

# **Generic Model Controller Tuning for Chemical Processes with Input Saturation**

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Any chemical processes are nonlinear and contain operating constraints, especially input saturation constraints. In recent years, a number of nonlinear control technologies have been developed, such as nonlinear control, based on the differential geometric approach (Kravaris and Kantor, 1990), nonlinear model predictive control (Patwardhan et al., 1990) and generic model control (GMC) (Lee and Sullivan, 1988). Here we are interested in using the GMC controller, which was developed with the specific objective of incorporating the nonlinear process model directly into the control algorithm. The formulation of the GMC controller is relatively straightforward. Importantly, this approach has several advantages that are desirable for designing nonlinear control systems:

- 1. The nonlinear (reduced) process model is directly incorporated in the control algorithm, allowing for the inherent nonlinearity of processes to be taken into account;
- The relationship between feedforward and feedback control is explicitly accounted for in the GMC algorithm.

For processes without constraints, tuning for the GMC controller is simple. In this case, the shape and speed of the response to a setpoint change need to be chosen by the user. That is, the tuning procedure proposed by Lee and Sullivan (1988) is to first choose the desired shape of the response and then set the timing of the response in relation to the known or estimated plant speed of response (Lee, 1993). But for constrained processes, this tuning approach is not suitable. Brown et al. (1990) proposed a nonlinear programming (NLP) algorithm to handle the constraint problem for input, state and output variables. But this algorithm is not easy for process engineers to use.

Flathouse and Riggs (1996) proposed an auto tune variation (ATV) tuning procedure for GMC controllers without constraints. Since the ATV procedure measures the process response at the cross-over frequency, ATV testing is not possible for a pure first-order process or an integrating process, neither of which has a cross-over frequency.

This paper examines GMC controller tuning with input saturation constraints, since input saturation is very common in chemical processes with flow control valves. It is arranged as follows. Firstly, a general form of nonlinear model, with a discrete GMC control algorithm is discussed. GMC controller tuning with input saturation constraints is then developed. The following section discusses an experimental application of this strategy to a laboratory pressure tank. Finally, some conclusions are given. Control in the face of process input constraints is very common and of great practical importance in the processing industries. Generic Model Control (GMC) is a model-based control framework for both linear and nonlinear systems. In this paper, a constrained GMC controller tuning approach using a nonlinear least squares technique is proposed. This tuning approach is simple to apply. For a SISO GMC control system with input saturation, the tracking performance is significantly improved by adding a simple heuristic switching strategy. The effectiveness of the proposed controller tuning approach is demonstrated using dynamic simulations and MIMO real-time experiments.

La régulation des procédés sujette aux contraintes d'entrée est très répandue et d'un grand intérêt pratique dans les industries de procédés. Le contrôle par modèles génériques (GMC) est un cadre de régulation basé sur des modèles pour des systèmes linéaires et non linéaires. Dans cet article, on propose une méthode de réglage des contrôleurs GMC qui fait appel à une technique de moindres carrés non linéaires. Cette méthode de réglage est simple à appliquer. Pour un système de contrôle SISO de type GMC avec saturation à l'entrée, la performance du suivi est considérablement améliorée par l'ajout d'une stratégie de commutation heuristique simple. L'efficacité de cette méthode de réglage des contrôleurs est démontrée par des simulations dynamiques et des expériences MIMO en temps réel.

Keywords: controller tuning, generic model control, nonlinear control, nonlinear least squares.

### Nonlinear Control Strategy with Input Saturation Control Algorithm

We consider a nonlinear system described by differential equations of the type:

$$\dot{x} = f(x, u, d)$$

$$y = h(x)$$
(1)

where x is state vector of dimension n, u is input vector of dimension m, d the disturbance vector of

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suitable dimension, and y is the output vector of suitable dimension. According to the GMC basic principle (Lee and Sullivan, 1988), we develop the control algorithm, which consists of three terms (dynamic process model, proportional action term and integral action term, respectively)

$$\hat{H}_{x}\hat{f}(x,u,d) - K_{1}(y_{sp} - y) - K_{2}\int_{0}^{t} (y_{sp} - y)dt = 0$$
<sup>(2)</sup>

where  $\hat{H}_x = \partial \hat{h} / \partial x$ ,  $\hat{f}$  and  $\hat{h}$  represent the approximation to the true model (Equation 1),  $y_{sp}$  is the setpoint of the output,  $K_1$  and  $K_2$  are diagonal n × n tuning parameter matrices. The control strategy (Equation 2) directly imbeds an approximate dynamic nonlinear model.

The control algorithm (2) is generally implicit. Hence it is solved on-line by some iterative numerical method. If  $\hat{f}(x,u,d)$  is linear with respect to u, e.g.,  $\hat{f}(x,u,d)$ , =  $\hat{g}(x,d)u$ , then Equation (2) becomes an explicit control algorithm:

$$u = \left[\hat{H}_{x}\hat{g}(x,d)\right]^{-1} - \left[K_{1}(\gamma_{sp} - \gamma) + K_{2}\int_{0}^{t} (\gamma_{sp} - \gamma)dt\right]$$
(3)

As written, Equation (3) gives the continuous form of the GMC algorithm. In order to use GMC in a discrete system, the integral must be evaluated numerically using a suitable approximation. Therefore, a discrete time version of Equation (3) is:

$$u_{k} = \left[\hat{H}_{x}\hat{g}(x_{k}, d_{k})\right]^{-1} \left[K_{1}(y_{sp} - y_{k}) + K_{2}\int_{0}^{k} (y_{sp} - y_{k})\Delta t\right]$$
(4)

where  $\Delta t$  is the sampling interval.

The values of the tuning elements of  $K_1$  and  $K_2$  can be determined by the relationships (Signal et. al, 1992):

$$K_{1(i,i)} = \frac{2\xi_i}{\tau_i}$$
  $K_{2(i,i)} = \frac{1}{\tau_i^2}$  (5)

 $\xi_i$  and  $\tau_i$  determine the shape and speed of the reference trajectory, respectively. The reference trajectory for a step change in the setpoint has a pseudo-second-order response. Yamuna et al. (1991) showed that the formula can be used to accurately calculate the reference trajectory for any values of  $\xi_i$  and  $\tau_i$ . If the model is perfect and there are no constraints, provided that a sufficiently small sampling interval  $\Delta t$  is chosen, the closed-loop response will follow the reference trajectory, that is, the close-loop system will have a time constant  $\tau_i$  and a damping coefficient  $\xi_i$ .

For unconstrained processes, tuning is simple. That is, the shape and speed of the response to a setpoint change can be chosen by the user. When particular values of  $\xi_i$  and  $\tau_i$ , which correspond to the reference trajectory, are selected,  $K_1$  and  $K_2$  can be easily computed from Equation (5). But for constrained processes, this tuning approach is not effective and an alternative method which explicitly handles the constraints is required.

#### **Constrained Tuning Using Least Squares**

Input saturation found in many chemical processes can be expressed as:

$$u_{L} \le u \le u_{U} \tag{6}$$



Figure 1. Control system with input saturation.

This type of constraint is usually caused by the physical limitations of the equipment, such as valves.

A control system with saturation is shown in Figure 1. If the GMC algorithm is directly used for this system with the input constraints, these constraints will limit the control action determined by Equation (2), resulting in a slower output response than expected.

We can apply ISE (Integral of the Square Error) as a measure of controller performance.

$$ISE = \int_{0}^{t_{f}} e^{2} dt \tag{7}$$

where, e is error shown in Figure 1. The discrete form of Equation (7) is:

$$ISE = \sum_{k=0}^{N} e_k^2 \Delta t \tag{8}$$

where N is the final simulation time. We can apply nonlinear least squares to minimize the following constrained objective, which gives rise to an increase in controller performance:

$$J = \min_{K1, K2} \sum_{K=0}^{N} e_k^2 \Delta t$$
 (9)

with  $u_L \le u \le u_{U'}$  and  $K_1$  and  $K_2$  are the two tuning parameters for the GMC controller. Because of the fact that we need to minimize *J* over all time steps from 0 to *N*, this is a multiobjective optimal problem. We can select initial values using the tuning procedure proposed by Lee and Sullivan (1988). This nonlinear least squares problem is solved by the following procedures in the MATLAB and SIMULNK environment:

- 1. Develop a process model for the controlled system;
- 2. Construct a GMC control algorithm;
- 3. Implement the nonlinear least squares strategy with a calculation of the control criterion as shown in Equation (9);
- 4. Choose the initial values of the matrices  $K_1$  and  $K_2$ .
- 5. Run the closed-loop control system to achieve an optimal tuning of the parameters in the control algorithm.

#### A Heuristic Switching Strategy

For the tracking problem, input constraints simply limit the control action determined by Equation (2), which results in a slow output response. For the SISO system we can improve the control performance by a simple heuristic switching strategy. When an increase in the setpoint gives rise to an error larger than some reference switch value, we set the control action to



Figure 2a. Comparison results of three control schemes.

its upper saturation value  $u = u_{U'}$  where u is the controller output, otherwise, we have  $u = u_{GMC'}$  where  $u_{GMC}$  is the GMC controller output. If the setpoint decreases and the error is less than some reference switch value, then we have  $u = u_L$ , otherwise, we set the controller output equal to the GMC controller output.

The reference error switch value used to switch between these various controller outputs, is itself a tuning variable. However, from our experience it is easy to select this value by trial and error, and a precise value is not required. We use  $e_{switch} \cong 0.02 \gamma_{sp}$  as an initial guess, and adjust online.

For MIMO systems this simple heuristic switching strategy is difficult to apply, because MIMO systems can be highly coupled. So for the MIMO control algorithm we used the least squares tuning values without heuristic switching.

#### Simulation Studies

The following SISO system is used to test the optimal tuning and the heuristic switching strategy of GMC with input constraints on the control variable.

Consider the nonlinear process:

 $\dot{y} = -0.25y^2 + 0.5u + 1.75uy \tag{10}$ 

that has been modeled as:

$$\dot{y}_m = -0.25 y_m^2 + 0.5 u + 1.75 u y_m \tag{11}$$

that is, there is no mismatch for model and process.

The input saturation control values are that the maximum value is 0.3 and the minimum value is -0.3. We used MATLAB/SIMULINK to solve the nonlinear least squares problem and to provide optimal tuning used for the GMC controller. The sampling interval  $\Delta_t$  was chosen as 0.1. The optimal tuning results we obtained as  $K_{1LS} = 0.5013$  and  $K_{2LS} = 0.1015$ .

For comparison, we calculated the conventional unconstrained GMC controller parameters according to the tuning procedure proposed by Lee and Sullivan (1988). We chose  $\xi = 40$  and  $\tau = 6$ , that is,  $K_{1G} = 0.3$  and  $K_{2G} = 6.25 \times 10^{-4}$ .



Figure 2b. Control action of three control schemes.

We made the comparison with 3 cases to evaluate the effectiveness of this proposed tuning strategy. The first case shows a unit step response using the standard GMC controller tuning without any special consideration given to constraints. We labeled this GMC in Figures 2a and 2b. The second case uses optimal least squares parameters but without the heuristic switching strategy added. This result is labeled GMC\_LS. The last case uses the tuning parameters from case 1 and adds the heuristic switching strategy. The controller output maximum was  $u_U = 0.3$  and the reference switching error calculated as above to be  $e_{switch} = 0.02$ . This result is labeled GMC\_H. It is clearly superior to the previous two results, and especially to the standard case GMC, which uses the tuning procedure proposed by Lee and Sullivan (1988) and gives rise to a sluggish response. The GMC\_LS result is also superior to the standard case GMC.

# Nonlinear Control of a Pressure Tank Pressure Tank

The laboratory system is a pressure tank (Figure 3) through which the air flows from a regulated supply. Control valves are installed on both the inlet and the outlet of the tank. The pressure in the tank and the outlet flow rate are measured and transmitted to a computer. Data collection and system control are accomplished by use of a microcomputer with a Data Translation input-output (I/O) interface board.

Here the pressure in the tank and the outlet flow rate are controlled variables, and the control valve stem positions on both the inlet and the outlet of the tank are the manipulated



Figure 3. Lab pressure tank.

variables. This is a two-input and two-output system. The variation in both inputs will influence both controlled variables. From the process design, it can be seen that this is a strongly interactive system. In fact, a Relative Gain Analysis (RGA) of the process shows that the RGA elements are all close to . This implies that a control scheme based on multi-loop PID controllers would be very difficult to achieve (Zhu and Jutan, 1994).

Lee et al. (1979) described the pressure tank model. So we can easily give the physical model:

$$\dot{m} = \frac{C_{vmax}l}{41616}\sqrt{\rho(-\Delta P)}$$
(12)

where  $\rho$  and  $\Delta P$  are density of air and differential pressure respectively, and *I* is the valve opening. We give:

$$\dot{P}_{v} = \frac{K_{vi}l_{i}}{V}\sqrt{\frac{RT}{M}}\sqrt{P_{v}(P_{i}-P_{v})} - \frac{K_{vo}l_{o}}{V}\sqrt{\frac{RT}{M}}\sqrt{P_{o}(P_{v}-P_{o})}$$
(13)

where  $K_{vi} = C_{vimax}/41616$  is a flow parameter for the inlet valve, and  $K_{vo} = C_{vomax}/41616$  is a flow parameter for the outlet valve.  $P_v$ ,  $P_i$  and  $P_o$ , are the pressures in the pressure tank, inlet and outlet, respectively, and  $I_i$  and  $I_o$  are the fractional openings of the inlet valve and the outlet valve, respectively. Applying Equations (12) and (13), we can obtain an expression for the volumetric flowrate at the outlet:

$$\dot{V}_{o} = K_{vo} I_{o} \sqrt{\frac{RT}{M}} \sqrt{\frac{P_{v} - P_{o}}{P_{o}}}$$
(14)

where  $V_0$  is the volumetric flowrate at the outlet.

#### Nonlinear Control for the Pressure Tank

The controlled variables are, pressure in the pressure tank and the volumetric flowrate at the outlet. The manipulated variables are the fractional openings of the inlet valve and the outlet valve. The input saturation values are the maximum opening 100% and the minimum opening 0%. The relationship between the controlled variables and valve opening is nonlinear, moreover,



Figure 4a. Control action of simulated pressure tank with setpoint change on both pressure and flowrate.

Table 1. Process data and tuning parameters.

R = 8314.4 J/kmol·K $P_o = 101 \text{ kPa}$  $T = 25^{\circ}\text{C}$  $P_i = 377 \text{ kPa}$ M = 29 kg/kmol $V = 0.0215 \text{ m}^3$  $K_{11} = 1.1488 \text{ s}^{-1}$  $K_{21} = 0.3622 \text{ s}^{-1}$  $K_{12} = 0.0011 \text{ s}^{-2}$  $K_{22} = 6.0454 \times 10^{-5} \text{ s}^{-2}$  $K_{vo} = 3.82 \times 10^{-6} \text{ (kg/s)}(\text{m}^3/\text{kg}\cdot\text{Pa})^{1/2}$  $\Delta t = 1 \text{ s}$ 

there is also severe hysteresis in the valves which is not directly modeled. This pressure tank represents a challenging multivariable real time control problem for our algorithm.

The GMC controller for the pressure tank can be obtained by applying Equations (3), (13) and (14):

$$L_{k} = G^{-1} \left[ K_{1} (\gamma_{sp} - \gamma_{p}) + K_{2} \sum_{0}^{k} (\gamma_{sp} - \gamma_{p}) \Delta t \right]$$
(15)

where  $L_{K}$  and  $y_{p}$  are the outputs of the controller and the pressure tank, respectively.  $K_{1}$  and  $K_{2}$  are tuning constant matrices.  $\Delta_{t}$  is sampling interval. So we have:

$$L_{k} = \begin{bmatrix} I_{i} \\ I_{o} \end{bmatrix}, \quad \gamma_{p} = \begin{bmatrix} P_{v} \\ \dot{V}_{o} \end{bmatrix}, \quad K_{1} = \begin{bmatrix} k_{11} & 0 \\ 0 & k_{12} \end{bmatrix}, \quad K_{2} = \begin{bmatrix} k_{21} & 0 \\ 0 & k_{22} \end{bmatrix}$$

$$G = \begin{bmatrix} \frac{K_{vi}}{V} \sqrt{\frac{RT}{M}} \sqrt{P_{v}(P_{i} - P_{v})} & -\frac{K_{vo}}{V} \sqrt{\frac{RT}{M}} \sqrt{P_{o}(P_{v} - P_{o})} \\ 0 & K_{vo} \sqrt{\frac{RT}{M}} \sqrt{\frac{P_{v} - P_{o}}{P_{o}}} \end{bmatrix}$$
(16)

We conducted steady state tests to calculate the valve parameters  $K_{vi}$  and  $K_{vo}$  at the 50% opening position for the inlet valve and the outlet valve. We set up a simulation of the pressure tank in SIMULINK and minimized the objective *J*, using nonlinear least squares to obtain the optimal tuning of the GMC controller under the given valve opening constraints. The process data and tuning parameters are shown in Table 1.



Figure 4b. Control action of simulated pressure tank with setpoint change on both pressure and flowrate.

The simulation results are shown in Figures 4a and 4b. The control performance is seen to be very good. Initial dynamics are due to random initial conditions or setpoint changes. For testing the robustness of this control strategy under realistic operating conditions, it is important to conduct real time control experiments.

### **Experimental Results**

The real time control system is implemented in the MATLAB/SIMULINK environment. The tuning parameters used were the optimal values calculated above and shown in Table 1.

We performed a real-time experiment in which the setpoint for pressure was changed from 176.9 kPa to 201.4 kPa at t = 250 s and then back to 176.9 kPa at t = 800 s. The setpoint for flowrate is also changed from 0.85 m<sup>3</sup>/h to 1.42 m<sup>3</sup>/h at t = 250 s and then back to 0.85 m<sup>3</sup>/h at t = 800 s. This tests the performance of the control system over different operating ranges of this nonlinear system.

Figure 5 shows the control results for the pressure tank, in which step changes are applied to both controlled variables. Additional experimental control results are given in Figures 6



Figure 5a. Control results of real time pressure tank with setpoint change on both pressure and flowrate.



Figure 6a. Control results of real time pressure tank with setpoint change on flowrate.

and 7, in which step changes are individually applied to the controlled variables of pressure and flow rate, respectively. The control results in Figures 6 and 7 show the good decoupling performance of our GMC controller.

Because of valve hysteresis and sticking, the operating point is not easily reproducible, leading to poor repeatability in the dynamic data, and difficulties in model identification. This also causes the values of  $K_{vi}$  and  $K_{vo}$  to vary with the change of the inlet and outlet valve positions. The results between simulation and real time control are thus somewhat different. However the control performance remains good, in spite of varying valve coefficients in different ranges.

## Conclusions

An optimal tuning strategy for GMC controller with input saturation constraints is proposed. It is demonstrated via simulation that a simple heuristic switching strategy can be applied to improve setpoint tracking for SISO systems. The optimal tuning strategy (with constraints but without heuristic switching) is applied to a MIMO model of a pressure tank, followed by an implementation on the real time experimental process. The



Figure 5b. Control results of real time pressure tank with setpoint change on both pressure and flowrate.



Figure 6b. Control results of real time pressure tank with setpoint change on flowrate.



Figure 7a. Control results of real time pressure tank with setpoint change on flowrate.



Figure 7b. Control results of real time pressure tank with setpoint change on flowrate.

control performance of the optimally tuned system with input constraints is shown to be very satisfactory. This extends the use of the GMC algorithm to input saturation processes which are commonly found in chemical process control.

# Nomenclature

- d disturbance of nonlinear controller
- е error of control system
- $K_1$  $K_2$  $K_{vi}$  $K_{vo}$ GMC controller constant or matrix 1, (s<sup>-1</sup>)
- GMC controller constant or matrix 2, (s<sup>-2</sup>)
- flow coefficient for inlet valve, (kg/s)(m<sup>3</sup>/kg·Pa)<sup>1/2</sup>
- flow coefficient for outlet valve, (kg/s)(m<sup>3</sup>/kg·Pa)<sup>1/2</sup>
- stem position for inlet valve, (%)
- stem position for outlet valve, (%)
- mass of air, (kg)
- М molecular weight of air, (kg/kmol)
- Р absolute pressure, (kPa)
- $\Delta P$ differential pressure, (Pa) R
- gas constant, (J/kmol·K) temperature, (°C) Τ
- t time, (s)

- sampling interval, (s) Δt
- manipulated variable u
- upper limited value of manipulated variable uL
- lower limited value of manipulated variable u<sub>U</sub>
- V volume, (m<sup>3</sup>)
- Ń flowrate. (m<sup>3</sup>/s) controlled variable y

#### Greek Symbols

- GMC tuning constant ξ
- density of air, (kg/m<sup>3</sup>) ρ
- GMC tuning constant, (s) τ

#### Subscripts

- order of matrix
- setpoint sp
- k sampling time for discrete system

#### Superscripts

- derivative ۸
- approximation

#### Abbreviations

- RGA relative gain analysis
- MIMO multi-input and multi-output (systems)
- SISO single-input and single-output (systems)
- GMC\_LS GMC with optimal strategy based on least squares
- GMC with heuristic switching strategy GMC H
- GMC generic model control
- ATV auto tune variation (tuning procedure)
- ISE integral of the square error

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Manuscript received Novmber 1, 2000; revised manuscript received August 15, 2001; accepted for publication February 6, 2002.