, and Laplace transformation with respect to the time and radial and vertical coordinates, respectively. For multi-layered problems, the transfer matrix method is utilized to obtain the general solutions. The calculated results confirm the importance and the need to account for the thermal stresses in design and analysis of flexible pavement.

INTRODUCTION

It has been recognized that critical stresses in flexible pavements result from traffic load. The analysis of such stresses has conventionally been performed based on the theory of an elastically multilayered half-space problem, which can be obtained by closed analytical solutions. It has also been recognized that the effect of temperature is important; for example, low temperature cracking of flexible pavements in one of the open problems in pavement engineering. Many research works have considered the calculation of thermal stresses in concrete pavements [e.g., Harik (1992) and Choubane and Tia (1993)]. However, very little effort has dealt with the field of pavement stresses caused by temperature.

This paper has three objectives. The first objective is to present general solutions of thermal stress for a single layer. The Laplace transformation, Hankel transformation, and Laplace transformation with respect to time and radial and vertical coordinates, respectively, are used to obtain explicit general solutions. The second objective is to derive a set of fundamental solutions for the elastic multilayered half-space problem with variable temperature. Complete explicit solutions are presented in the domain of Laplace transformation and Hankel transformation. The final objective is to show the importance of thermal stress in the design and analysis of flexible pavements by the calculated results.

GOVERNING EQUATIONS AND GENERAL SOLUTION FOR SINGLE LAYER

The calculation model of a flexible pavement consists of a multilayer and is treated as an axial symmetric elastic layered half-space problem as shown in Fig. 1. The governing equations can be expressed by displacements and the variation of temperature as a basic unknown as follows:

$$\frac{1}{1-2\mu}\frac{\partial e}{\partial r} + \nabla^2 U - \frac{U}{r^2} = \frac{2(1-\mu)}{1-2\mu}\alpha\frac{\partial T}{\partial r} \tag{1a}$$

$$\frac{1}{1-2\mu}\frac{\partial e}{\partial z} + \nabla^2 W = \frac{2(1-\mu)}{1-2\mu}\alpha\frac{\partial T}{\partial z} \tag{1b}$$

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The thermal transfer equation for the flexible pavement can be expressed as follows:

$$\lambda\nabla^2 T = \frac{\partial T}{\partial t} \tag{2}$$

The constitutive relations can be expressed as

$$\sigma_r = 2G\left(\frac{\mu}{1-2\mu}e + \frac{\partial U}{\partial r}\right) - \alpha\frac{ET}{1-2\mu} \tag{3a}$$

$$\sigma_\theta = 2G\left(\frac{\mu}{1-2\mu}e + \frac{U}{r}\right) - \alpha\frac{ET}{1-2\mu} \tag{3b}$$

$$\sigma_z = 2G\left(\frac{\mu}{1-2\mu}e + \frac{\partial W}{\partial z}\right) - \alpha\frac{ET}{1-2\mu} \tag{3c}$$

$$\tau_{zr} = G\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial r}\right) \tag{3d}$$

where

$$e = \frac{\partial U}{\partial r} + \frac{U}{r} + \frac{\partial W}{\partial z};\quad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2};\quad G = \frac{E}{2(1+\mu)}$$

where E and G = modulus and shear modulus of pavement materials, respectively; μ and α = Poisson's ratio and expanding parameter, respectively; U and W represent the radial and vertical displacements of pavement, respectively; T = function of temperature; and λ = parameter of thermal transfer. To obtain the solutions, the following equations should be solved:

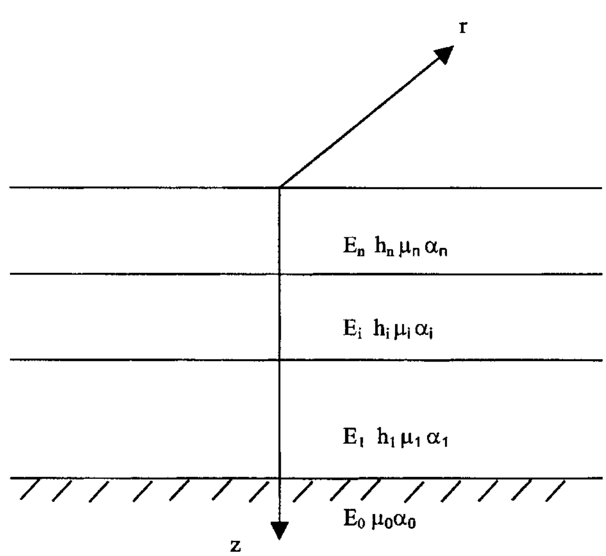

FIG. 1. Calculated Model of Flexible Pavement

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$$\frac{1}{1-2\mu}\frac{\partial e}{\partial r} + \nabla^2 U - \frac{U}{r^2} = \frac{2(1-\mu)}{1-2\mu}\alpha\frac{\alpha T}{\alpha r} \tag{4a}$$

$$\frac{1}{1-2\mu}\frac{\partial e}{\partial z} + \nabla^2 W = \frac{2(1-\mu)}{1-2\mu}\alpha\frac{\alpha T}{\partial z}; \quad \lambda\nabla^2 T = \frac{\partial T}{\partial t} \tag{4b,c}$$

To omit the variable of time, the Laplace integral transformation is utilized. Let

$$\hat{F}(r, z, s) \int_{-\infty}^{\infty} F(r, z, t)e^{-st}\, dt \tag{5}$$

and the inverse transformation is

$$F(r, z, t) = \int_{\alpha-i\infty}^{\alpha+i\infty} \hat{F}(r, z, s)e^{st}\, ds \tag{6}$$

From the integral transformation of (5), (4) becomes

$$\frac{1}{1-2\mu}\frac{\partial \hat{e}}{\partial r} + \nabla^2 \hat{U} - \frac{\hat{U}}{r^2} = \frac{2(1-\mu)\alpha}{1-2\mu}\frac{\partial \hat{T}}{\partial r} \tag{7a}$$

$$\frac{1}{1-2\mu}\frac{\partial \hat{e}}{\partial z} + \nabla^2 \hat{W} = \frac{2(1-\mu)\alpha}{1-2\mu}\frac{\partial \hat{T}}{\partial z}; \quad \lambda\nabla^2 T^2 = s\hat{T} \tag{7b,c}$$

where the initial temperature is assumed as zero. Now (7) becomes a coupled ordinary differential equation group of the second order. To solve (7) the Hankel integral transformation, as follows, should be used:

$$\bar{U}(\zeta, z, s) = \int_0^{\infty} r\hat{U}(r, z, s)J_1(\zeta r)\, dr \tag{8a}$$

$$\bar{W}(\zeta, z, s) = \int_0^{\infty} r\hat{W}(r, z, s)J_0(\zeta r)\, dr \tag{8b}$$

$$\bar{\tau}_{zr}(\zeta, z, s) = \int_0^{\infty} r\hat{\tau}_{zr}(r, z, s)J_1(\zeta r)\, dr \tag{8c}$$

$$\bar{\sigma}_z(\zeta, z, s) = \int_0^{\infty} r\hat{\sigma}_z(r, z, s)J_0(\zeta r)\, dr \tag{8d}$$

The inverse transformation is as follows:

$$\hat{U}(\zeta, z, s) = \int_0^{\infty} \zeta\bar{U}(r, z, s)J_1(\zeta r)\, d\zeta \tag{9a}$$

$$\hat{W}(\zeta, z, s) = \int_0^{\infty} \zeta\bar{W}(r, z, s)J_0(\zeta r)\, d\zeta \tag{9b}$$

$$\hat{\tau}_{zr}(\zeta, z, s) = \int_0^{\infty} \zeta\bar{\tau}_{zr}(r, z, s)J_1(\zeta r)\, d\zeta \tag{9c}$$

$$\hat{\sigma}_z(\zeta, z, s) = \int_0^{\infty} \zeta\bar{\sigma}_z(r, z, s)J_0(\zeta r)\, d\zeta \tag{9d}$$

After using the integral transformation [(8)], (7) becomes

$$\frac{\partial^2\bar{U}}{\partial z^2} - \frac{2(1-\mu)}{1-2\mu}\zeta^2\bar{U} - \frac{1}{1-2\mu}\zeta^2\frac{\partial\bar{W}}{\partial z} = -\frac{2(1-\mu)}{1-2\mu}\alpha\zeta\bar{T} \tag{10a}$$

$$\frac{2(1-\mu)}{1-2\mu}\frac{\partial^2\bar{W}}{\partial z^2} - \zeta^2\bar{W} + \frac{\zeta}{1-2\mu}\frac{\partial\bar{U}}{\partial z} = \frac{2(1-\mu)}{1-2\mu}\alpha\frac{\partial\bar{T}}{\partial z} \tag{10b}$$

$$\lambda\frac{\partial^2\bar{T}}{\partial z^2} - \lambda\zeta^2\bar{T} = s\bar{T} \tag{10c}$$

Again, using the Laplace integral transformation to the variable of z and the inverse integral transformation, which are shown, respectively, by (11) and (12)

$$\tilde{F}(r, p, s) = \int_{-\infty}^{+\infty} \bar{F}(r, z, t)e^{-pz}\, dz \tag{11}$$

$$\bar{F}(r, z, s) = \int_{\alpha-i\infty}^{\alpha+i\infty} \tilde{F}(r, p, s)e^{pz}\, dp \tag{12}$$

Thus (10) can be written as follows:

$$\left(p^2 - \frac{2(1-\mu)}{1-2\mu}\zeta^2\right)\tilde{U} - \frac{\zeta}{1-2\mu}p\tilde{W} + \frac{2(1-\mu)}{1-2\mu}\alpha\tilde{T}$$
$$= p\bar{U}(\zeta, 0, s) + \bar{U}'(\zeta, 0, s) - \frac{1}{1-2\mu}\zeta\bar{W}(\zeta, 0, s) \tag{13a}$$

$$\frac{\zeta}{1-2\mu}p\tilde{U} + \left(\frac{2(1-\mu)}{1-2\mu}p^2 - \zeta^2\right)\tilde{W} - \frac{2(1-\mu)}{1-2\mu}\alpha p\tilde{T}$$
$$= \left[\frac{\zeta}{1-2\mu}\bar{U}(\zeta, 0, s) + \frac{2(1-\mu)}{1-2\mu}\bar{W}'(\zeta, 0, s) + \frac{2(1-\mu)}{1-2\mu}p^2\bar{W}(\zeta, 0, s) - \frac{2(1-\mu)}{1-2\mu}\alpha\bar{T}\right] \tag{13b}$$

$$[\lambda(p^2 - \zeta^2) - s^2]\tilde{T} = \lambda p\bar{T}(\zeta, 0, s) + \lambda\bar{T}'(\zeta, 0, s) \tag{13c}$$

Eq. (3) can be written as follows:

$$\bar{\sigma}_z(\zeta, z, s) = 2G\left[\frac{\mu}{1-2\mu}\zeta\bar{U}(\zeta, z, s) + \frac{1-\mu}{1-2\mu}\bar{W}'(\zeta, z, s)\right] - \frac{\alpha E}{1-2\mu}\bar{T}(\zeta, z, s) \tag{14a}$$

$$\bar{\tau}_{zr}(\zeta, z, s) = G[\bar{U}'(\zeta, z, s) - \zeta\bar{W}(\zeta, z, s)] \tag{14b}$$

In (14) let $z = 0$ and put it into (13); then omit $U'(\zeta, 0, s)$ and $W'(\zeta, 0, s)$. The following equations can be obtained:

$$\left(p^2 - \frac{2(1-\mu)}{1-2\mu}\zeta^2\right)\tilde{U} - \frac{\zeta}{1-2\mu}p\tilde{W} = -\frac{2(1-\mu)}{1-2\mu}\alpha\tilde{T}$$
$$+ p\bar{U}(\zeta, 0, s) - \zeta\frac{2\mu}{1-2\mu}\bar{W}(\zeta, 0, s) + \frac{2(1-\mu)}{E}\bar{\tau}_{zr}(\zeta, 0, s) \tag{15a}$$

$$\frac{\zeta}{1-2\mu}p\tilde{U} + \left(\frac{2(1-\mu)}{1-2\mu}p^2 - \zeta^2\right)\tilde{W} = \frac{2(1-\mu)}{1-2\mu}\alpha p\tilde{T}$$
$$+ \zeta\bar{U}(\zeta, 0, s) + \frac{2(1-\mu)}{1-2\mu}p\bar{W}(\zeta, 0, s) + \frac{2(1-\mu)}{E}\bar{\sigma}_z(\zeta, 0, s) \tag{15b}$$

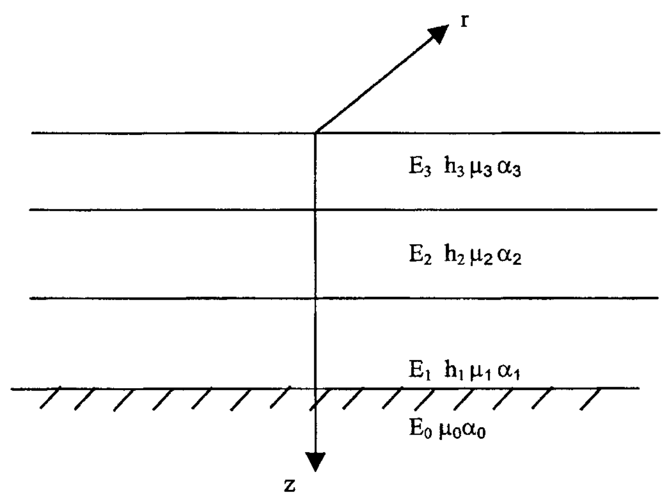

FIG. 2. Computing Model of Practical Pavement

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$$[\lambda(p^2 - \zeta^2) - s^2]\tilde{T} = \lambda p\bar{T}(\zeta, 0, s) + \lambda\bar{T}'(\zeta, 0, s) \quad (15c)$$

Having solved (15) and by using the inverse integral transformation [(12)], the solutions of (15) can be written in the form of a matrix as follows:

$$\begin{pmatrix} \bar{U}(\zeta, z, s) \\ \bar{W}(\zeta, z, s) \\ \bar{\tau}_{zr}(\zeta, z, s) \\ \bar{\sigma}_z(\zeta, z, s) \\ \bar{T}(\zeta, z, s) \\ \bar{T}'(\zeta, z, s) \end{pmatrix} = \begin{bmatrix} \phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} & \Phi_{16} \\ \Phi_{21} & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} & \Phi_{26} \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} & \Phi_{45} & \Phi_{46} \\ \Phi_{51} & \Phi_{52} & \Phi_{53} & \Phi_{54} & \Phi_{55} & \Phi_{56} \\ \Phi_{61} & \Phi_{62} & \Phi_{63} & \Phi_{64} & \Phi_{65} & \Phi_{66} \end{bmatrix} \begin{pmatrix} \bar{U}(\zeta, 0, s) \\ \bar{W}(\zeta, 0, s) \\ \bar{\tau}_{zr}(\zeta, 0, s) \\ \bar{\sigma}_z(\zeta, 0, s) \\ \bar{T}(\zeta, 0, s) \\ \bar{T}'(\zeta, 0, s) \end{pmatrix} \quad (16)$$

where Φ_{ij} ($i = 1, 2, \ldots, 6$; $j = 1, 2, \ldots, 6$) = transfer function. Eq. (16) can be expressed as follows:

$$\bar{Y}(\zeta, z, s) = [\Phi]\bar{Y}(\zeta, 0, s), \quad z \subset [0, h_j] \quad (17)$$

where

$$\bar{Y}(\zeta, z, s) = [\bar{U}(\zeta, z, s), \bar{W}(\zeta, z, s), \bar{\tau}_{zr}(\zeta, z0, s), \bar{\sigma}_z(\zeta, z, s), \bar{T}(\zeta, z, s), \bar{T}'(\zeta, z, s)]^T$$

$$\bar{Y}(\zeta, 0, s) = [\bar{U}(\zeta, 0, s), \bar{W}(\zeta, 0, s), \bar{\tau}_{zr}(\zeta, 0, s), \bar{\sigma}_z(\zeta, 0, s), \bar{T}(\zeta, 0, s), \bar{T}'(\zeta, 0, s)]^T$$

where $[\Phi]$ = transfer matrix; and $Y(\xi, 0, s)$ = boundary condition of top layer. Eq. (16) establishes the relationship between the quantities of the upper surface and lower surface of the j layer by the transfer matrix $[\Phi]$.

GENERAL SOLUTION FOR MULTILAYER

Eq. (17) is suitable to any layer, particularly for the case of $z = zj$ when equation becomes

$$\bar{Y}(\zeta, z_j, s) = [\Phi_j]\bar{Y}(\zeta, z_{j-1}, s) \quad (18)$$

Letting j in (18) be equal to 1 and n, respectively, (18) can be written as follows:

$$\bar{Y}(\zeta, z_1 s) = [\Phi_1]\bar{Y}(\zeta, z_0, s) \quad (19a)$$

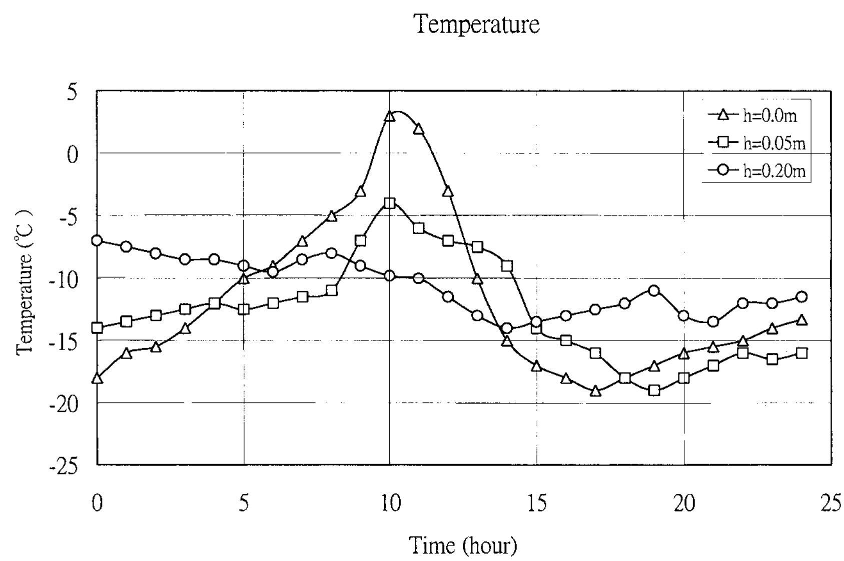

FIG. 3. Calculated Results of Pavement Temperature

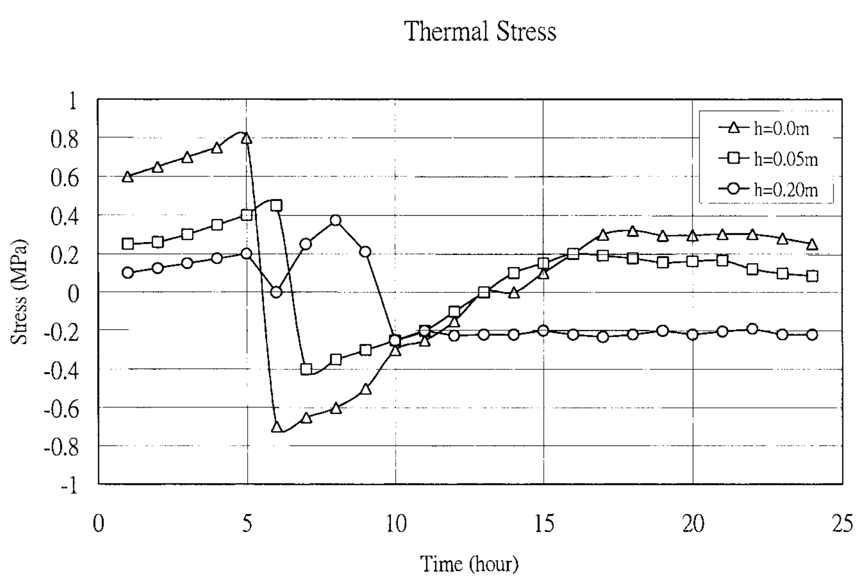

FIG. 4. Calculated Results of Pavement Thermal Stress

$$\bar{Y}(\zeta, z_n s) = [\Phi_n]\bar{Y}(\zeta, z_{n-1}, s) \qquad (19b)$$

At the interfaces, the compatibility conditions can be written as follows:

$$\bar{Y}(\zeta, z_j s) = \bar{Y}(\zeta, z_{j+1}, s) \qquad (20)$$

Eq. (20) shows the equality of the interface stresses and the continuity of the interface displacements. Using (19) and (20) repeatedly, the following formula for the multilayered problem can be obtained:

$$\bar{Y}(\zeta, z_n, s) = [\Phi_{n-1}]\cdot[\Phi_{n-2}]\cdots[\Phi_2]\cdot[\Phi_1]\bar{Y}(\zeta, 0, s)$$
$$= \left(\prod_{j=1}^{n-1}[\Phi_j]\right)\bar{Y}(\zeta, 0, s) \qquad (21)$$

The quantities of the top and bottom layers of flexible pavements have been linked together by (21). Usually, τ_{zr} and σ_z, of top and bottom layers, are known. Actually, (21) is a set of simultaneous equations in terms of layer displacements. All of the calculations in the above procedure are multiplication of matrices, which can be programmed.

NUMERICAL EXAMPLES

To prove the correctness of the formulations in this paper, a practical flexible pavement, shown in Fig. 2, is calculated. The material properties are indicated below:

- Asphalt surface: E_3 = 2,500 MPa; h_3 = 0.15 m; λ_3 = 1.0; α_3 = 0.0022
- Cement stabilized sand base: E_2 = 1,500 MPa; h_2 = 0.20 m; λ_2 = 1.2; α_2 = 0.0028
- Lime stabilized soil subbase: E_1 = 700 MPa; h_1 = 0.30 m; λ_1 = 1.1; α_1 = 0.0026
- Soil subbase: E_0 = 500 MPa; λ_0 = 1.0; α_0 = 0.0030

The calculation results are shown in Figs. 3 and 4.

SUMMARY AND CONCLUSIONS

A set of general solutions are presented in the Laplace and Hankel transformation spaces for equations governing symmetric deformations of flexible pavements with variable temperature. These general solutions are used to calculate the stresses and displacements of flexible pavements caused by temperature. Through the analysis and calculation, the main findings in this paper are summarized as follows:

- Through the theoretical analysis, it proves that maximum stresses of flexible pavements caused by temperature often appears at the top surface of the pavement.
- The magnitude and direction of stress caused by temperature changes with time; for example, at some time the stress is positive, whereas at other times the stress is negative. The reversal of stress direction can cause the fatigue damage of pavement and also is one of the reasons for the cracking of flexible pavement in cold regions.
- The temperature effects have been considered in the material selection. The results in the present paper have clearly demonstrated that temperature effects should also be considered in the analysis and design for flexible pavements.

ACKNOWLEDGMENTS

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APPENDIX. REFERENCES

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