An Analytical Approach to Evaluate the Coefficients of Thermal Expansion of Textile Composite Materials

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An analytical approach is developed to evaluate the coefficients of thermal expansion (CTE) of textile reinforced composites. At the micro level, a cylindrical composite model is employed to model the fiber/matrix thermal and mechanical interactions. The effects of voids and fiber coating on the thermal expansion coefficients of composites are considered at this level. The cylindrical model was then embedded in a macro hybrid finite element solution structure to calculate the value of the CTE for textile composites. AS-4/epoxy balanced plain weave textile composites were manufactured. Five different fiber volume fractions were tested for CTE. Evaluation of the thermal expansion coefficients using the current model was compared to experimental data for in-plane and out-of-plane directions.

1. INTRODUCTION

Thermal expansion of composite materials has received considerable attention over the past few years. This is due to the need to minimize thermal residual stresses in composite materials during curing or during the life of the composite component. Thermal residual stresses can have an adverse effect on the strength and fatigue life of structural components. If composite parts are subjected to operational cyclic temperature variations, they may be prone to failure owing to thermal fatigue.

Voids can form during the manufacturing process of composite, especially in ceramic composites. Voids can have an effect not only on the mechanical properties of a composite, but also on its thermal properties. Voids cause stress concentrations which reduce the failure strength of components. A uniform heat flow (linear temperature distribution) induces no stress concentration in an infinite homogeneous elastic body. However if the body contains voids which almost have no thermal conductivity, the temperature field will be perturbed around the voids. The resulting nonlinear temperature distribution can cause localized thermal stresses.

Thermal expansion coefficients of unidirectional fiber reinforced composites has been modeled by many workers (1–8). In general, these studies investigated the effect of volume fractions of fibers and matrix, thermal expansion coefficients of constituent materials, fiber-matrix interface, and mechanical properties of the constituents on the overall CTE of the composite.

In 1967, Levin (1), using an extension of Hill's method (2), showed that a simple relationship can be established between the effective thermal expansion coefficient and the effective elastic moduli of the composite. Based on Levin's approach, Rosen and Hashin (3) derived bounds for the values of CTE using the effective thermal expansion coefficients and the specific heats of the composite anisotropic constituents. In 1968, Schapery (4) constructed a complementary and potential energy principle of thermo-elasticity theory in conjunction with a procedure for minimizing the difference between upper and lower bounds proposed by Levin (1). Chamberlain (5) developed a relationship for transverse thermal expansion coefficient, considering each fiber as being surrounded by matrix in the form of a thick-walled cylinder. The influence of internal stresses, due to the thermo-mechanical mismatch between the fiber and the matrix, and the thermal expansion behavior of unidirectional fiber-reinforced ceramics has been considered by Hsueh and Becher (6).

None of the above mentioned models have studied the effect of voids on the thermal expansion coefficients of textile composites. Furthermore, extension of these models to include the effect of voids and 3-phase composites (i.e., fiber/coating/matrix) has not been investigated.

2. ANALYTICAL APPROACH

In the present work an analytical approach was developed to evaluate the coefficients of thermal expansion (CTE) of textile reinforced composites. First, a repeat unit cell of the composite was graphically modeled using a previously developed geometric model (15). Second, the unit cell was divided into a smaller set of hexahedral brick elements with fibers and matrix around each integration point (14). At the micro level, a cylindrical composite model was employed to model the fiber/matrix thermal and mechanical interactions. The effect of voids and fiber coating on the thermal expansion coefficients of composites was also considered at this level. Although this approach was derived for 2- and 3-phase materials, it could be easily extended to model n-phase materials with different internal structures and inclusions/voids.

The Micro-Level Cylindrical Model

The stress strain behavior for anisotropic materials can be represented, in tensor form, as follows:

$$\varepsilon_{ij} = S_{ijkl}\,\sigma_{kl} + \alpha_{ij}\,\Delta T$$

$$\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl} - \Gamma_{ij}\,\Delta T$$

where σ is the stress vector, ε is the strain vector, C is the stiffness tensor, S is the compliance tensor ($S = C^{-1}$), α is the vector of CTE, Γ is the vector of thermal stresses ($\Gamma = C_{ijkl}\,\alpha_{kl}$) and ΔT is the temperature increment.

For fibrous composites, the coefficient of thermal expansion is also a function of the volume fraction of constituent materials, their CTE, their relative stiffness and the interface characteristics between the constituents. Fiber coating could have a large effect on the CTE of composites.

In order to derive the stress-strain relationships, the following assumption were made: i) steady state heat flow, ii) temperature of fiber, coating and matrix temperature is constant and equal and iii) all interfaces are perfectly bonded.

In the cylindrical composite model shown in *Fig. 1*, if we treat the fiber as a transversely isotropic material, and the matrix and the coating as isotropic materials, the basic equations of stress, strain and displacement, in polar coordinates, can be represented as follows (11):

$$u_{ir} = A_{2i}\frac{1}{r}\int_0^r T_i r\,dr + C_{1i}r + \frac{C_{2i}}{r} - \upsilon_i C_{3i} r/E_i \quad (1a)$$

$$\sigma_{ir} = -A_{1i}\frac{1}{r^2}\int_0^r T_i r\,dr + A_{3i}C_{1i} - \frac{A_{4i}C_{2i}}{r^2} \quad (1b)$$

$$\sigma_{i\theta} = -A_{1i}\frac{1}{r^2}\int_0^r T_i r\,dr - A_{1i}T_i + A_{3i}C_{1i} + \frac{A_{4i}C_{2i}}{r^2} \quad (1c)$$

$$\sigma_{iz} = -A_{1i}T_i + 2\upsilon_i A_{3i}C_{1i} = -C_{3i} \quad (1d)$$

$$\varepsilon_{ir} = \frac{du_{ir}}{dr} \quad (1e)$$

$$\varepsilon_{i\theta} = \frac{u_{ir}}{r} \quad (1f)$$

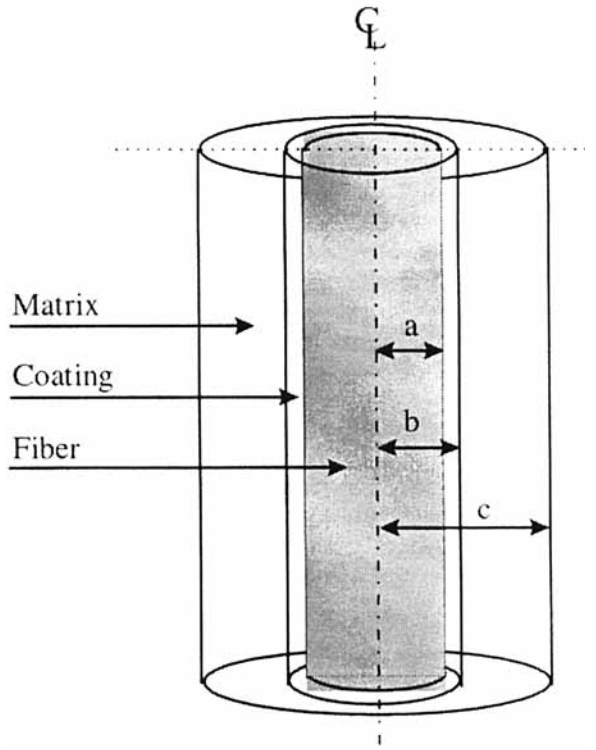

Fig. 1. Cylindrical model used to calculate CTE for 2-phase and 3-phase fibrous composites.

$$\varepsilon_{iz} = \frac{C_{3i}}{E_i} \quad (1g)$$

where i refers to fiber (f), matrix (m) or coating (c); $0 \le r \le c$, u_{ir}, is radial displacement; σ_{ir}, $\sigma_{i\theta}$ and σ_{iz} are stresses in radial, tangential and axial directions, respectively; ε_{ir} and $\varepsilon_{i\theta}$ ε_{iz} are strains in radial, tangential and axial directions, respectively; α_{ia} and α_{it} are axial and transverse thermal expansion coefficients, respectively; E_i, and υ_i are axial Young's modulus and Poisson's ratio, respectively; C_{1i}, C_{2i} and C_{3i} are constants that need to be solved; and,

$$A_{1i} = \frac{(\alpha_{it} + \upsilon_i\alpha_{ia})E_i}{1-\upsilon_i^2},\ A_{2i} = \frac{\alpha_{it} + \upsilon_i\alpha_{ia}}{1-\upsilon_i},$$

$$A_{3i} = \frac{E_i}{(1+\upsilon_i)(1-2\upsilon_i)},\ A_{4i} = \frac{(\alpha_{it}\upsilon_i + \alpha_{ia})E_i}{1-\upsilon_i^2}$$

Notice that because of the symmetry about the axes and the uniformity in the axial direction of this fiber/coating/matrix system, all shear strains and stresses are zero.

To calculate the axial thermal expansion coefficient, where the fibers control the composite behavior (especially in polymer composites), we assume the boundary conditions to be as follows:

$$u_{fr} = 0 \quad at\ r = 0 \quad (2a)$$

$$\sigma_{fr} = \sigma_{cr} \quad at\ r = a \quad (2b)$$

$$\sigma_{mr} = \sigma_{cr} \quad at\ r = b \tag{2c}$$

$$\sigma_{mr} = 0 \quad at\ r = c \tag{2d}$$

$$\varepsilon_{fz} = \varepsilon_{cz} = \varepsilon_{mz} \tag{2e}$$

Solving the previous equations, for a specific material configuration such as fiber/interface/matrix or fiber/matrix only) under the specified boundary conditions, we can obtain the constants C_{1i}, C_{2i}, C_{3i} and calculate the stresses and strains in *Eq 1*.

Notice that the number of the constituents of the composite will determine the size of the problem. For example, for a 2-phase composite (i.e., fiber and matrix only), the number of constants to be solved are 6 with 3 boundary conditions, while for a 3-phase composite (i.e., fiber/coating/matrix) the number of constants is 9 with 5 boundary conditions. These constants are needed to evaluate the stress and strain states and CTE in *Eq 1*. If we define the longitudinal thermal expansion coefficient of a composite as:

$$\alpha_a = \frac{\varepsilon_{iz}}{T_b}$$

where ε_{iz} is the strain of the fiber, coating or matrix (refer to *Eq 2e*) in the axial direction, then we can calculate the thermal expansion coefficient of the composite in the axial direction.

To calculate the transverse thermal expansion coefficient, where the matrix will control the composite behavior, we assume the boundary conditions to be as follows:

$$u_{fr} = 0 \quad at\ r = 0 \tag{3a}$$

$$\sigma_{fr} = \sigma_{cr} \quad at\ r = a \tag{3b}$$

$$u_{fr} = u_{cr} \quad at\ r = a \tag{3c}$$

$$\sigma_{mr} = \sigma_{cr} \quad at\ r = b \tag{3d}$$

$$u_{mr} = u_{cr} \quad at\ r = b \tag{3e}$$

$$\sigma_{mr} = 0 \quad at\ r = c \tag{3f}$$

Again, notice that the number of the constituents of the composite will determine the size of the problem. For a 2-phase composite, the number of constants to be solved are 6 with 4 boundary conditions, while for a 3-phase composite, the number of constants is 9 with 6 boundary conditions.

The transverse thermal expansion coefficient of the composite could be defined as the value of the strain at $r = c$ (the outer surface of the composite cylinder) at a unit increase in temperature:

$$\alpha_t = \frac{\varepsilon_{mr}}{T_c}$$

where ε_{mr} is the strain of the matrix in the radial direction. Using this equation, we can calculate the thermal expansion coefficient of the composite in the transverse direction.

Effect of Voids

Voids inside the matrix can be considered as inclusions that only affect the properties of matrix. The properties of matrix can be used to replace the original matrix properties in the general composite cylindrical model approach.

For the Matrix-Voids model, shown in *Fig. 2*, voids are assumed to be spherical and the heat transfer is assumed to be in steady state with the following temperature distribution (12):

$$T_m = T_m^a + \frac{T_m^a - T_m^b}{\left(\frac{1}{b} - \frac{1}{a}\right)}\left(\frac{1}{a} - \frac{1}{r}\right) \tag{4}$$

where T_m^a is the temperature in matrix at $r = a$, T_m^b is the temperature in matrix at $r = b$, and T_m is the temperature inside matrix.

Inclusion affects the elastic properties of composites (13). If the inclusion is taken as a void, the bulk modulus and shear modulus of the void will be zero (i.e. $K_v = G_v = 0$). The bulk and shear moduli of matrix containing voids can be obtained as follows (13):

$$\frac{K_{mv}}{K_m} = 1 + \frac{V_v}{\dfrac{3(1-V_v)K_m}{3K_m + 4G_m} - 1} \tag{5.a}$$

$$\frac{G_{mv}}{G_m} = 1 + \frac{V_v}{\dfrac{6}{5}\dfrac{(1-V_v)(K_m + 2G_m)}{3K_m + 4G_m} - 1} \tag{5.b}$$

where V_v is the volume fraction of voids, K and G are bulk and shear moduli respectively, mv denotes matrix containing voids, m denotes pure matrix.

On account of radial symmetry of the temperature distribution the radial stress component σ_r and the tangential component σ_t should satisfy the condition of equilibrium, in the radial direction of an element as follows (11):

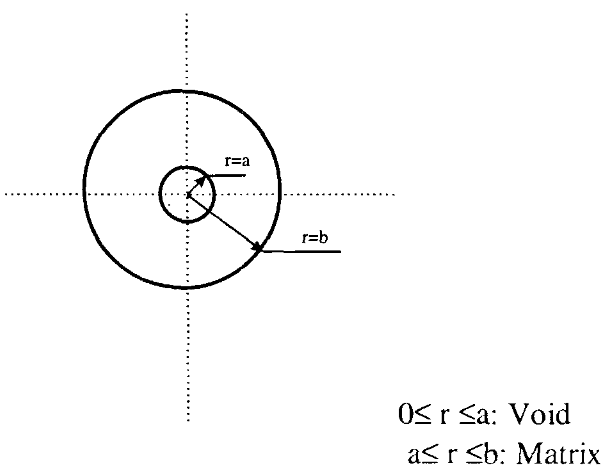

Fig. 2. Spherical model of matrix with void.

$$\frac{d\sigma_r}{dr} + \frac{d}{r}(\sigma_r - \sigma_t) = 0 \tag{6}$$

The stress-strain relations can be represented as follows (16):

$$\varepsilon_{mr} - \alpha_m T_m = \frac{1}{E_m}(\sigma_{mr} - 2\upsilon_m \sigma_{mt}) \tag{7.a}$$

$$\varepsilon_{mt} - \alpha_m T_m = \frac{1}{E_m}[\sigma_{mt} - \upsilon_m(\sigma_{mr} + \sigma_{mt})] \tag{7.b}$$

$$\varepsilon_{mr} = \frac{du_{mr}}{dr} \tag{7.c}$$

$$\varepsilon_{mt} = \frac{u_{mr}}{r} \tag{7.d}$$

where T_m is the temperature distribution in matrix, σ_{mr} and σ_{mt} are matrix radial and tangential stresses, respectively; ε_{mr} and ε_{mt} are matrix radial and tangential strains, respectively; u_{mr} is matrix radial displacement; and α_m, E_m, and υ_m are thermal expansion coefficient, Young's modulus and Poisson's ratio of matrix, respectively.

Solving *Eqs 6* and *7*, the expressions of stress and strain could be derived as:

$$u_{mr} = \frac{1+\upsilon_m}{1-\upsilon_m}\alpha_m \frac{1}{r^2}\int_a^r T_m r^2 dr + C_{1m} r + \frac{C_{2m}}{r^2} \tag{8.a}$$

$$\sigma_{mr} = -\frac{2\alpha_m E_m}{1-\upsilon_m}\frac{1}{r^3}\int_a^r T_m r^2 dr + \frac{E_m C_{1m}}{1-2\upsilon_m} - \frac{2E_m C_{2m}}{1+\upsilon_m}\frac{1}{r^2} \tag{8.b}$$

$$\sigma_{mt} = \frac{\alpha_m E_m}{1-\upsilon_m}\frac{1}{r^3}\int_a^r T_m r^2 dr + \frac{E_m C_{1m}}{1-2\upsilon_m} + \frac{E_m C_{2m}}{1+\upsilon_m}\frac{1}{r^2} - \frac{\alpha_m E_m T_m}{1-\upsilon_m} \tag{8.c}$$

where C_{1m} and C_{2m} are constants of integration to be determined from the following boundary conditions:

$$\sigma_{mr} = 0 \quad \text{at} \quad r = a \quad \text{and} \quad r = b \tag{9}$$

Substituting *Eq 8* into *Eq 9*, C_{1m} and C_{2m} can be obtained as follows:

$$C_{2m} = A_{2m}\frac{\frac{1}{b^3}\int_a^b T_m r^2 dr}{\frac{1}{a^3} - \frac{1}{b^3}} = A_{2m}\frac{a^3}{b^3 - a^3}\frac{2b^3 - ab^2 - ba^2}{6}$$

$$C_{1m} = \frac{2A_{1m}C_{2m}}{a^3}$$

where $A_{1m} = \dfrac{1-2\upsilon_m}{1+\upsilon_m}$, $A_{2m} = \dfrac{1+\upsilon_m}{1-\upsilon_m}\alpha_m$, $\dfrac{a^3}{b^3} = V_\upsilon$, V_υ is the volume fraction of voids in matrix only. The thermal expansion coefficient of matrix with voids can be defined as:

$$\alpha_{m\upsilon} = \frac{\varepsilon_{mr}}{T_m^b} \quad \text{at } r = b$$

In the case of existence of voids in the composite, the properties of the matrix with voids $E_{m\upsilon}$, $G_{m\upsilon}$, $\upsilon_{m\upsilon}$ and $\alpha_{m\upsilon}$ is used to replace the properties of the matrix without voids E_m, G_m, υ_m and α_m in the cylindrical model outlined in the previous section.

Hybrid Finite Element Analysis

This model was previously developed (14, 15) to predict the elastic and thermal properties of textile composites. This is a two-part model. First a geometrical model is used to construct the textile preform and characterize the relative volume fractions and spatial orientation of each yarn in the composite space. Data acquired from the geometrical analysis is used by a hybrid finite element approach to model the composite behavior.

The geometrical model starts by modeling the preform forming process in a typical textile machine. An ideal fabric geometrical representation is constructed by calculating the location of a set of spatial points "knots" that can identify the yarn center-line path within the preform space. This is followed by incorporating a B-spline function to approximate a smooth yarn centerline path relative to the identified knots. The B-spline function is chosen as the approximation function due to its ability to minimize the radius of curvature along its path and its C^2 continuity. Constructing a 3-D object (i.e. yarn) by sweeping a cross section along the smooth centerline forming the yarn surface carries out the final step in this model.

A repeat unit cell of the modeled preform is identified from the geometric modeling and used to represent a complete yarn or tow pattern. A hybrid finite element approach is used to divide the unit cell into smaller subcells. Each subcell is an hexahedral brick element with fibers and matrix around each integration point. A virtual work technique is applied to the FE solution to calculate the properties of the repeat unit cell. The unit cell properties are considered to be representative of the composite properties.

The heterogeneous solution, although successful in cutting down the number of elements, has a limitation. This limitation is bounded by the difference in the fiber and matrix properties the can cause instabilities in the numerical solution. To overcome such limitation a homogenization operation is carried out at the micro-scale level. This is done around each integration point in the FE mesh. *Figure 3* shows a schematic presentation of the finite element division scheme and the micro-level homogenization.

3. EXPERIMENTAL WORK

Materials

Balanced plain weave fabric performs made from 3K AS-4 graphite yarns with 5 ends/cm were purchased from BP Chemical Company. CIBA-GEIGY Araldite

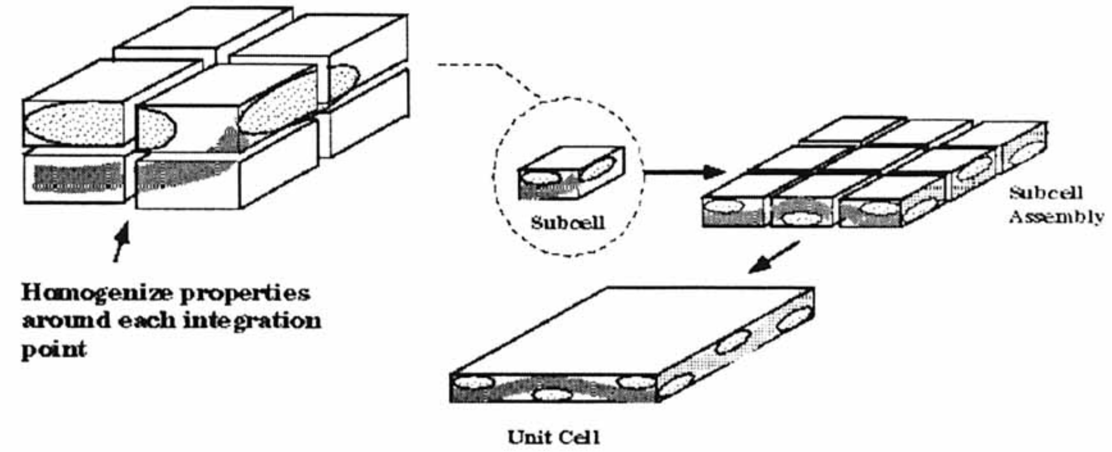

Fig. 3. Finite element divisions of a unit cell and micro-level homogenization.

epoxy resin and HY 956 hardener were used as the matrix material. The mixing ratio was 4:1 for epoxy resin to hardener at room temperature. Fabric layers were manually placed in 30.48 × 38.1 cm molds and impregnated with epoxy resin. The composite was cured in a compression molding machine at 80°C for 2 hours and then post cured at 120°C for 1.5 hours in an oven. The balanced plain weave composite was manufactured with five different volume fractions by applying different pressure in the compression-molding machine (14).

Testing Procedure and Experimental Setup

Testing samples (3.8 cm × 3.8 cm × sample thickness) were cut and placed into an oven. Four samples were tested for each volume fraction. Two strain gauges were placed on the surface of each sample, in the in-plane and the out-of-plane directions. A temperature sensor was placed in the in-plane direction. These two strain gauges and temperature sensor were connected to a data acquisition system. Temperature of the oven was increased from room temperature to 115.6°C gradually.

A standard titanium silicate isotropic sample was used instrumented with strain gauges to determine the thermally induced apparent strain. The average experimental thermal expansion coefficient of the standard sample was $\alpha_2 = -11.01 \times 10^{-6}$ (mm/mm/°C). The standard sample has an actual thermal expansion coefficient $\alpha_1 = 0.031 \times 10^{-6}$ (mm/mm/°C). The thermally induced apparent strain was calculated as:

$$\begin{aligned}\varepsilon_{apparent} &= \varepsilon_2 - \varepsilon_1 = (\alpha_2 - \alpha_1)\Delta T \\ &= -11.04 \times 10^{-6}\,\Delta T\end{aligned}$$

where $\varepsilon_{apparent}$ is the thermally induced apparent strain, ε_1 is the actual strain, ε_2 is the experimental strain, and ΔT is the temperature change. The electrical resistance of the metallic foil strain gauges changes as the temperature increases leading to the thermally induced apparent strain.

4. RESULTS AND DISCUSSION

The experimental in-plane and out-of-plane thermal expansion coefficients for balanced plain weave carbon/epoxy textile composites are listed in *Table 1*. From this table we can conclude that with the increase of the fiber volume fraction, the in-plane thermal expansion coefficient decreases. This means that fibers control the in-plane strain, making the composite stronger in this direction.

The out-of-plane CTE experimental values, for all five different fiber volume fractions, are larger than the thermal expansion coefficients of each component of the composite. Micrographic images, similar to the one shown in *Fig. 4*, show that the composite under investigation suffered from large laminar cracks. These cracks could occur during curing of the composite as a result of thermal residual stresses or during the CTE testing. Debond sites can cause the out-of-plane elastic modulus to sharply decrease and the out-of-plane CTE to increase.

The hybrid finite element analysis model presented in this paper was used to predict the CTE for these samples for the in-plane direction only. Since we were not able to quantify the exact dimensions of different interfacial gaps, the model was not used to predict the out-of-plane properties. Weave crimp angles were measured from micrographic images and listed in *Table 2*. With the increase of fiber volume fractions,

Table 1. Experimental CTE for Balanced Plain Weave Carbon/Epoxy Composite.

Fiber Volume Fraction (%)	In-Plane CTE ($\times 10^{-6}$ mm/mm/°C)	Out-of-Plane ($\times 10^{-6}$ mm/mm/°C)
26.8	12.15 ± 2.7	106.55 ± 1.32
37.5	4.97 ± 0.82	107.79 ± 3.8
47.5	4.32 ± 0.23	112.98 ± 8.26
54.0	4.01 ± 1.34	113.12 ± 3.52
55.7	2.28 ± 0.11	92.96 ± 5.53

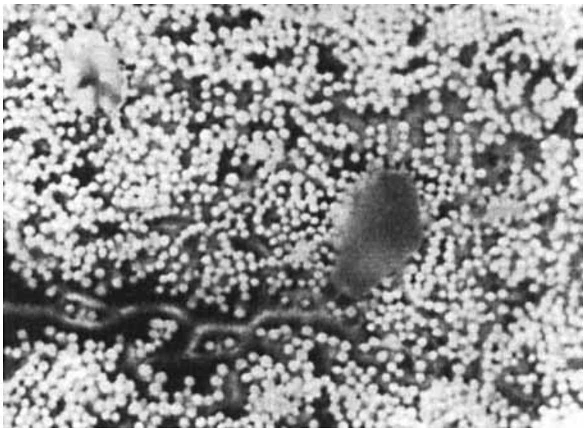

Fig. 4. Microscopic image of carbon/epoxy balanced plain weave composite with V_f = 37.5%, ×1400.

the crimp angles decrease. This is expected since more volume fraction typically means higher pressure in composite manufacturing. The pressure help "flatten" the crimp angle of the yarns. Elastic and thermal properties of carbon fibers and epoxy matrix at room temperature as listed in the literature provided by the manufacturers and used in the modeling process are listed in *Table 3*. It is important to notice that these material properties may change within the temperature range under investigation.

Table 2. Fiber Volume Fraction and Crimp Angles.

Fiber volume fraction (%)	26.8	37.5	47.1	54	55.7
Crimp angle (degrees)	5.53	5.35	5.16	4.95	4.81

Table 3. Properties of Fiber and Matrix.

Material Properties	AS4 Graphite	Epoxy
Young's modulus (GPa)	221	3
Shear modulus (GPa)	13.8	1.11
Poisson's ratio	0.2	0.35
Thermal expansion coefficients		
Axial (mm/mm/°C)	-0.2×10^{-6}	55×10^{-6}
Transverse (mm/mm/°C)	10×10^{-6}	

Predictions using the current model are shown in *Fig. 5*, and compared to experimental data. From this *Figure*, we can see that the theoretical in-plane thermal expansion coefficients are in good agreement with experimental results except the first point. This agreement proves that the existence of interfacial gaps have little to no effect on the in-plane modulus and CTE of the composites.

5. CONCLUSION

In this work, a theoretical model is developed to predict the thermal expansion coefficients for unidirectional fiber reinforced composites. This model is based on a cylindrical composite approach. Stress-strain behavior and temperature distribution in matrix and fibers are used to construct the model. For

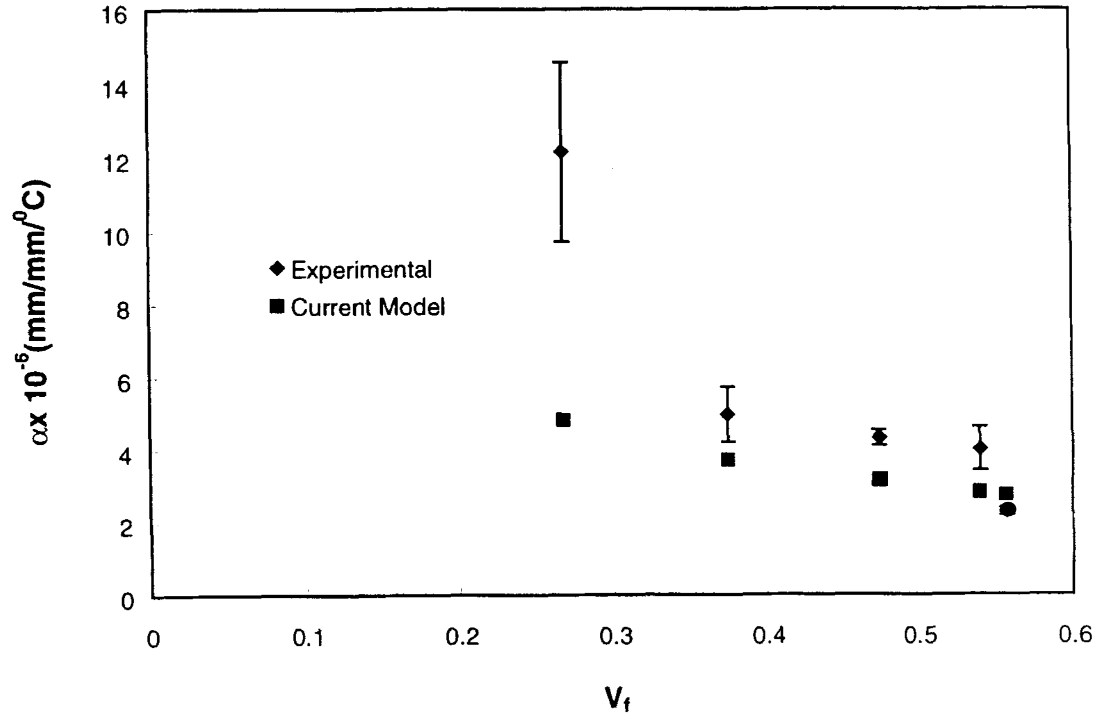

Fig. 5. In-plane CTE for carbon/epoxy balanced plane weave composite with change of volume fraction of fiber V_f in (%).

longitudinal and transverse thermal expansion coefficients, different boundary conditions are employed to solve their stress-strain equations. Furthermore, the effect of voids on the thermal expansion coefficients of composites is also considered via alteration of the properties of matrix material. This new model is embedded within a hybrid finite-element analysis model to calculate thermal expansion coefficients of textile composites.

AS4 graphite/epoxy balanced plain weave composites with five different fiber volume fractions were tested for in-plane and out-of-plane thermal expansion coefficients. For in-plane thermal expansion coefficients, the theoretical predictions matched the experimental results well for both composites. The experimental out-of-plane thermal expansion coefficients for all five different fiber volume fractions were larger than the thermal expansion coefficients of each component of the composite. These large values could be attributed to the interfacial gaps and fiber/matrix debonding that may have occurred during curing of the composite.

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