Model of the Tensile Stress-Strain Behavior of Fabrics

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ABSTRACT

Currently, objective measurements of textile fabrics refer to mechanical and physical properties measured by testing devices such as KES (Kawabata's evaluation system) and FAST (fabric assurance by simple testing). Textile scientists and engineers can easily obtain the values of these properties, but their use for developing models of fabric behavior is rare, although this is the simplest and most correct way to do it. Here, we develop an analytical model of the tensile force dependent on strain for a fabric sample, integrating the Kawabata parameters obtained by means of this test. The empirical stress-strain curves of some fabrics are presented and compared with curves obtained analytically.

In improving the quality of fabrics, the mechanical property of force-extension is very significant. This property is evaluated in tensile experiments by means of several objective measurement technologies and particularly by the most sophisticated of them, the KES (Kawabata's evaluation system). But a difficulty arises as to how to develop a theoretical model representing the behavior of fabrics during this test.

Peirce [7] was the first to attack this difficulty. He developed a model based on the microstructure of fabrics. His model supposed an initial structure of a plain weave fabric composed of uniform yarns with circular cross sections, inextensible, incompressible, and perfectly flexible. Although his model is incomplete, it constitutes the basis of research in this field. Several improved models were developed, in particular by Olofsson [5, 6], Grosberg and Kedia [1], and Kawabata *et al.* [2, 3].

Olofsson's model takes into account the fact that the cross section of the yarn is not necessarily circular and that the yarns are in a complex state of deformation. His model is evaluated by considering yarn geometry to be related to external forces and reaction forces in the fabric and by assuming a relation between the curve of the yarn in the fabric and its released state. He introduces the form factor as a new parameter. Grosberg and Kedia considered two extreme cases of biaxial tension in fabrics: the state where the yarns are considered to be initially straight, and the fully relaxed state, in which the crimped shape has been set into the yarns. In their model, they took account of the rigidity of yarn bending to calculate fabric properties. In Kawabata's model [2], the structure of the fabric is identical to that of Peirce, except that he represented it in a different way to solve the problem of biaxial tension. Kawabata [3] used the same structure as for biaxial tension [2], with the aim of solving the problem in the uniaxial case. He took into account the bending of the transverse yarn, assuming the yarn to be perfectly flexible in the direction where the load is applied. He used an empirical approach to evaluate the behavior of uniaxial and biaxial tension.

An analysis of models of the Peirce type shows that they are not able to describe the shearing behavior of fabrics. Consequently they are unable to correctly describe the behavior of fabric tension deformation. Most of these models do not account for the effect of hysteresis, which is considerable in fabric behavior. Moreover, they are based on some assumptions that do not represent the physical reality of the fabric, which makes these models incomplete in spite of their strong point, being based on a microscopic study.

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In this work, we have developed a new fabric model for the tensile deformation test, based on the empirical results of Kawabata. This model integrates Kawabata parameters, which indirectly account for all the special properties of fabric—nonlinear, anisotropic, and hysteretic behaviors.

The paper is structured in three sections. In the first, we recall the definitions of Kawabata parameters in the tensile test. The second section is subdivided in two: in the first, we propose an approximation of linearity according to the strain in both cases of the extension and recovering process, while in the second, we develop the analytical model of tensile force according to the strain also in both cases of extension and recovery. The third section is devoted to a comparison of the empirical and analytical results as well as their discussion.

Tension Test (KES-FBI) [4]

To apply the Kawabata tensile test to a fabric sample, we lay it flat between two horizontal grips (one fixed and the other mobile) 5 cm apart. The useful dimension of the sample is thus 5 cm long and 20 cm wide. The mobile grip moves at speed v_t, either during extension or return, at a low and constant rate (0.1 mm/s $\leq v_t \leq$ 0.2 mm/s) in order to exert continuously increasing tension up to a (500 gf/cm) threshold, before allowing it to return to its initial position. The tension in the direction of the width is limited because of the significant width of the sample. The shape of the stress-strain diagram, illustrated in Figure 1, is generally obtained for the two orthogonal directions, warp and weft.

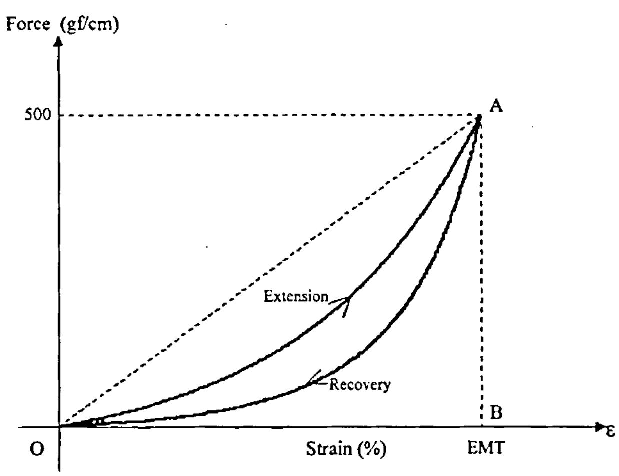

FIGURE 1. Shape of the tensile diagram (test of the Kawabata KES-FB1).

From the diagram, we obtain four parameters (proposed by Kawabata), defined as follows:

1. EMT = maximum strain (in %): the relative extension corresponding to the force limits f_{max} = 500 gf/cm.

2. WT = tensile energy per unit of area (in gf · cm/cm^2), expressed by the following formula:

$$WT = \int_0^{EMT} f(\epsilon) d\epsilon \quad , \tag{1}$$

where f = tensile force in the case of extension by a unit of length (in gf/cm), and ϵ = strain (in %).

3. LT = tensile linearity: the ratio of WT and $W0T$, it is a dimensionless parameter that characterizes the behavior of the tensile test of the sample. If this number is equal to 1, the behavior is linear; if it is higher or lower than 1, the tensile diagram is, respectively, concave or convex:

$$LT = \frac{WT}{W0T} \quad , \tag{2}$$

where

$$W0T = \frac{f_{max} \times EMT}{2} \quad , \tag{3}$$

with f_{max} = 500 gf/cm, and $W0T$ is the surface of triangle OAB (Figure 1.). OA is the theoretical tensile curve when LT = 1.

4. RT = resilience (in %), the capacity of restitution of the tensile energy during the recovery process; it is the ratio of surface WT' to surface WT, formulated by

$$RT = \frac{WT'}{WT} \times 100 \quad , \tag{4}$$

where

$$WT' = -\int_{EMT}^{0} f'(\epsilon) d\epsilon \quad , \tag{5}$$

with f = tensile force in the case of the recovery process by a unit of length (in gf/cm).

Tensile Linearity Approximation and Tensile Force Modeling

The tensile diagram given by the Kawabata test (KES-FB1) illustrated in Figure 1 shows that the behavior of fabric in the tensile test is nonlinear and presents hysteresis. Indeed, to develop an analytical model of the tensile force taking account of all these effects, we have chosen to model the functions (forces) measured directly using the Kawabata parameters LT, WT, and RT.

TENSILE LINEARITY APPROXIMATION ACCORDING TO DEFORMATION

A digitalization of the tensile diagrams in extension and recovery enables us to evaluate the linearity ratios (in the extension and recovering process) according to the strain and to plot the curves (see Figure 2.) Knowing that the linearity of the tensile deformation of fabrics (in extension and recovery) is always lower or equal to 1, in the interval $[0, EMT]$, linearity functions for both cases can be respectively approximated as follows:

$$LT(\epsilon) \cong B\epsilon + 1 \quad \text{with } B \text{ is a constant} < 1 \quad , \tag{6}$$

$$LT'(\epsilon) \cong B'\epsilon + 1 \quad \text{with } B' \text{ is a constant} < 1 \quad . \tag{7}$$

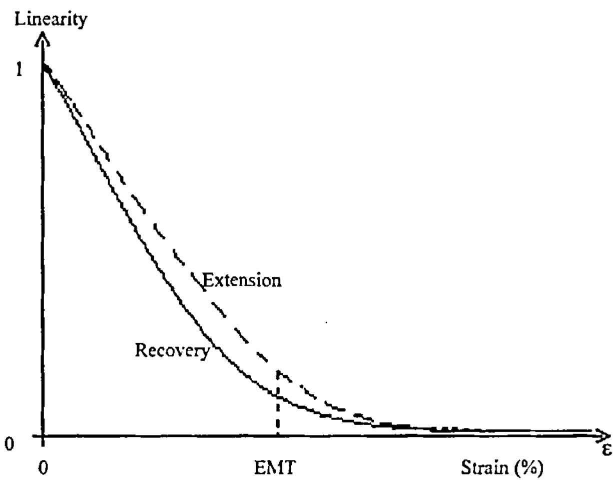

FIGURE 2. Shape of the curves of linearity according to strain (extension and recovery process).

MODELING TENSILE FORCE

Resolution of the Problem for Extension

According to the portion of extension of the tensile diagram (Figure 1), f is a function with only one variable, which verifies the following conditions: f is a monotonous function on $[0, EMT]$, with EMT a real constant, f is a positive function on $[0, EMT]$, $f(0) = 0$, and $f(EMT) = f_{max}$, where

$$EMT = \frac{2WT}{LTf_{max}} \quad . \tag{8}$$

Neglecting the approximation error, we will consider henceforth that approximation 6 is an equality.

From Equations 1, 2, 3, and 6, we have

$$2\int_0^{\epsilon} f(x)dx = \epsilon f(\epsilon)(B\epsilon + 1) \quad . \tag{9}$$

Now since f is continuous, derivable, and the derivative is continuous, the derivation of Equation 9 with respect to ϵ gives

$$\frac{\left(\dfrac{\partial f(\epsilon)}{\partial \epsilon}\right)}{f(\epsilon)} = \frac{1}{\epsilon} - 3\,\frac{B}{(B\epsilon + 1)} \quad \text{with } \epsilon \neq 0 \quad . \tag{10}$$

Knowing that f is a positive function, integrating Equation 10 between ϵ and EMT, we obtain, since $f(EMT) = f_{max}$,

$$\log(f(\epsilon)) = \log\left(\frac{\epsilon}{(B\epsilon + 1)^3}\right) + \log\left(f_{max}\frac{(BEMT + 1)^3}{EMT}\right) \quad . \tag{11}$$

In Equation 11, we set

$$A^2 = f_{max}\frac{(BEMT + 1)^3}{EMT} \quad ,$$

with

$$LT = BEMT + 1 \quad \text{and} \quad EMT = \frac{2WT}{LTf_{max}} \quad .$$

We have

$$A^2 = \frac{f_{max}{}^2 LT^4}{2WT} \quad \text{and} \quad B = (LT - 1)\frac{LTf_{max}}{2WT} \quad . \tag{12}$$

Finally, the tensile force f in the case of extension is established starting from Equations 11 and 12:

$$f(\epsilon) = \frac{f_{max}{}^2 LT^4 \epsilon}{2WT\left((LT - 1)\dfrac{LTf_{max}}{2WT}\epsilon + 1\right)^3} \qquad \epsilon \in [0, EMT] \quad . \tag{13}$$

Resolution of the Problem for the Recovery Process

In this case, the resolution of the problem is similar to the preceding one: consider a function f' with only one variable, which verifies the same conditions as f [according to the portion of the recovery process in the tensile diagram (Figure 1)] and replace the constants A^2 and B with the following constants A'^2 and B':

$$A'^2 = \frac{f_{max}{}^2 LT'^4}{2WT'} \quad \text{and} \quad B' = (LT' - 1)\frac{LT' f_{max}}{2WT'} \quad . \quad (14)$$

Knowing that

$$LT' = \frac{WT'}{W0T} \quad , \quad (15)$$

we obtain according to Equations 2, 4, and 15,

$$\frac{LT}{WT} = \frac{LT'}{WT'} \quad \text{and} \quad LT' = \frac{LTRT}{100} \quad . \quad (16)$$

Hence, by replacing 16 in 14, we find

$$A'^2 = \left(\frac{RT}{100}\right)^3 A^2 \quad \text{and} \quad B' = \left(\frac{LTRT}{100} - 1\right)\frac{LTf_{max}}{2WT} \quad . \quad (17)$$

Finally, we obtain the tensile force f' in the case of the recovery process, starting from Equation 11, where B is replaced by B' and where $A'^2 = f_{max}\frac{(B'EMT+1)^3}{EMT}$, and using Equation 17, we get

$$f'(\epsilon) = \frac{f_{max}{}^2 RT^3 LT^4 \epsilon}{2WT\left((LTRT - 100)\frac{LTf_{max}}{2WT}\epsilon + 100\right)^3} \qquad \epsilon \in [0, EMT] \quad . \quad (18)$$

Comparing the Experimental and Theoretical Results

The experimental results of uniaxial tension on two fabric samples are shown in Figures 3 and 4 (continuous lines EC_1 and EC_2). The two fabric samples are very different from the point of view of their mechanical behavior. The first one is rigid in the two orthogonal directions, warp and weft, while the second one is rigid in the weft direction, but very elastic in the warp direction. The mechanical properties (Kawabata parameters) obtained from the curves resulting from the experimental tests of the two samples are presented in Table I.

By introducing these parameters into Equations 13 and 18, we can get the tensile force according to the strain, and then obtain the theoretical tensile curves of the two fabric samples. These are illustrated by discontinuous lines TC_1 and TC_2 (Figures 3 and 4), while the experimental curves are shown for comparison by continuous lines EC_1 and EC_2. A similar agreement between the experimental and theoretical results can be obtained for other samples.

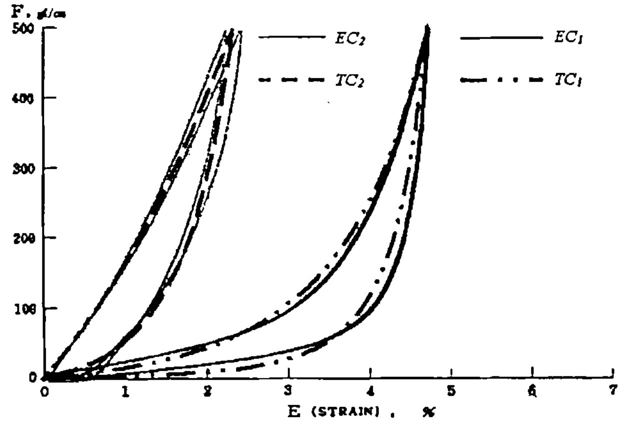

FIGURE 4. Experimental (continuous line EC_1 and EC_2) and theoretical (discontinuous line TC_1 and TC_2) tensile diagrams in the warp and weft directions for sample 2.

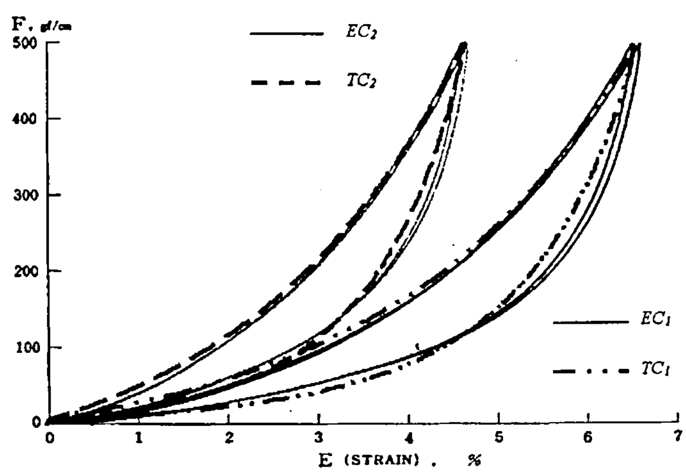

FIGURE 3. Experimental (continuous line EC_1 and EC_2) and theoretical (discontinuous line TC_1 and TC_2) tensile diagrams in the warp and weft directions for sample 1.

TABLE I. Kawabata parameters in the tensile test for the two fabric samples

Sample	No. 1		No. 2	
Direction	Warp	Weft	Warp	Weft
LT, no unit	0.626	0.696	0.450	0.866
WT, gf · cm²/cm	20.50	16.10	53.2	10.10
RT, %	62.93	64.29	49.06	56.44

NOTATION

TC_1 theoretical curve in the warp direction
TC_2 theoretical curve in the weft direction
EC_1 experimental curve in the warp direction
EC_2 experimental curve in the weft direction

As shown by the comparison (Figures 3 and 4) of the experimental and theoretical data for the two samples, the fit is good between the empirical and theoretical tensile curves even though they are based on a simple analytical model with simple assumptions. However, this comparison also shows a slight difference between these curves on the portion of the recovery process in the warp direction. This difference is due either to errors of precision at the time of determining the Kawabata parameters or to an error in the theoretical expression for the linearity $LT(\epsilon)$, which is approximated by a linear function. If we calculate the Kawabata parameters from the theoretical tensile curves, we obtain the same values as those evaluated from experimental tensile curves. However, we automatically obtain the same EMT. The validity of this model is thus relatively limited to the test conditions of KES and more particularly to a fabric sample 5 cm long and 20 cm wide.

Experimental and theoretical tensile curves are traced (Figures 3 and 4) by using the same reduction of scale for each fabric in each testing condition (warp and weft directions), to enable the comparison of empirical and theoretical plots.

Conclusions

The analytical model presented in this work is based on the empirical results of the Kawabata (KES) tensile tests and on a linear approximation of $LT(\epsilon)$. Integration of the Kawabata parameters in this model allows it to be more precise when modeling fabric tensile deformation. This precision can be verified by comparing the experimental and theoretical results that we have presented.

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