Hyper-Distributed Hyper-Parallel Self-Organizing Dynamic Scheduling Based on Solitary Wave

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Abstract This paper presents a new soliton approach to hyper-distributed hyper-parallel self-organizing dynamic scheduling for task allocations among rational autonomous agents in a multi-agent system (MAS). This approach can overcome many drawbacks of other mechanisms currently used for coalition formation and cooperation in MAS. The thorny problems, such as overabundant bid, social behaviors, colony intelligence, variable neighbors, and interdependency, can easily be treated by using the proposed approach, whereas they are very difficult for other conventional approaches. The simulation on a distributed transport scheduling system shows the soliton approach featured by hyper-parallelism, effectiveness, openness, dynamic alignment and adaption.

Keywords soliton, distributed artificial intelligence, multi-agent system, hyper-distributed hyper-parallel problem-solving, dynamic task allocation

1 Introduction

Basically, there are two categories in distributed artificial intelligence (DAI): distributed problem-solving (DPS) and multi-agent system (MAS), the former being concerned with how to increase the whole outcome of the system via the cooperation among individual agents, whereas the latter trying to increase its own personal utility of each individual through the cooperation. The following cooperation/coordination paradigms are usually used in most MAS so far[1–4]:

• Hybrid cooperation with both distributed and concentrated manners, where each agent only plays a role designated or assigned by some agents of higher level, and thus the cooperation takes place without extra overhead for communication or inference;

• Purely distributed cooperation, where agents form coalitions through repeated mutual negotiations according to their own benefits and scopes with respect to either the environment or other agents, and then acquire their fair share of interests with other agents within the same coalition in the light of a previously reached agreement.

No matter which cooperation paradigm is used, it is necessary to determine the cooperative members in advance and to form the coalition among many agents before executing the given tasks. That is the case with the non-super-additive coalition algorithm oriented to DSP in [3] and the multiagent negotiation under time constraints in [4]. There are many shortcomings in the coalition methods currently used in MAS:

• The proper coalition could never be formed until a large amount of calculation and communication is done in nearly exhaustive way, so that considerable overhead of time and resources is required.

• Once a process of task allocation has ended, owing to incomplete matching between a coalition and the corresponding task, some capabilities that agents have or tasks require always remain to be further handled.

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But there is no dynamic strategy being ready at all times to deal with the problem.

• Once a coalition aimed at a given task forms, no matter how the environment changes, the correspondence between the coalition and the task is almost fixed, being lack of colony intelligence.

• Tasks generally play a passive role in MAS, selected by autonomous agents rather than actively selecting agents.

To counter the problems mentioned above, this paper makes use of and extends the competitive wave principles proposed in [5–8], and then presents a new soliton approach to hyper-distributed hyper-parallel self-organizing dynamic cooperation/coordination among rational autonomous agents of MAS, which can overcome many difficulties encountered in other mechanisms for the coalition and the cooperation. The soliton approach can also be used to implement the algebraic modelling of [9] for MAS distributed problem-solving. By the approach, the coalition formation and the cooperation concurrently occur, so that all the coalitions needn't be built in advance of executing tasks via cooperations. Moreover, through a special soliton — competitive waves with controllable propagation speeds and adjustable amplitudes, a lot of thorny problems, such as remainder capabilities, overabundant bid, dynamic coalitions, social behaviors, colony intelligence, variable neighbors, and interdependency, could easily be treated. The simulation on a distributed transportation scheduling system shows the soliton approach characterized by hyper-parallelism, effectiveness, openness, dynamic alignment and adaption.

2 Soliton Modelling for MAS Task Allocation

The solitary wave has both particle and wave properties, and is a universal phenomenon in nature and physics. Particularly, its energy is concentrated in a relatively small region, and its waveform and/or wave speed could recover, called as elastic dissemination, when the waves interact mutually. The competitive wave[5–8], as a special solitary wave, is a nonlinear wave which propagates concurrently in nonlinear media in such a way that the propagation paths and the speeds all depend on the competition results between waves. Only the competition winner wave along a hyper-edge can continually propagate further, whereas the loser wave along a hyper-edge is deprived of propagating forwards unless the wave along the hyper-edge becomes a winner again in the competition turn that follows. There are no interference between waves confluent to the same wave node and no reflection from either wave node or hyper-edge. The waveform remains rectangular without any distortion due to dispersion or diffusion. The wave amplitude decreases in inverse proportion to the propagation distance. The wave speed can change with the received wave amplitude and introduced heuristic knowledge. By virtue of the characteristics of solitary competitive wave, a hyper-distributed hyper-parallel self-organizing dynamic modelling for MAS task allocation will be constructed as follows.

Definition 1. *Given a task set $\mathcal{G}(t) = \{g_k | k = 1, \dots, m\}$, and an agent set $\mathcal{A}(t) = \{a_i | i = 1, \dots, n\}$ at time slot t, each task $g \in \mathcal{G}(t)$ with required capability vector $\mathbf{e}_g(t)$ and payment $r_g(t)$, and each agent $a \in \mathcal{A}(t)$ with owned capability vector $\varphi_a(t)$, there are binary relations: $\mathcal{H}(t) \subseteq \mathcal{A}(t) \times \mathcal{A}(t)$, $\mathcal{H}'(t) \subseteq \mathcal{G}(t) \times \mathcal{A}(t)$, and $\mathcal{D}(t) \subseteq \mathcal{G}(t) \times \mathcal{G}(t)$, which represent the accessible neighbor relations between agents, between agents and tasks, and between tasks, respectively. Their entries are defined by*

$$\mathcal{H}_{ij}(t) = \begin{cases} 1, & \text{if } a_j \in \mathcal{N}_i(t) \\ 0, & \text{if } a_j \notin \mathcal{N}_i(t) \end{cases}, \quad \mathcal{H}'_{ki}(t) = \begin{cases} 1, & \text{if } g_k \in \mathcal{N}'_i(t) \\ 0, & \text{if } g_k \notin \mathcal{N}'_i(t) \end{cases},$$

$$\mathcal{D}_{kk'}(t) = \begin{cases} 1, & \text{if } g_k \mapsto g_{k'} \\ 0, & \text{otherwise} \end{cases},$$

where $\mathcal{N}_i(t)$ and $\mathcal{N}'_i(t)$ express the agent neighbors exerted by a_i's social actions and the task neighbors participated in by a_i, respectively, and $g_k \mapsto g_{k'}$ holds iff $g_{k'}$ is a precedent condition of g_k.

Hereupon, at a given time slot t, there are the following sets of directed edges:

$$\mathbf{E}_1(t) = \{a_i \xrightarrow{t(a_i)} g_k | a_i \in \mathcal{A}(t), g_k \in \mathcal{G}(t), t \geq t(a_i)\},$$

$$\mathbf{E}_2(t) = \{g_k \xrightarrow{t(g_k)} a_i | g_k \in \mathcal{G}(t), a_i \in \mathcal{A}(t), t \geq t(g_k)\},$$

$$\mathbf{E}_3(t) = \{a_i \xrightarrow{t(a_i)} a_j | a_i, a_j \in \mathcal{A}(t), t \geq t(a_i)\},$$

$$\mathbf{E}_4(t) = \{g_k \xrightarrow{t(g_k)} g_{k'} | g_k, g_{k'} \in \mathcal{G}(t), t \geq t(g_k)\}.$$

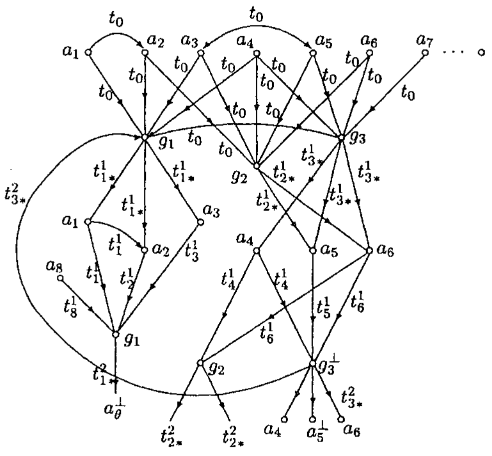

Fig.1. Solitary wave modelling for MAS task allocation.

Then we dynamically establish a special implicit directed AND/OR graph $\mathbf{G}(\mathbf{N}(t), \mathbf{E}(t))$ along which concurrent solitary waves will propagate, where $\mathbf{E}(t) = \mathbf{E}_1(t) \cup \mathbf{E}_2(t) \cup \mathbf{E}_3(t) \cup \mathbf{E}_4(t)$, and $\mathcal{A}(t) \cup \mathcal{G}(t) \subseteq \mathbf{N}(t)$, as shown in Fig.1. Each node $u \in \mathbf{N}(t)$ has its capability vector, $\varphi_u(t)$ or $(e_u(t), r_u(t))$, as defined in Definition 1, while each directed edge $u \xrightarrow{t(u)} v$ in $\mathbf{E}$ has a generalized distance $d(u \xrightarrow{t(u)} v)$ which can be defined according to the problem under consideration, as described later in the section on simulation. At a given time slot t, there is such a set $\mathbf{F}(t)$ of nodes, called the wavefront of $\mathbf{G}$, that $\mathbf{F}(t) = \{u | u \in \mathbf{N}(t)$, for $\forall v \in \mathbf{N}(t)$, $\nexists u \xrightarrow{t(u)} v \in \mathbf{E}_1 \cup \mathbf{E}_2\}$, and there is, at most, one occurrence of a node with the same name in $\mathbf{F}(t)$. Only the nodes in $\mathbf{F}(t)$ are able to spread new edges further at the next time slot. When there is wave arrival at node $u \in \mathbf{F}(t)$, node u will, according to its own belief or criteria, autonomously select and combine edges from the set $\Omega_u^*(t)$ of all the input edges to it so as to form a set $\Omega_u(t)$ of hyper-edges input to node u, to decide the optimal hyper-edge $\mathcal{L}_u^*(t)$ within $\Omega_u(t)$, and to yield a set $\mathcal{Q}_u(t)$ of its output edges at the next time slot.

In the process for a node u in $\mathbf{F}(t)$ to form its output edges $\mathcal{Q}_u(t)$, the following rules $\mathbf{R}$ should be observed:

- If u is an agent node a_i, $\varphi_{a_i}(t) \neq 0$, and $\mathcal{H}'_{ki}(t) = 1$ or $\mathcal{H}_{ij}(t) = 1$, then there is $a_i \xrightarrow{t(a_i)} g_k \in \mathcal{Q}_{a_i}(t)$ or $a_i \xrightarrow{t(a_i)} a_j \in \mathcal{Q}_{a_i}(t)$, respectively, where $t(a_i) \geq t$.
- If u is a task node g_k, $\mathcal{D}_{k'k}(t) = 1$ or $a \xrightarrow{t(a)} g_k \in \mathcal{L}_{g_k}^*(t)$, and all the precedent tasks of g_k have finished, i.e., for $\forall g \in \{g | g \xrightarrow{t(g)} g_k \in \Omega_{g_k}^*(t)\}$ there is $e_g(t) = 0$ (hereafter referred to as $g^\perp$), then there is $g_k \xrightarrow{t(g_k)} g_{k'} \in \mathcal{Q}_{g_k}(t)$ or $g_k \xrightarrow{t(g_k)} a \in \mathcal{Q}_{g_k}(t)$, respectively, where $t(g_k) \geq t$.

As shown in Fig.2, at time t_{3*}^1 the hyper-edge composed of $a_4 \xrightarrow{t_0} g_3$, $a_5 \xrightarrow{t_0} g_3$, and $a_6 \xrightarrow{t_0} g_3$ forms as the winner scrambles for node g_3 with other possible hyper-edges; at time t_5^1 node a_5 selects $\Omega_{a_5}(t_5^1) = \{g_3 \xrightarrow{t_{3*}^1} a_5,\ g_2 \xrightarrow{t_{2*}^1} a_5\}$, $\mathcal{Q}_{a_5}(t_5^1) = \{a_5 \xrightarrow{t_5^1} g_3^\perp\}$, where $g_3^\perp$ means task g_3 has already finished, i.e., $e_{g_3}(t) = 0$; node g_1 cannot determine its hyper-edges and cannot refract wave along $\mathcal{Q}_{g_1}(t)$ until the wave along $g_3^\perp \xrightarrow{t_{3*}^2} g_1$ arrives at g_1; at time t_{3*}^1, node a_4 located in edge $g_3 \xrightarrow{t_{3*}^1} a_4$ becomes a wavefront node, so a_4's remainder capability can bid for task g_2 via edge $a_4 \xrightarrow{t_4^1} g_2$, and for task g_3 once again via $a_4 \xrightarrow{t_4^1} g_3^\perp$, so that $\mathcal{Q}_{a_4}(t_4^1) = \{a_4 \xrightarrow{t_4^1} g_3^\perp,\ a_4 \xrightarrow{t_4^1} g_2\}$.

On the basis of dynamically generated $\mathbf{G}(\mathbf{N}(t), \mathbf{E}(t))$, below we will discuss the issues about the amplitude, speed and time period for solitary waves to propagate along $\mathbf{G}(\mathbf{N}(t), \mathbf{E}(t))$. Without loss of generality, suppose the following.

- The time spent by a node $u \in \mathbf{F}(t)$ in combining hyper-edge set $\Omega_u(t)$, determining optimal hyper-edge $\mathcal{L}_u^*(t)$ and yielding its output edges $\mathcal{Q}_u(t)$ can be neglected in comparison with the time for wave propagation along hyper-edges.
- The below discussion on the solitary wave propagation only focuses on that along $\mathbf{E}_1(t) \cup \mathbf{E}_2(t)$ for the time being.

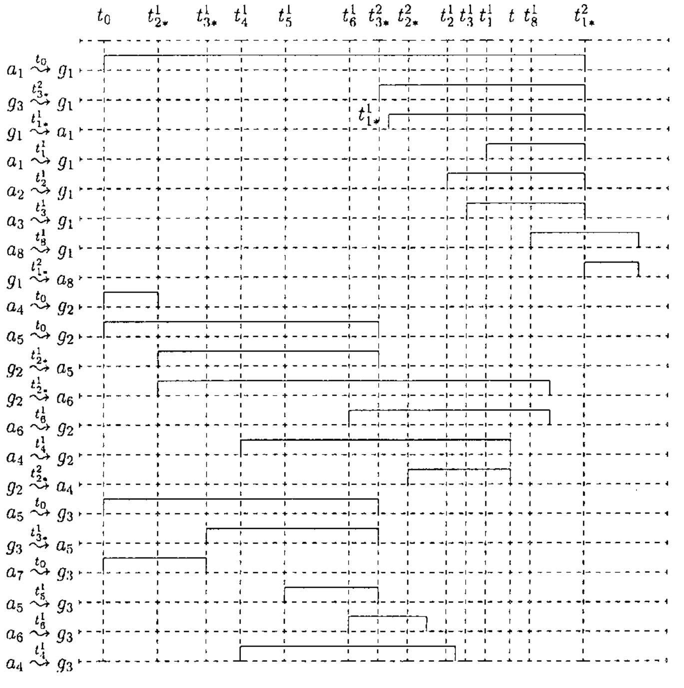

Fig.2. Solitary wave propagations along implicit AND/OR graph.

• The divergence of the intrinsic transmission delay in wave media (nodes, edges, and hyper-edges) can be neglected as compared with the wave propagation time.

• Waves can propagate along hyper-path $\mathcal{P}_u(t)$ to node u at time t, iff the waves of all the hyper-edges in $\mathcal{P}_u(t)$ are the competition winners at time t.

Definition 2. *$\mathcal{P}_u(t) = \mathcal{J}_u(t) \sim \mathcal{L} \sim u$ represents such a hyper-path having existed before time t from wave sources $\mathcal{J}_u(t)$ along hyper-edge $\mathcal{L} \in \Omega_u(t)$ up to wave node u that in $\Omega_{u'}(t)$ there is one and only one hyper-edge $\mathcal{L}'$ which belongs to $\mathcal{P}_u(t)$, where u' is any nonsource node within $\mathcal{P}_u(t)$, hereafter referred to as simply $\mathcal{P}_u(t) = \mathcal{J}_u(t) \sim u$. Hyper-path $\mathcal{P}(t) = \mathcal{J}_v(t) \sim v \sim u$ represents the cascade of hyper-path $\mathcal{P}_v(t) = \mathcal{J}_v(t) \sim v$ and edge $v \xrightarrow{t(v)} u \in \mathcal{Q}_v(t)$.*

Definition 3. *The amplitude attenuation $\delta(\mathcal{J}_u(t) \sim \mathcal{L} \sim u, t)$ for waves to pass hyper-path $\mathcal{P}_u(t) = \mathcal{J}_u(t) \sim \mathcal{L} \sim u$ and to arrive at u at time t is defined by*

$$\delta(\mathcal{J}_u(t) \sim \mathcal{L} \sim u, t) = \sum_{v \xrightarrow{t(v)} u \in \mathcal{L},\ \mathcal{L}' \in \mathcal{P}_u(t)} [\delta(\mathcal{J}_v(t) \sim \mathcal{L}' \sim v, t) + \beta d(v, u)] \tag{1}$$

where $d(v,u)$ is the generalized distance of edge $v \xrightarrow{t(v)} u$; and β is a positive coefficient. And for wave source u, there is $\delta(\mathcal{J}_u(t) \sim \mathcal{L} \sim u, t) = 0$ at any time t.

The waves arrive along hyper-path $\mathcal{P}_u(t)$ at node u with the amplitude

$$\alpha(\mathcal{J}_u(t) \sim \mathcal{L} \sim u, t) = \begin{cases} \vec{a} - \delta(\mathcal{J}_u(t) \sim \mathcal{L} \sim u, t), & \text{if } \lambda > 0 \\ \theta, & \text{otherwise} \end{cases} \tag{2}$$

where

$$\lambda = \sum_{v \xrightarrow{t(v)} u \in \mathcal{L}, \mathcal{J}_v(t) \sim \mathcal{L}' \sim v \in \mathcal{P}_u(t)} [\alpha(\mathcal{J}_v(t) \sim \mathcal{L}' \sim v, t) - \beta d(v, u)] - (p-1)\vec{a}, \tag{3}$$

with θ being a threshold value close to 0, p being the number of edges in $\mathcal{L}$, and $\vec{a}$ being constant wave amplitude gushed from a wave source.

Definition 4. *When the waves arrive along $\mathcal{P}_v(t_v) = \mathcal{J}_v(t_v) \sim \mathcal{L}' \sim v$ at node v just at time t_v, and the edge $v \xrightarrow{t(v)} u$ is chosen as an output of node v, supposing $t_v \approx t(v)$, the refraction wave could propagate along $v \xrightarrow{t(v)} u$ towards u at the speed*

$$\mathcal{S}_{\mathcal{P}_v(t_v)}(v \sim u, t \geq t_v) = \begin{cases} \mathcal{S}_0 d(v,u)/\partial(t), & \text{if } \partial(t) > 0 \\ 0, & \text{if } \partial(t) \leq 0 \end{cases} \tag{4}$$

$$\partial(t \geq t_v) = \beta d(v,u) + h_v(u) - \mathcal{S}_0 t_v + \delta(\mathcal{P}_v^*(t), t) \tag{5}$$

where $\mathcal{S}_0$ is a positive constant; $h_v(u)$ is a heuristic generalized distance estimated for hereafter propagation from v via u to wave sinks $\aleph$. If $u \in \aleph$, then $h_v(u) = 0$; and $\delta(\mathcal{P}_v^*(t), t)$ is the smallest attenuation among the waves that arrive at v along some hyper-paths until time t, namely $\delta(\mathcal{P}_v^*(t), t) = \min_{\mathcal{P}_v(t_v)}\{\delta(\mathcal{P}_v(t_v), t_v) | t_v \leq t\} = \min_{\mathcal{L} \in \Omega_v(t), k \xrightarrow{t(k)} v \in \mathcal{L}} [\delta(\mathcal{P}_k^*(t), t) + \beta d(k,v)]$.

Definition 5. *The time period $T(\mathcal{P}_u(t))$ for waves to pass through $\mathcal{P}_u(t) = \mathcal{J}_u(t) \sim \mathcal{L} \sim u$ up to node u is equal to*

$$t_u = T(\mathcal{P}_u(t)) = \sum_{v \xrightarrow{t(v)} u \in \mathcal{L}, \mathcal{P}_v(t) \in \mathcal{P}_u(t)} [T(\mathcal{P}_v(t)) + T(v \xrightarrow{t(v)} u)] = T(\mathcal{P}_u(t_u)) \tag{6}$$

where $T(v \xrightarrow{t(v)} u)$ is the period for waves from $\mathcal{P}_v(t)$ to arrive at u via $v \xrightarrow{t(v)} u$.

3 Concurrent Algorithm and Properties

Hyper-Distributed Hyper-Parallel Algorithm MTL

Step 1. Provide MTL with agent set $\mathcal{A}(t_0) = \{a_i | i = 1, \ldots, n\}$ as wave sources, and task set $\mathcal{G}(t_0) = \{g_k | k = 1, \ldots, m\}$, each a_i with capability vector $\varphi_i(t_0)$ and each g_k with capability $e_k(t_0)$ and payment $r_k(t_0)$.

Cobegin 1.

Costep 2. Parallelly update wavefront $\mathbf{F}(t)$ at time t, and at the begining let $\mathbf{F}(t_0) = \mathcal{A}(t_0) \cup \mathcal{G}(t_0)$; For $\forall a_i, g_k \in \mathbf{F}(t)$, once waves along a hyper-edge arrive at them,

Costep 3. Parallelly construct $a_i \xrightarrow{t} a_j \in \mathcal{Q}_{a_i}(t)$ and $g_k \xrightarrow{t} g_{k'} \in \mathcal{Q}_{g_k}(t)$ according to $\mathcal{H}_{ij}(t)$ and $\mathcal{D}_{kk'}(t)$, respectively;

Costep 4. a_i with remainder capability parallelly builds wave edge $a_i \xrightarrow{t} g_k \in \mathcal{Q}'_{a_i}(t)$ according to $\mathcal{H}'_{ki}(t)$ to make a new bid for g_k;

Costep 5. For every $u \in \mathbf{F}(t)$, by using rules $\mathbf{R}$, parallelly form hyper-edge set $\Omega_u(t)$, select the optimal hyper-edge $\mathcal{L}_u^*(t)$ from $\Omega_u(t)$, and, if $u \in \mathcal{G}(t)$, then build $\mathcal{Q}_u(t)$ corresponding to $\mathcal{L}_u^*(t)$;

Costep 6. Once $\mathcal{Q}_u(t)$ is established parallelly, parallelly refract input waves of $\mathcal{L}_u^*(t)$ into edges of $\mathcal{Q}_u(t)$ immediately, at the speed decided by (4) and (5) and with the wave amplitude attenuation by (1)–(3); source node u always gushes out constant amplitude wave from u along each edge of $\mathcal{Q}_u(t_0)$ in $\mathbf{E}_1(t)$;

Costep 7. Parallelly calculate capability decrements, $\Delta\varphi_i(t)$ practically consumed by $a \xrightarrow{t(a)} g_k$ within $\mathcal{L}_{g_k}^*(t)$, and $\Delta e_k(t)$ practically contributed by $g_k \xrightarrow{t(g_k)} a_i$ within $\mathcal{Q}_{g_k}(t)$, where $t \geq t(a_i)$, $t \geq t(g_k)$, $a_i \in \mathbf{F}(t)$. Parallelly modify capability vectors, $\varphi_i(t)$ and $e_k(t)$, and if for $\forall k$, $e_k(t) = 0$ or for $\forall i$, $\varphi_i(t) = 0$, then MTL successfully finishes; otherwise go to Costep 2.

Coend 1.

In what follows, (1)–(6) are always observed. The validation and some properties for MTL to find out optimal solution are given by the following theorems.

Lemma 1. *If $\delta(\mathcal{P}_v^*(t), t)$ doesn't change with $t \geq t_v$, then the period for wave to pass through $\mathcal{P}_v(t_v)$ and edge $v \xrightarrow{t(v)} u$ up to u is equal to the period for wave to pass through the optimal hyper-path $\mathcal{P}_v^*(t)$ and $v \xrightarrow{t(v)} u$ to u, where t_v is the time for waves along $\mathcal{P}_v(t_v)$ to arrive at node v.*

Proof. Because $\delta(\mathcal{P}_v^*(t),t)$ still remains constant after t_v, by (4) and (5), $\partial(t \geq t_v)$ remains unchanged and the wave from $\mathcal{P}_v(t_v)$ will refract at a constant speed along $v \xrightarrow{t(v)} u$, where by

$$T(v \xrightarrow{t(v)} u) = d(v,u)/\mathcal{S}_{\mathcal{P}_v(t_v)}(v \sim u, t \geq t_v)$$
$$= [\delta(\mathcal{P}_v^*(t), t \geq t_v) + \beta d(v,u) + h_v(u)]/\mathcal{S}_0 - t_v,$$

and

$$T(\mathcal{P}_v(t_v) \sim u) = t_v + T(v \xrightarrow{t(v)} u) = [\delta(\mathcal{P}_v^*(t), t \geq t_v) + \beta d(v,u) + h_v(u)]/\mathcal{S}_0 \quad (7)$$

Let $\mathcal{P}_v(t_v^\triangle)$ be the optimal hyper-path to v until t_v, along which waves arrive at node v just at time $t_v^\triangle$, that is $t_v^\triangle \leq t_v$, $\mathcal{P}_v(t_v^\triangle) = \mathcal{P}_v^*(t_v)$, and $\delta(\mathcal{P}_v(t_v^\triangle), t_v^\triangle) \leq \delta(\mathcal{P}_v(t_v), t_v)$. Obviously, for $t_v^\triangle \leq t \leq t_v$, there is $\delta(\mathcal{P}_v(t_v^\triangle), t_v^\triangle) \leq \delta(\mathcal{P}_v(t), t)$, which means that $\delta(\mathcal{P}_v^*(t), t \geq t^\triangle)$ also does not change after $t_v^\triangle$, and thus $T(\mathcal{P}_v(t_v) \sim u) = T(\mathcal{P}_v(t_v^\triangle) \sim u) = T(\mathcal{P}_v^*(t) \sim u)$ holds for $t \geq t_v^\triangle$. □

Lemma 2. *If $\delta(\mathcal{P}_v^*(t),t)$ does not change with $t \geq t^*$, for $\forall v$, $v \xrightarrow{t(v)} u \in \mathcal{L} \in \Omega_u(t)$, t^* being the earliest time for waves to reach v along some hyper-path, then the period for waves to propagate along $\mathcal{P}_u(t) = \mathcal{J}_u(t) \sim \mathcal{L} \sim n$ up to u is equal to that for waves to propagate along the optimal hyper-path up to $\mathcal{L}$ and then to pass via $\mathcal{L}$ up to u.*

Proof. By (6) and (7), there is

$$t_u = T(\mathcal{P}_u(t)) = T(\mathcal{P}_u(t_u)) = \sum_{v \xrightarrow{t(v)} u \in \mathcal{L}} [\delta(\mathcal{P}_v^*(t), t \geq t_v^*) + \beta d(v,u) + h_v(u)]/\mathcal{S}_0$$
$$= \sum_{v \xrightarrow{t(v)} u \in \mathcal{L}} [\delta(\mathcal{P}_v(t_v^*), t_v^*) + \beta d(v,u) + h_v(u)]/\mathcal{S}_0 \quad (8)$$

□

Lemma 3. *Under the same condition as Lemma 2, and if $t_v^* \geq t_u^*$, then the optimal hyper-path $\mathcal{P}_u^*(t)$ never contains edge $v \xrightarrow{t(v)} u$, where t_v^* is the earliest time for waves to arrive at v.*

Proof. By $t_v^* = \min_{\mathcal{P}_v(t_v)}[T(\mathcal{P}_v(t_v))]$ and by (8), t_v^* is equal to the time for wave to propagate along the optimal hyper-path $\mathcal{P}_v^*(t)$ to v. Thus from $t_v^* \geq t_u^*$, it can be derived that

$$t_u^* = T(\mathcal{P}_u^*(t)) = T(\mathcal{P}_u(t_u^*)) = \sum_{v' \xrightarrow{t(v')} u \in \mathcal{L}} [\delta(\mathcal{P}_{v'}(t_{v'}^*), t_{v'}^*) + \beta d(v',u) + h_{v'}(u)]/\mathcal{S}_0$$
$$= \sum_{v' \xrightarrow{t(v')} u \in \mathcal{L}} [t_{v'}^* + T(v' \xrightarrow{t(v')} u)] \leq t_v^* < t_v^* + T(v \xrightarrow{t(v)} u),$$

and then $v' \neq v$, $v \xrightarrow{t(v)} u \notin \mathcal{L}$, $v \xrightarrow{t(v)} u \notin \mathcal{P}_u^*(t)$. □

Lemma 4. *For $\forall v$, if $\delta(\mathcal{P}_v^*(t),t)$ never changes when $t \geq t_v^*$, then the wave with the maximum amplitude will arrive earliest at $t_u^* = [\delta(\mathcal{P}_u^*(t_u^*), t_u^*) + h(u)]/\mathcal{S}_0$, where $v \xrightarrow{t(v)} u \in \mathcal{L}$, $\mathcal{L} \in \Omega_u(t)$.*

Proof. By assumption $\delta(\mathcal{P}_v^*(t), t \geq t_v^*) = \delta(\mathcal{P}_v(t_v^*), t_v^*) = c(v)$, $c(v)$ being a constant, and by (8), there are

$$t_u = T(\mathcal{P}_u(t_u)) = \sum_{v \xrightarrow{t(v)} u \in \mathcal{L}} [\delta(\mathcal{P}_v^*(t), t \geq t_v^*) + \beta d(v,u)]/\mathcal{S}_0 + h(u)/\mathcal{S}_0$$
$$= \sum_{v \xrightarrow{t(v)} u \in \mathcal{L}} [c(v) + \beta d(v,u)]/\mathcal{S}_0 + h(u)/\mathcal{S}_0,$$

and

$$\delta(\mathcal{P}_u^*(t), t \geq t_u^*) = \min_{\mathcal{P}_u(t_u)} \{\delta(\mathcal{P}_u(t_u), t_u \geq t_u^*)\} = \min_{\mathcal{L} \in \Omega_u(t)} \sum_{v \xrightarrow{t(v)} u \in \mathcal{L}} [\delta(\mathcal{P}_v^*(t), t \geq t_v^*) + \beta d(v,u)].$$

By Lemma 3, only the case $t_v^* < t_u^*$ needs considering, and therefore

$$\delta(\mathcal{P}_u^*(t), t \geq t_u^*) = \min_{\mathcal{L}\in\Omega_u(t)} \sum_{v \xrightarrow{t(v)} u\in\mathcal{L}} [\delta(\mathcal{P}_v^*(t), t > t_v^* + \beta d(v,u)]$$
$$= \min_{\mathcal{L}\in\Omega_u(t)} \sum_{v \xrightarrow{t(v)} u\in\mathcal{L}} [c(v) + \beta d(v,u)] = \min_{\mathcal{L}\in\Omega_u(t)} [\mathcal{S}_0 t_u - h(u)] = \mathcal{S}_0 t_u^* - h(u)$$

holds true, namely, when $t \geq t_u^*$, $\delta(\mathcal{P}_u^*(t), t)$ is also unchanged. Hence, $t_u^* = [\delta(\mathcal{P}_u^*(t),\ t \geq t_u^*) + h(u)]/\mathcal{S}_0 = [\delta(\mathcal{P}_u^*(t_u^*), t_u^*) + h(u)]/\mathcal{S}_0$, and at t_u^* the minimum attenuation wave, i.e., the maximum amplitude wave, arrives at u earliest. □

Lemma 5. *When $t \geq t^*(v)$, it is true that $\delta(\mathcal{P}_v^*(t), t)$ doesn't change with time for any wave node v.*

Proof. First, define the maximum intermediate node number $\xi(u)$ of node u in a hyper-path $\mathcal{P}_u(t)$ as follows. If u is a wave source, $\xi(u) = 0$ and if $\max_{v \xrightarrow{t(v)} u\in\mathcal{L}, \mathcal{L}\in\mathcal{P}_u(t)} \{\xi(v)\} = k$, then $\xi(u) = k + 1$.

By induction for $\xi(u)$, if $\xi(u) = 0$, it is obvious that $\delta(\mathcal{P}_u^*(t), t) = 0$ for $t \geq t_0$ and the lemma holds true. When $\xi(u) = 1$, any ancestor node v of u is a wave source, thus, $\delta(\mathcal{P}_u^*(t), t) = \min_{\mathcal{L}\in\Omega_u(t)} \sum_{v \xrightarrow{v(t)} u\in\mathcal{L}} [\delta(\mathcal{P}_v^*(t), t) + \beta d(v,u)] = \min_{\mathcal{L}\in\Omega_u(t)} \sum_{v \xrightarrow{t(v)} u\in\mathcal{L}} \beta d(v,u)$ is unchanged with time and the lemma is true for $\xi(u) = 1$. By the induction assumption for $\xi(u) = k$, one needs to prove that the lemma is also true for $\xi(u) = k + 1$. For $\xi(u) = k + 1 \geq 2$, the set $\omega(u)$ of the father nodes of u can be divided into such two subsets, $\omega_1(u)$ and $\omega_2(u)$, that if $v \in \omega(u)$ and $t_v^* < t_u^*$ then $v \in \omega_1(u)$, otherwise $v \in \omega_2(u)$. By the induction assumption about $\xi(u) \leq k$ and by Lemma 3, it is sure that $\delta(\mathcal{P}_v^*(t), t \geq t_v^*) = c(v)$ is a constant, and there is

$$\delta(\mathcal{P}_u^*(t), t \geq t_u^*) = \min_{\mathcal{L}\in\Omega_u(t)} \sum_{v \xrightarrow{t(v)} u\in\mathcal{L}} [\delta(\mathcal{P}_v^*(t), t \geq t_u^*) + \beta d(v,u)]$$
$$= \min_{\mathcal{L}\in\Omega_u(t)} \sum_{v \xrightarrow{t(v)} u\in\mathcal{L}, v\in\omega_1(u)} [c(v) + \beta d(v,u)].$$

Therefore $\delta(\mathcal{P}_u^*(t), t \geq t_u^*)$ remains unchanged with time, and the conclusion holds for $\xi(u) = k + 1$. □

Theorem 1. *The wave along the hyper-path $\mathcal{P}_v^*(t)$ with the minimum attenuation will arrive at node u earliest, and the arrival time is $t_u^* = [\delta(\mathcal{P}_u^*(t),\ t \geq t_u^*) + h(u)]/\mathcal{S}_0$.*

Proof. It is straightforward from Lemmas 3, 4 and 5. □

Theorem 2. *Waves that pass along $\mathcal{P}_v(t)$ and $\mathcal{P}_v^*(t)$ to v at time t_v and t_v^*, respectively, $t_v - t_v^* < [\beta d(v,u) + h_v(u) - h(v)]/\mathcal{S}_0$, will arrive at the next node u at the same time.*

Proof. By Lemma 4 and Theorem 1, we have $t_v^* = [\delta(\mathcal{P}_v^*(t),\ t \geq t_v^*) + h(v)]/\mathcal{S}_0$. Thus, $\beta d(v,u) + h_v(u) - \mathcal{S}_0 t_v + \delta(\mathcal{P}_v^*(t), t \geq t_v^*) > 0$, namely $\partial(t \geq t_v^*) > 0$ can be derived. Furthermore, by Definition 9 and Lemma 1, $T(\mathcal{P}_v(t) \sim u) = [\delta(\mathcal{P}_v^*(t), t \geq t_v^*) + \beta d(v,u) + h_v(u)]/\mathcal{S}_0 = T(\mathcal{P}_v^*(t) \sim u)$ holds true. If $t_v - t_v^* \geq [\beta d(v,u) + h_v(u) - h(v)]/\mathcal{S}_0$, then $\partial(t \geq t_v) \leq 0$, and $\mathcal{S}_{\mathcal{P}_v(t_v)}(v \sim u, t \geq t_v) = 0$, which means the wave from $\mathcal{P}_v(t)$ fails to propagate along $v \xrightarrow{t(v)} u$.. □

Theorem 3. *If MTL selects a heuristic value $h_v(u)$ so that $[h_v(u) - h(v)]/d(v,u)$ is a constant, then the wave propagates at an identical speed along any edge.*

Proof. By Theorem 1, $\mathcal{S}_0 t_v^* = \delta(\mathcal{P}_v^*(t),\ t \geq t_v^*) + h_v(u)$ holds, and

$$\mathcal{S}_{\mathcal{P}_v(t_v)}(v \sim u, t \geq t_v^*) = \mathcal{S}_0 d(v,u)/[\beta d(v,u) + h_v(u) - \mathcal{S}_0 t_v^* + \delta(\mathcal{P}_v^*(t), t \geq t_v^*)]$$
$$= \mathcal{S}_0 d(v,u)/[\beta d(v,u) + h_v(u) - h(v)] = \mathcal{S}_0/(\beta + c)$$

is a positive constant. □

Theorem 4. *The period time for Algorithm MTL to find out the optimal solution is independent of the heuristic value.*

Proof. By Theorem 1 and $h(u) \equiv 0$ for any wave sink node u, there is the time $t_u^* = [\delta(\mathcal{P}_u^*(t),\, t \geq t_u^*) + h(u)]/\mathcal{S}_0 = \delta(\mathcal{P}_u^*(t),\, t \geq t_u^*)/\mathcal{S}_0$. Here $\delta(\mathcal{P}_u^*(t),\, t \geq t_u^*)$ has nothing to do with the heuristic value $h(v)$ of any wave node v. □

Theorem 5. *The larger the heuristic values, the less the complexity of wave nodes required for finding out the optimal solution.*

Proof. Let the set of nodes via which waves have passed by time t be $\mathcal{N}(t)$, and $h_1(u) \leq h_2(u)$ for wave node u. By Theorem 1, $t_u^*(1) = [\delta(\mathcal{P}_u^*(t),\, t \geq t_u^*(1)) + h_1(u)]/\mathcal{S}_0$ and $t_u^*(2) = [\delta(\mathcal{P}_u^*(t),\, t \geq t_u^*(2)) + h_2(u)]/\mathcal{S}_0$ hold true. Because $\delta(\mathcal{P}_u^*(t), t \geq t_u^*(1)) = \delta(\mathcal{P}_u^*(t), t \geq t_u^*(2))$ is irrelative to $h(u)$, there is $t_u^*(1) - h_1(u) = t_u^*(2) - h_2(u)$ and thus $t_u^*(2) \geq t_u^*(1)$, which implies the waves with $h_2(u)$ cannot propagate so fast as the waves with $h_1(u)$ can. Moreover, according to Theorem 4, irrespective of $h_1(u)$ or $h_2(u)$, the Algorithm MTL spends the same time in finding the optimal hyper-path, whereby $\mathcal{N}_2(t) \subseteq \mathcal{N}_1(t)$. □

Theorem 6. *Algorithm MTL has the time complexity $O(L)$ to find the existential solution, where L is the total distance of the optimal hyper-path.*

Proof. By Theorem 1, the wave reaches a wave sink node u at time t_u^* which is directly proportional to L. □

4 Simulation and Conclusions

By using the solitary wave modelling and the algorithm MTL, the simulation experiments on a distributed transportation scheduling system are carried out, where every distributed transportation company and every distributed warehouse, as a hauler (agent) and a cargo owner (task) respectively, manage to pursue their own maximum profits or minimum costs via negotiation and then via coordination under various constraints, such as volume of road haulage, freight charges, hauling capacity, the number of trucks, service quality, price of goods, haul cycle, freight distance, order of transport priority, and so forth. All the constraints in addition to some interferential social behaviors among individual haulers are taken into account in the simulation. Particularly, the generalized distance $d(v,u)$ used in algorithm MTL is synthetically defined as follows:

$$\begin{aligned} d(a_i, g_k) =& w_1\|\varphi_i(t)\|/\sum_{j=1}^{m}\|\varphi_{ij}(t)\| + w_2\|\varphi_i(t)\|/\|\varphi_{ik}(t)\| \\ &+ w_3\|e_k(t)\|/\|\varphi_{ik}(t)\| + w_4\|\varphi_{ik}(t)\|/r_{ik}(t); \\ d(g_k, a_i) =& w_1'\|e_k(t)\|/\sum_{i=1}^{n}\|\varphi_{ik}(t)\| + w_2'\|e_k(t)\|/\|\varphi_{ik}(t)\| \\ &+ w_3'p(d_{ik}, v_{ik}, \|\varphi_{ik}(t)\|) + w_4'f(q_i), \end{aligned}$$

where $\|\cdot\|$ is the norm value of capability vector; $w_1, \ldots, w_4$ and $w_1', \ldots, w_4'$ are weight coefficients; p is a function to calculate the time period for a_i to finish subtask g_{ik} corresponding to $\|\varphi_{ik}\|$, d_{ik} and v_{ik} being the freight distance and freight velocity, respectively; and f is a function of service quality q_i of company a_i.

It is owing to both controllable speed and amplitude of wave propagation that the complex transport scheduling problem is effectively solved in a hyper-distributed hyper-parallel self-organizing way. Fig.3 illustrates some simulation results. Here for simplic-

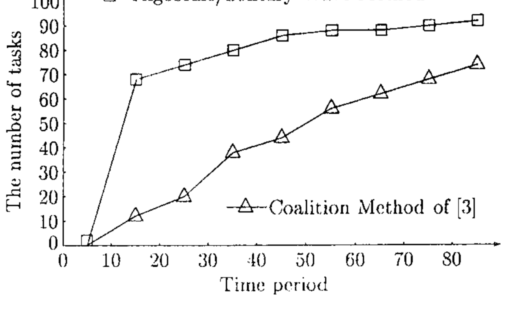

Fig.3. Simulation results on distributed self-organizing dynamic scheduling for transport problem.

ity, in the simulation, the capacity totally required for an agent to bid for several tasks once must be less than the capacity the agent owns currently, that is the overabundant bid, which can also be dealt with by MTL algorithm in principle, is unallowable.

The conclusions are summarized as follows:

- The solitary competitive wave approach and the algorithm MTL can implement hyper-distributed hyper-parallel self-organizing dynamic scheduling for MAS task allocation.
- The proposed approach is featured with many advantages over other conventional problem-solving methods for MAS. Specially, it is as easy as natural to deal with the interdependency, remainder capability, dynamic coalition, social behaviors, colony intelligence, etc.
- The solitary wave model is essentially different from general implicit AND/OR graph used to search state space. In the classical search of implicit AND/OR graph, there are two phases: top-down search and bottom-up search, and in bottom-up phase all the hyper-edges are previously fixed. Moreover, the back-tracking is usually necessary. On the other hand, however, in the soliton model, the hyper-edges are dynamically constructed, and the back-tracking and two phases are not necessary, so that it is possible to handle the stochastic distributed social intervenient behaviors of MAS.

References

[1] Fujita S, Lesser V R. Centralized task distribution in the presence of uncertainty and time deadline. In *Int. Conf. Multi-Agent Systems (ICMAS-96)*, Tokyo, 1996, pp.87–94.

[2] Ito A, Yano H. The emergence of cooperation in a society of autonomous agents — The prisoner's dilemma game under the disclosure of contract histories. In *Int. Conf. Multi-Agent Systems (ICMAS-95)*, San Francisco, 1995, pp.201–208.

[3] Onn Shehory, Sarit Kraus. Methods for task allocation via agent coalition formation. *Artificial Intelligence*, 1998, (101): 165–200.

[4] Sarit Kraus, Jonathan Wilkenfeld, Gilad Zlotkin. Multiagent negotiation under time constraints. *Artificial Intelligence*, 1995, (75): 297–345.

[5] Shuai D X. Concurrent competitive wave approach to hyper-distributed hyper-parallel AI processing. *Journal of Computer Science and Technology*, 1997, 12(6): 543–554.

[6] Shuai D X. Asynchronous heterogeneous mechanism for hyper-distributed hyper-parallel AI processing. *Journal of Computer Science and Technology*, 1999, 14(3): 234–241.

[7] Shuai D X. Asynchronous superimposition mechanism of concurrent competitive waves for hyper-distributed hyper-parallel heuristic problem solving. *Journal of Computer Science and Technology*, 1997, 12(4): 330–336.

[8] Shuai D X. Synchronous-homogeneous concurrent propagation mechanism of competitive waves for hyper-distributed hyper-parallel AI processing. In *Proc. IEEE MFI'96*, Washington, U.S.A., 1996, pp.312–319.

[9] Shuai D X, Gu J, Deng Z D. Hyper-distributed hyper-parallel algebraic approach to MAS problem-solving. In *Proc. 13th Int. Conference on Parallel and Distributed Computing Systems (PDCS-2000)*, Las Vegas, U.S.A., August, 2000, pp.478–483.

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