Journal of Advanced Transportation, Vol. 37, No. 1, pp. 119-138
www.advanced-transport.com

Power Unit – Cargo Space Link In Inland Waterway Navigation

Zoran Radmilovic
Zlatko HRLE
Jasna Muskatirovic

This paper deals with transportation technology regarding links between power unit or motor ship and ship cargo space. These links can be divided into two groups: rigid and flexible. Rigid link, established between power unit and cargo space, is dominant in maritime and road transport (sea ships and trucks), and occasionally in transport on inland waterways (self-propelled barges). Flexible link is used in railroad transport (between the locomotive and railroad units), partially in road transport (systems with trailers and semi-trailers), and in inland waterway transport (push-towing, pulling systems and combinations of these systems).

The main goal of this research is determination of possible link types and organization of fleet for transport on inland waterways in case of flexible link.

Introduction

Rigid link power unit has less exploitation time, since it has to wait along with ship cargo space at loading and unloading points (ports), depending on technology used, transportation process geography, and other operations (customs controls, change in transport conditions, etc.).
In the use of flexible link between power unit and cargo space, possibilities for higher exploitation time of power unit exist. This is true only for time periods during ship cargo space operations. For example, the motorboat and locomotive do not have to wait on tow or railcar units for cargo loading and unloading.

The river fleet, as a function of transport technology, may be presented as a self-propelled motor cargo ship; a system of pushboat and

Zoran Radmilovic, Zlatko Hrle and Jasna Muskatirovic are in the Faculty of Transport and Traffic Engineering, University of Belgrade, Belgrade, Serbia, Yugoslavia
Received: September 2000; Accepted: October 2002

barge tows; pull-tug and pulled barge tows; and a combination of the above mentioned systems, as it is shown in Fig. 1.

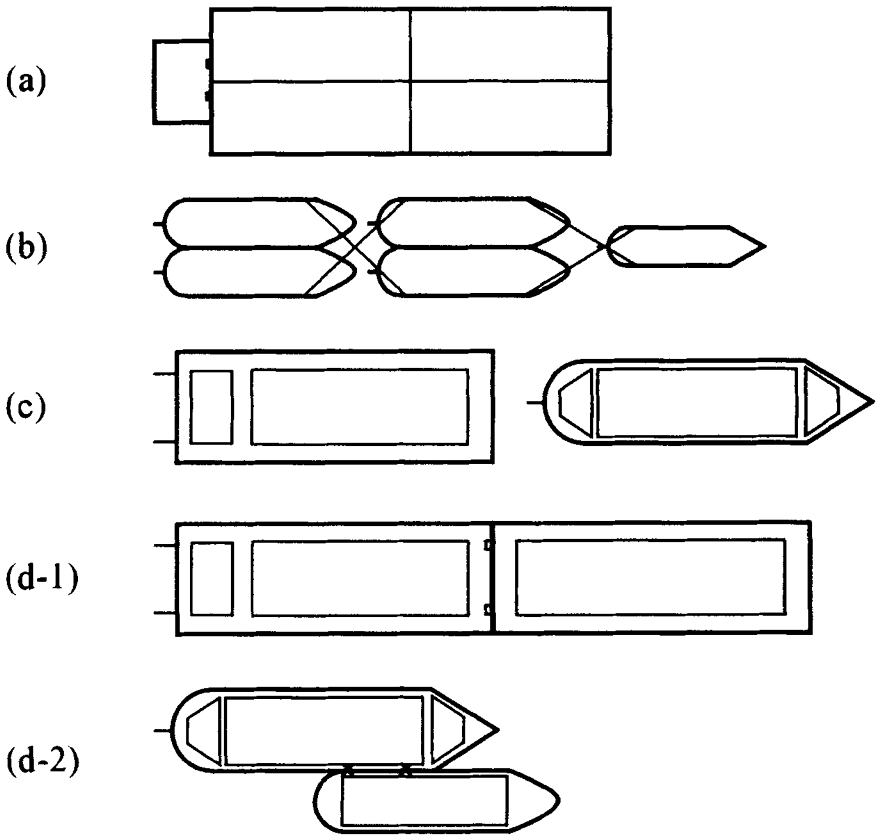

Fig. 1. Transport fleet and types of vessels in inland navigation
(a) - Pushed barge tow with pushboat;
(b) - Pulled barge tow with pull tug;
(c) - Self-propelled barge or motor cargo ship;
(d-1) - Self-propelled pushed barge with pushed barge in tow;
(d-2) -Self-propelled pulled barge with pulled barge in tow

The self-propelled barge system is the simplest transport system, in an organizational sense, since the link that exists between the power unit and the cargo space is rigid. This system is the most frequent by used system in maritime transport, because these ships transport most of the total volume of cargo.

The pull-towing and push-towing systems are similar systems regarding transport operation, since they have non-rigid connections between the power unit (pushboat or pull tug) and the cargo space (pushed and the pulled barge tows).

Power unit - cargo space link in the inland waterway transport can be, according to the current exploitation, divided into the following groups:

1. Non-disrupted link is established between the power unit and cargo space during loading/unloading process and when the ship moves. The pushboat is forced to wait only in cases when navigational or other conditions have been changed. It does not take over other tows.
2. Semi-non-disrupted link is established between the power unit and cargo space, when the pushboat can take over other tows.
3. Disrupted link is understood to be established between the power unit and cargo space in cases when the pushboat changes pushed barge tows at loading/unloading spaces or when the ship is en route.

Two principles are obeyed in the definition of transportation organization. The first principle defines the number of ships or number of tows in operation. The second principle defines the number of barges in tow as a constant value or a variable.

Constant number of barges means the number of barges does not change at loading and unloading points and during the navigation.

The variation in number of barges in tow means the tow size can change at loading/unloading points and during navigation.

Determination of the kind of link between the power unit and cargo space depends on the navigation conditions and characteristics of cargo flows. For example, for longer waterway reaches in the case where navigation conditions can be rapidly changed, decision between non-disrupted or disrupted link should be made having in mind waiting costs of power unit or cargo space. In the case of favorable navigation conditions and when cargo flows are high-tonnage flows, it is necessary to make a decision on the type of link between power unit and cargo space. The type of link should depend on coordination between transport processes, cargo operation, port services, etc., or the ratio between travel time and standing time at the loading and unloading points or ports. The main objective is to ensure maximum exploitation of inland waterway ships per time, cargo capacity, power, and achievement of maximum transportation capacity.

As it is mentioned, most frequent cases in inland waterway transport are:
- First case: coordination of river fleet operations in longer inland waterway sector, and,
- Second case: transport of high-tonnage cargoes such as liquid or/and dry bulk cargoes in shorter distances.

Power Unit – Cargo Space Link as Queueing System with Bulk Arrivals and Single Service

The power unit – cargo space link, such as river pushboat-barge tow link, can be considered as queueing systems with bulk arrivals, single service and unlimited queues at anchorage.

In this paper, bulk arrivals are presented by pushed barge tows. The basic objective of the application of multi-channel bulk queue systems is determination of steady-state probabilities and adequate efficiency measures. The steady-state probabilities refer to the probability that the pushboat is idle and the probability that the time spent in queue by a random barge of an arrival tow is greater than zero depending upon the utilization factor; pushboat occupancy; number of barges in tow; and number of pushboats.

In defining the queueing systems, it is assumed that:

1. The queueing systems are systems with infinite waiting capacity, where the sources of arrival patterns are not the integral parts of a system.
2. Ship arrivals may be either single (self-propelled barges), or in bulk (pushed barge tows), and random arrivals refer to Poisson's distribution.
3. All barge tows wait until served by the pushboat (during the travel time plus waiting times at the anchorage).
4. Service channels are pushboats with similar, or identical, and independent main characteristics. Service times (travel time with barge trains) are independently and identically distributed according to the negative exponential or Erlang's probability distribution, with k=1 (k is the number of phases of Erlang's distribution). In many cases, the service time distribution may differ (Erlang, when k>1; and normal, the same as the general probability distribution). However, the exponential distribution can be used as the first approximation and base for deriving parameters resulting from other distributions, [Radmilovic, 1992; Radmilovic *et al.*, 1996; Radmilovic *et al.*, 1998].

5. The tow size is a random variable. The number of barges in tow has the constant and geometric probability distribution.
6. The queue discipline is "First come, first served" (FCFS) by the tows and random within the tows.

In the extended Kendall's queueing notation, this system is covered by the $M^X/M/c(\infty)$, $M^X/E_K/c(\infty)$ and $M^X/G/c(\infty)$ symbols (Chaudhry, *et al.*, 1983) where:

M - Poisson interarrival and exponential service time distribution,
X - the random variable presenting number of barges in tow
c - number of pushboats
E_k - the Erlang service distribution with phases k, $k \geq 1$, and
G - the general probability distribution of service times.

At the moment, numerical calculation is possible for heterogeneous cases, but analytically explicit results can be achieved only for the constant and geometric distribution of the arrived tow size [Chaudhry *et al.*, 1983; Radmilovic, 1992]. Some explicit results from the $M^X/M/c(\infty)$ system can be extended to the $M^X/E_K/c(\infty)$ system by some of the approximation formulas; for example, by Lee and Longton's and Cosmetatos' formulas, [Noritake *et al.*, 1983].

Determination of Probability That Pushboat is Idle When Number of Barges in Tow is Constant

According to theoretical results, [Kabak, 1970; Chaudhry *et al*, 1983], it is assumed that the input process is the Poisson's process with the constant arrival rate λ for all values of the time t. The tow size X is a random variable, with the distribution given by $a_m = P(X=m)$, $m \geq 1$ (m- number of barges in tow). X has the mean or expected value $\bar{a}$ ($0 < \bar{a} = \sum_{m=1}^{\infty} m \cdot a_m < \infty$), variance ($0 < \sigma_a^2 < \infty$), and probability generating function ($A(Z) = \sum_{m=1}^{\infty} a_m \cdot Z^m$). The service rate for each pushboat in operative mode is μ, so the service rate of the pushboat system is $n\mu$, for $1 \leq n \leq c$, and $c\mu$, for $n \geq c$ (c - number of pushboats). The process of fleet operations is considered to be Markov's process with enumerable states.

In the reviewed queueing system, the limiting case or behavior is treated when the steady-state probabilities can be determined. If the steady-state probability is that the pushboat is idle, P_o, then the other probabilities and operating characteristics of the river push-towing fleet can be calculated, too.

Using mathematical derivations shown in Chaudhry *et al*, (1983) and Radmilovic, (1992), the following equation can be derived:

$$\sum_{n=0}^{c-1}(c-n)\cdot P_n = c(1-\rho) \tag{1}$$

where:

n - number of barges in the river pushboat operation system;

c - number of pushboats;

P_n - steady-state probability that n barges are in the river pushboat operation system;

ρ - utilization factor in queueing theory or pushboat occupancy as defined by [Nicolaou, 1967; Noritake *et al.*, 1983], or

$$\rho = \frac{\lambda \bar{a}}{c\mu} \tag{2}$$

where:

λ - average arrival rate of tows or single barges (barges/day or tows/day);

μ - average service rate of barges (barges/day);

$\bar{a}$ - average number of barges in tow (barges/tow).

From Eq.1, probability P_O can be easily determined, since that steady-state probability has to satisfy the boundary condition. By using Kabak's recurrence formula [Kabak, 1970], it is simple to obtain P_O, since the probabilities P_I, ...P_n are connected by the following relationship:

$$P_n = y(n)\sum_{k=0}^{n-1} P_k A_{n-k} \tag{3}$$

where

$$y(n) = \frac{\lambda}{\mu(n)}, \ \mu(n) = \mu \cdot \min(n,c), \text{ and} \tag{4}$$

$$A_{n-k} = 1 - \sum_{m=0}^{n-k-1} a_m, \ (A_1 = 1) \tag{5}$$

a_m - probability that tow of m barges is present in river fleet operation system.

For example, in the case of a river fleet with four pushboats, the probability P_O is obtained from Eq.1., in form

$$4P_o + 3P_1 + 2P_2 + P_3 = 4(1-\rho) \tag{6}$$

Using equations 3, 4 and 5, and having in mind that pushboat occupancy (Eq.2) can be written in form

$$\rho = \frac{\lambda \overline{a}}{4\mu} \tag{7}$$

probabilities can be written as:

$$P_1 = \frac{4}{\overline{a}} \rho P_o \tag{8a}$$

$$P_2 = \frac{2}{\overline{a}} \rho P_o \left(1 - a_1 + \frac{4}{\overline{a}} \rho \right) \tag{8b}$$

$$P_3 = \frac{4}{3} \frac{1}{\overline{a}} \rho P_o \left(1 - a_1 - a_2 + 6 \frac{\rho}{\overline{a}} (1 - a_1) + \frac{8}{\overline{a}^2} \rho \right) \tag{8c}$$

Finally, P_o can be calculated by substituting (Eq. 8a, b and c) into (Eq.6):

$$P_o = \frac{3\overline{a}^3 (1-\rho)}{8\rho^3 + 6\rho^2 \overline{a}(3 - a_1) + \rho \overline{a}^2 (13 - 4a_1 - a_2) + 3\overline{a}^3} \tag{9}$$

where

a_1, a_2 - probabilities of one self-propelled barge and/or tow with two barges present in the river pushboat operation system.

If in Eq.9, $\bar{a} = a_1 = 1$, and $a_m = 0$, the probability P_O is valid to classic queueing system with delay *M/M/4(∞)* in Kendall's notation.

In Tables 1, 2 and 3 the numerical results of the probability that a pushboat is idle, when the number of barges in tows is constant, are obtained from the $M^{X=\bar{a}} / M / c(\infty)$ system with delay, depending on the number of pushboats (c = 2, 4 and 5), the number of barges in tows ($\bar{a}$ =1, 2, 3, 4, 5, 6, 7, 8, 9 and 10), and the pushboat occupancy (ρ = 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9). The numerical results can easily be extended to other cases based on Eq.1-Eq.5.

Also, in Figs. 2., 3. and 4., these probabilities are presented as a function of pushboat occupancy, number of pushboats, and number of barges in tows, in the case when waiting time (V_q) of a random barge in an arriving tow is greater than zero.

Determination of Probability That Time Spent in Queue by Random Barge of Arrival Tow is Greater Than Zero

If V_q is the waiting time of a random barge in an arriving tow and if a tow containing m barges arrives to find the system in state n, $0 \le n \le c-1$, then for given values of m and n, the probability of immediate service for the barge under consideration is 1 if $1 < m \le c-n$, and $(c-n)/m$ if $m \ge c-n+1$ [Chaudhry, *et al.*, 1983].

Consequently, for given values of m and n,

$$P(V_q = 0|n) = \sum_{m=1}^{c-n} \frac{m \cdot a_m}{\bar{a}} + \sum_{m=c-n+1}^{\infty} \frac{c-n}{m} \frac{m \cdot a_m}{\bar{a}}, \quad 0 \le n \le c-1$$

and

$$P(V_q = 0 \mid n) = \frac{c - n - \sum_{m=1}^{c-n} (c-n-m) \cdot a_m}{\bar{a}} \tag{10}$$

where

$\frac{m \cdot a_m}{\bar{a}}$ the probability that barge under consideration arrives in a tow of size *m*, [Chaudhry *et al.*, 1983].

Since that probability $V_q > 0$ is complement to the probability of $V_q = 0$, the unconditional probability of $V_q > 0$ is

$$P(V_q > 0) = 1 - P(V_q = 0) \tag{11}$$

and

$$P(V_q > 0) = 1 - \sum_{n=0}^{c-1} P(V_q = 0|n) \cdot P_n \tag{12}$$

or by using Eq. 10

$$P(V_q > 0) = 1 - \frac{1}{\bar{a}} \sum_{n=0}^{c-1} \left[c - n - \sum_{m=1}^{c-n} (c - n - m) \cdot a_m \right] P_n \tag{13}$$

If the position of a random barge within the arrived tow is *J*, [Chaudhry *et al.*, 1983], then

$$P(J = j) \equiv r_j = \sum_{m=j}^{\infty} \frac{a_m}{\bar{a}} \tag{14}$$

or

$$P(J = j) = \frac{1 - \sum_{m=1}^{m=j} a_m}{\bar{a}} \tag{15}$$

In the case of a river pushboat operation system with four pushboats, the probability $P(V_q > 0)$, is obtained by substituting Eq.3, Eq.4, Eq.5 and Eq.15 into Eq.13. This can be expressed as:

$$P(V_q>0)=1-\frac{1}{\overline{a}}\left[4-\frac{3}{\overline{a}}-2\frac{1-a_1}{\overline{a}}-\frac{(1-a_1)(1-a_2)}{\overline{a}}\right]P_o-$$
$$-\frac{1}{\overline{a}}\left[3-\frac{2}{\overline{a}}-\frac{1-a_1}{\overline{a}}\right]\frac{4\rho}{\overline{a}}P_o-\frac{1}{\overline{a}}\left(2-\frac{1}{\overline{a}}\right)\cdot\left[\frac{2\rho}{\overline{a}}(1-a_1)+\frac{8\rho^2}{\overline{a}^2}\right]P_o- \quad (16)$$
$$-\frac{1}{\overline{a}}\left[\frac{4\rho}{3\overline{a}}(1-a_1-a_2)+\frac{8\rho^2}{\overline{a}^2}(1-a_1)+\frac{32\rho^3}{3\overline{a}^3}\right]P_o$$

In this case, the probability P_o is determined by Eq.9. If the river pushboat operation system has four pushboats, (c=4), and the number of barges in tow that arrive is $X=const=\overline{a}=6$, the probabilities of appearance of a self-propelled barge or tow with two barges are $a_1=a_2=0$ or consequently, $a_m=0$ except for $m=6$. From Table 2, the probability value of P_o=0.35604 for pushboat occupancy ρ =0.5 is obtained. After substitutions into Eq.16, $P(V_q>0)$=0.7381. This means a randomly selected barge in the tow of six barges has to wait for a pushboat with a high probability of 0.7381.

In Tables 1, 2 and 3, the steady-state probabilities, which represent the time the barge of an arrival tow will spend at the anchorage before being served, were computed depending on the pushboat occupancy (ρ = 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8 and 0.9), the number of pushboats (c= 2,4 and 5) and the number of barges in tows ($\overline{a}$ =1, 2, 3, 4, 5, 6, 7, 8, 9 and 10). Also, in Figs. 2, 3 and 4, these probabilities are presented as a function of pushboat occupancy, number of pushboats, and number of barges in tow.

Table 1. Probability that pushboat is idle (P_o) and probability that time spent in queue by random barge of arrived tow is greater then zero $(P(V_q>0))$ $M^x/M/2(\infty)$ delayed system, $X=\bar{a}$ =const.

ρ	$\bar{a}$	1	2	3	4	5	6	7	8	9	10
0.1	P_o	0.8182	0.8571	0.8710	0.8781	0.8823	0.8852	0.8873	0.8889	0.8901	0.8911
	$P(V_q>0)$	0.0182	0.3143	0.4968	0.6049	0.6753	0.7246	0.7610	0.7889	0.8110	0.8289
0.2	P_o	0.6667	0.7273	0.7500	0.7619	0.7692	0.7742	0.7778	0.7805	0.7826	0.7843
	$P(V_q>0)$	0.0667	0.3818	0.5500	0.6476	0.7108	0.7548	0.7873	0.8122	0.8319	0.8478
0.3	P_o	0.5385	0.6087	0.6364	0.6512	0.6604	0.6667	0.6712	0.6747	0.6770	0.6796
	$P(V_q>0)$	0.1385	0.4522	0.6040	0.6907	0.7464	0.7852	0.8137	0.8355	0.8528	0.8668
0.4	P_o	0.4286	0.5000	0.5294	0.5454	0.5555	0.5625	0.5676	0.5714	0.5745	0.5769
	$P(V_q>0)$	0.2286	0.5250	0.6588	0.7341	0.7822	0.8156	0.8402	0.8589	0.8737	0.8858
0.5	P_o	0.3333	0.4000	0.4286	0.4444	0.4545	0.4615	0.4667	0.4706	0.4737	0.4762
	$P(V_q>0)$	0.3333	0.6000	0.7143	0.7778	0.8182	0.8461	0.8668	0.8823	0.8947	0.9048
0.6	P_o	0.2500	0.3077	0.3333	0.3478	0.3571	0.3636	0.3684	0.3721	0.3750	0.3773
	$P(V_q>0)$	0.4500	0.6770	0.7704	0.8218	0.8543	0.8768	0.8932	0.9058	0.9157	0.9238
0.7	P_o	0.1765	0.2220	0.2432	0.2553	0.2632	0.2687	0.2727	0.2759	0.2783	0.2804
	$P(V_q>0)$	0.5765	0.7556	0.8270	0.8660	0.8905	0.9075	0.9199	0.9293	0.9368	09428
0.8	P_o	0.1111	0.1429	0.1579	0.1668	0.1724	0.1765	0.1795	0.1818	0.1837	0.1852
	$P(V_q>0)$	0.7111	0.8357	0.8842	0.9104	0.9269	0.9382	0.9465	0.9528	0.9578	0.9618
0.9	P_o	0.0526	0.0690	0.0769	0.0816	0.0847	0.0870	0.0886	0.0899	0.0909	0.0917
	$P(V_q>0)$	0.8526	0.9172	0.9419	0.9551	0.9634	0.9691	0.973	0.9784	0.9789	0.9809

Table 2. Probability that pushboat is idle (P_o) and probability that time spent in queue by random barge of arrived tow is greater then zero ($P(V_q>0)$) $M^x/M/4(\infty)$ delayed system, $X=\bar{a}$ =const.

ρ	$\bar{a}$	1	2	3	4	5	6	7	8	9	10
0.1	P_o	0.6703	0.7405	0.7818	0.8093	0.8264	0.8381	0.8465	0.8530	0.8580	0.8621
	$P(V_q>0)$	0.0008	0.2590	0.3625	0.4214	0.4877	0.5452	0.5929	0.6323	0.6652	0.6929
0.2	P_o	0.4491	0.5469	0.6077	0.6494	0.6762	0.6949	0.7087	0.7193	0.7277	0.7346
	$P(V_q>0)$	0.0096	0.2860	0.4039	0.4727	0.5378	0.5917	0.6355	0.6714	0.7012	0.7261
0.3	P_o	0.3002	0.4014	0.4679	0.5147	0.5460	0.5682	0.5848	0.5977	0.6081	0.6165
	$P(V_q>0)$	0.0370	0.3305	0.4556	0.5283	0.5900	0.6394	0.6789	0.7110	0.7375	0.7596
0.4	P_o	0.1993	0.2911	0.3549	0.4011	0.4328	0.4558	0.4733	0.4870	0.4981	0.5071
	$P(V_q>0)$	0.0907	0.3909	0.5158	0.5875	0.6441	0.6882	0.7231	0.7511	0.7741	0.7933
0.5	P_o	0.1304	0.2069	0.2630	0.3048	0.3342	0.3560	0.3729	0.3861	0.3969	0.4058
	$P(V_q>0)$	0.1739	0.4655	0.5833	0.6500	0.7000	0.7381	0.7679	0.7917	0.8111	0.8273
0.6	P_o	0.0831	0.1422	0.1880	0.2230	0.2483	0.2674	0.2822	0.2941	0.3039	0.3120
	$P(V_q>0)$	0.2870	0.5528	0.6571	0.7154	0.7574	0.7889	0.8132	0.8326	0.8484	0.8614
0.7	P_o	0.0502	0.0923	0.1265	0.1533	0.1732	0.1885	0.2005	0.2102	0.2182	0.2249
	$P(V_q>0)$	0.4286	0.6511	0.7364	0.7833	0.8162	0.8405	0.8592	0.8739	0.8859	0.8958
0.8	P_o	0.0273	0.0536	0.0760	0.0940	0.1076	0.1183	0.1268	0.1337	0.1394	0.1443
	$P(V_q>0)$	0.5964	0.7592	0.8203	0.8536	0.8764	0.8930	0.9056	0.9156	0.9237	0.9303
0.9	P_o	0.0113	0.0235	0.0343	0.0433	0.0502	0.0557	0.0602	0.0638	0.0668	0.0694
	$P(V_q>0)$	0.7877	0.8759	0.9083	0.9259	0.9376	0.9461	0.9526	0.9576	0.9617	0.9651

Table 3. Probability that pushboat is idle (P_o) and probability that time spent in queue by random barge of arrived tow is greater then zero $(P(V_q>0))$ $M^x/M/5(\infty)$ delayed system, $X=\bar{a}$ =const.

ρ \ $\bar{a}$		1	2	3	4	5	6	7	8	9	10
0.1	P_o	0.6065	0.6872	0.7362	0.7693	0.7934	0.8100	0.8221	0.8314	0.8387	0.8445
	$P(V_q>0)$	0.0002	0.2537	0.3476	0.4044	0.4429	0.4900	0.5345	0.5739	0.6081	0.6378
0.2	P_o	0.3678	0.4716	0.5401	0.5884	0.6244	0.6500	0.6690	0.6838	0.6956	0.7052
	$P(V_q>0)$	0.0038	0.2699	0.3771	0.4444	0.4901	0.5385	0.5809	0.6176	0.6491	0.6762
0.3	P_o	0.2228	0.3223	0.3933	0.4456	0.4856	0.5148	0.5371	0.5546	0.5687	0.5803
	$P(V_q>0)$	0.0201	0.3030	0.4206	0.4934	0.5435	0.5895	0.6290	0.6625	0.6909	0.7151
0.4	P_o	0.1343	0.2182	0.2828	0.3325	0.3714	0.4000	0.4232	0.4413	0.4560	0.4683
	$P(V_q>0)$	0.0597	0.3545	0.4764	0.5500	0.6000	0.6429	0.6786	0.7083	0.7333	0.7545
0.5	P_o	0.0801	0.1452	0.1992	0.2424	0.2771	0.3037	0.3248	0.3419	0.3559	0.3677
	$P(V_q>0)$	0.1304	0.4243	0.5423	0.6131	0.6600	0.6983	0.7294	0.7551	0.7764	0.7944
0.6	P_o	0.0466	0.0937	0.1356	0.1704	0.1991	0.2216	0.2398	0.2546	0.2670	0.2775
	$P(V_q>0)$	0.2361	0.5112	0.6196	0.6818	0.7231	0.7556	0.7815	0.8026	0.8201	0.8348
0.7	P_o	0.0259	0.0573	0.0871	0.1128	0.1345	0.1520	0.1662	0.1781	0.1880	0.1965
	$P(V_q>0)$	0.3778	0.6137	0.7044	0.7554	0.7889	0.8145	0.8347	0.8510	0.8643	0.8755
0.8	P_o	0.0130	0.0314	0.0500	0.0667	0.0810	0.0928	0.1026	0.1108	0.1178	0.1238
	$P(V_q>0)$	0.5541	0.7304	0.7967	0.8333	0.8571	0.8750	0.8889	0.9000	0.9091	0.9167
0.9	P_o	0.0050	0.0130	0.0217	0.0297	0.0367	0.0426	0.0476	0.0518	0.0554	0.0585
	$P(V_q>0)$	0.7625	0.8596	0.8954	0.9150	0.9276	0.9369	0.9440	0.9497	0.9543	0.9582

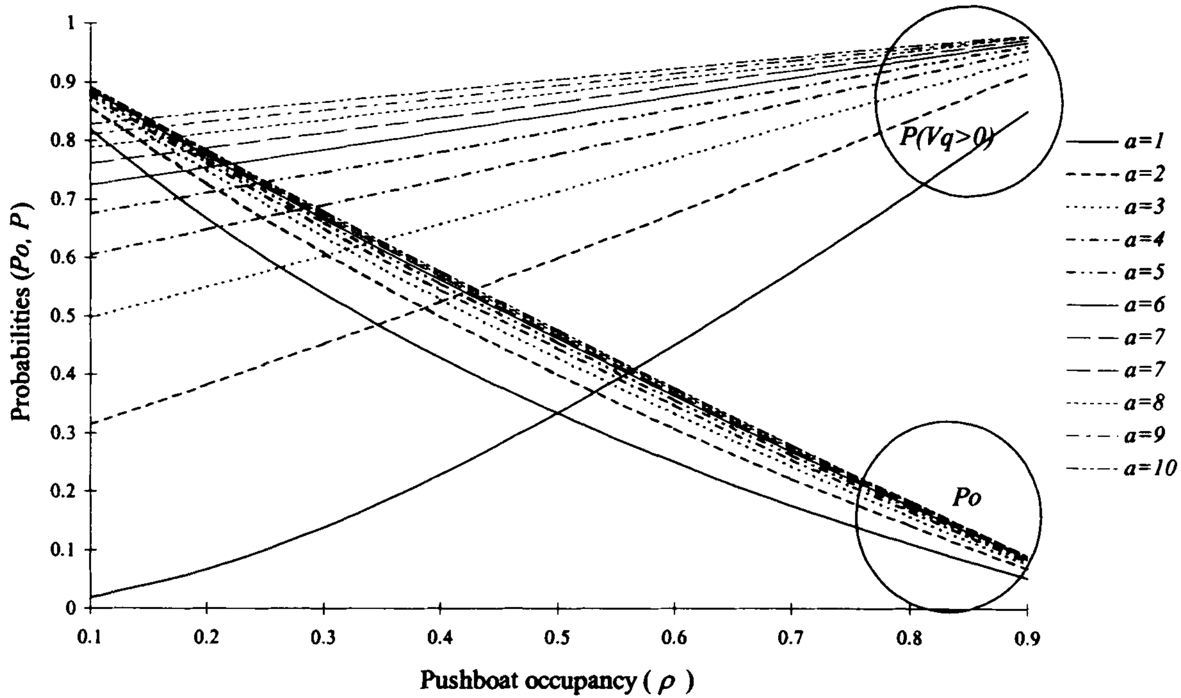

Fig.2. Queueing analysis solutions for probability that pushboat is idle (P_o) and probability that time spent in queue by random barge of arrived tow is greater then zero *($P(V_q>0)$) $M^X/M/2(\infty)$, $X=\bar{a}$* =1,2,3,4,5,6,7,8,9,10, delayed system

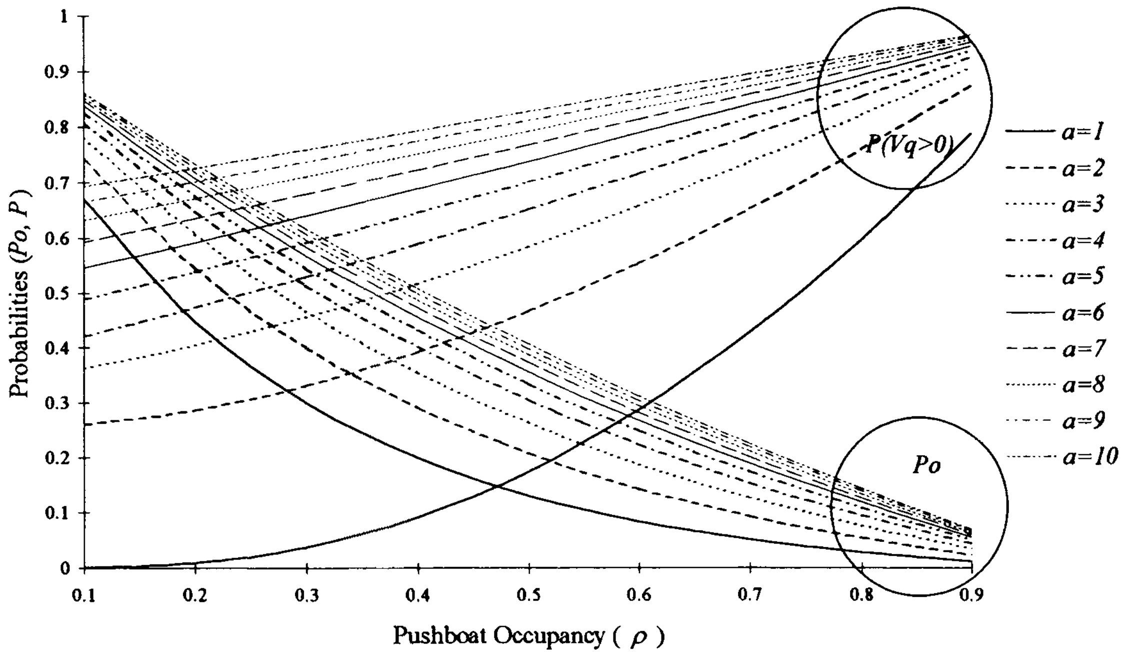

Fig. 3. Queueing analysis solutions for probability that pushboat is idle (P_o) and probability that time spent in queue by random barge of arrived tow is greater then zero ($P(V_q>0)$) $M^x/M/4(\infty)$, $X=\overline{a}=1,2,3,4,5,6,7,8,9,10,$ delayed system

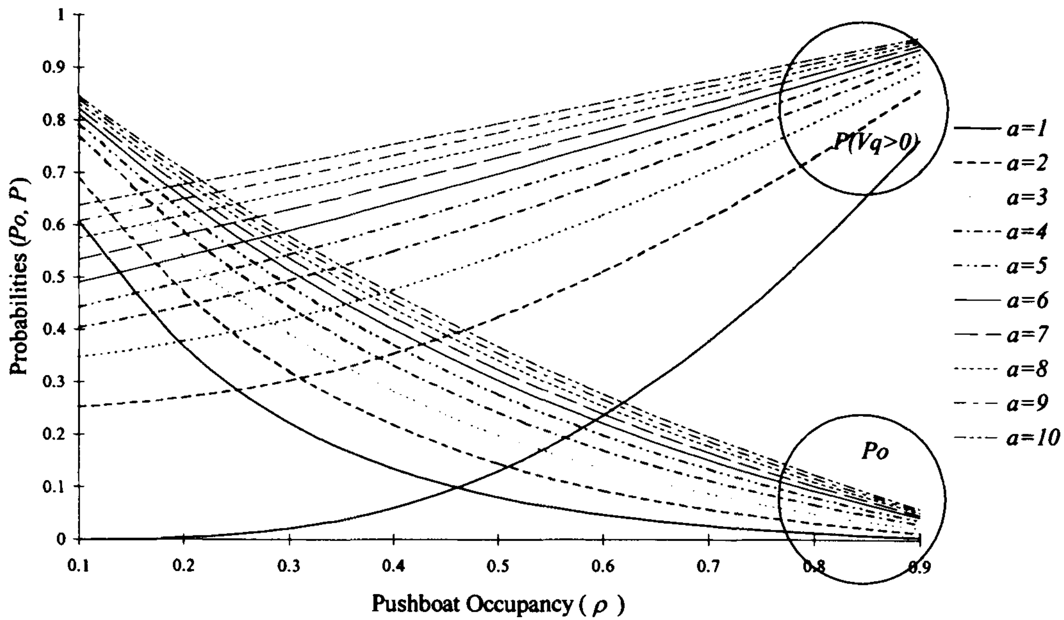

Fig. 4. Queueing analysis solutions for probability that pushboat is idle (P_o) and probability that time spent in queue by random barge of arrived tow is greater then zero $(P(V_q>0))$ $M^X/M/5(\infty)$, $X=\bar{a}$ =1,2,3,4,5,6,7,8,9,10, delayed system

Conclusions

The main conclusions, related to the application of bulk-arrival queueing systems in power unit – cargo space link, are as follow:

1. The power unit – cargo space link, such as river pushboat-barge tow link, is observed as a queueing system with delay, in the push-towing system. This link may be non-disrupted, semi-non-rupted, or disrupted, in the push-towing operations.
2. Up to now, explicit results of the probability that the pushboat is idle and the probability that time spent in queue by random barge of arrival tow is greater than zero, were obtained by the $M^{X=const}/M/c(\infty)$ queueing system. However, the $M^{X}/M/c(\infty)$ system with delay can be used to obtain some other operating parameters, such as: a) the average number of barges in queue; b) average number of barges with the pushboat or the average number of busy pushboats; c) the mean waiting time of barges at the anchorage; or d) the average waiting time of barges in queue in units of average service time or waiting time/service time ratio, etc.
3. Diagrams shown in Figs. 2, 3 and 4, and computational results in Tables 1, 2 and 3, could be used for the analysis of pushboat cycle time and barge tow cycle time, depending on the tow size, mean arrival rate, average service time, number of pushboats and pushboat occupancy. For different combinations of input values, one can easily reach the solutions for the analysis of pushboats' and barge-tows' cost.
4. The computational results and diagrams could be used for the service planning, which are provided by pull tugs and pulled barge tows.
5. With the constant pushboat occupancy and the increase of number of barges in tow, the probability that the time spent in queue by random barge of arrival tow is greater than zero ($P(V_q>0)$), increases on a higher rate than the probability that the pushboat is idle (P_O).
6. With a smaller number of barges in tow or a smaller tow size, the probabilities P_O and $P(V_q>0)$ decrease at a higher rate with the increase of the number of pushboats, as presented in Tables 1, 2 and 3 and in Figs. 2, 3 and 4.
7. The probabilities P_O and $P(V_q>0)$ can be used to find the optimal number of pushboats and the optimal tow size depending on the associated cost of pushboats and barge tows.

Presented models and results are convenient for different analyses, planning and development of river pushboat operations, particularly for push-towing and pull-towing systems. Nevertheless, the conveniences of this methodology are the simple application and estimates of existing conditions and the planning of pushboat requirement, river fleet management, and better decision-making for dispatchers and managers in river shipping companies.

The results obtained in this paper are restrictive since that the assumptions about interarrival and service time distributions, as well as the tow size probability distribution, must be verified before application in river pushboat operation systems. In other words, the variables that affect these results are numerous and it is not possible to take all of them into account.

References

Chaudhry, M.L., Templeton, J.G.C., 1983, *A First Course in Bulk Queues*, John Wiley and Sons, Inc., New York

Frankel, E.G., 1977, *Criteria and Requirements for Port Development and Operations -Models and Analytical Techniques*, Massachusetts Institute of Technology, Cambrige, Mass.

Hrle, Z., 1996, *Contribution to Determination of River Fleet Transportation Capacity*, Ph.D. thesis, Faculty of Transport and Traffic Engineering, Belgrade, (in Serbian)

Kabak, I.W., 1970, Blocking and delays in $M^{(x)}/M/c$ bulk arrival queueing systems, *Management Science*, 17(1), pp. 112-115

Nicolaou, S.N., 1967, Berth planning by evaluation of congestion and cost, *Journal of Waterways and Harbour Division*, ASCE, 93(4), pp. 107-132.

Noritake, M., Kimura, S., 1983, Optimum number and capacity of seaport berths, *Journal of Waterway, Port, Coastal and Ocean Engineering*, ASCE, 109(3), pp. 323-339.

Radmilovic, Z.R., 1992, Ship-berth link as bulk queueing system in ports, *Journal of Waterway, Port, Coastal and Ocean Engineering*, ASCE 118(5), pp. 474-495

Radmilovic, Z.R., Colic, V., Hrle, Z., 1996, Some aspects of storage and bulk queueing systems in transport operations, *Transportation Planning and Technology*, Vol. 20, pp. 67-81.

Radmilovic, Z., Colic, V., Hrle, Z., Muškatirovic, J., 1998, *A River Pushboat-Barge Tow Link as Bulk Queueing System*, 8th World Conference on Transport Research, Book of Abstracts, Antwerp, p. 51

Appendix

The following symbols are used in this paper:

A_{n-k}	– variable in Kabak's reccurent relation defined by (5);
$A(Z)$	– probability generating function of a_m;
$\bar{a}$	– mean or average number of barges in tow or arrival tow size;
a_m	– probability distribution function of arrival tow size, $m \geq 1$;
c	– number of pushboats;
E_k	– Erlang distribution with phases k, $k \geq 1$;
G	– general probability distribution of service times;
k	– number of phases of Erlang distribution;
M	– Poisson interarrival and exponential service time distribution;
m	– number of barges in tow;
n	– number of barges in the river pushboat operation system;
P_n	– steady-state probability that n barges are in the river pushboat operation system;
P_0	– steady-state probability that river pushboat operation system is idle;
$P(J=j)$	– probability that the position of the random barge within the arrival tow is *J*;
$P(V_q=0)$	– probability that time spent in queue by random barge of arrival tow is zero;
$P(V_q>0)$	– probability that time spent in queue by random barge of arrival tow is greater zero;
$(V_q=0/n)$	– conditional probability that time spend in queue by random barge is zero with *n* barges in river pushboat operation system;
X	– random variable of number of barges in tow;
$y(n)$	– variable in Kabak's recurrent relation defined by (4);
V_q	– waiting of a random barge in an arriving tow;
Z	– complex variable;
λ	– mean arrival rate of tows or self-propelled barge;
μ	– mean service time of barge tows in river pushboat operation system;
ρ	– pushboat occupancy or utilization factor in queueing theory;
σ^2_a	– variance of random variable of arrival tow size or number of barges in tow, *X*.