Deposition Velocities for Newtonian Slurries in Turbulent Flow

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In this contribution both new and previously published deposition velocity data for aqueous Newtonian slurries are incorporated in a correlation which complements the earlier predictions of Durand, Wilson and A.D. Thomas. Most of the new data has been obtained for slurries of narrowly sized sand particles of median diameter ranging between 90 and 420 µm, in pipes of diameter between 0.05 and 0.5 m. The data were obtained in laboratory investigations under isothermal flow conditions.

Dans ce travail de contribution, de nouvelles données et des données publiées antérieurement sur la vitesse de déposition de suspensions newtoniennes aqueuses sont introduites dans une corrélation qui vient compléter les prédictions antérieures de Durand, Wilson et A.D. Thomas. La plupart des nouvelles données ont été obtenues pour des suspensions de particules de sable presque monodisperses dont le diamètre varie entre 90 et 420 µm, dans des conduites d'un diamètre compris entre 0,05 et 0,5 m. Ces données ont été obtenues lors de recherches en laboratoire dans des conditions d'écoulement isothermes.

Keywords: deposition velocity, slurry flow, solid–liquid flow, pipeline flow.

Because the deposition condition represents the lower limit to operating velocities for most slurry transport systems, prediction of deposition velocities is an essential step in pipeline design. The increasing scale of mining operations has led to use of larger pipe diameters and higher solids concentrations for tailings transportation. Prudent design of these pipelines has stimulated laboratory investigations of deposition velocities so that the data base available for selecting operating conditions continues to develop.

Of the numerous correlations which have been proposed for predicting deposition velocities of Newtonian slurries, those of Wilson (1979) and A.D. Thomas (1979) are most notable because they incorporate theoretical concepts and are applicable in principle for a broad spectrum of particle, pipe and fluid properties. Since 1979, theoretical advances in understanding deposition velocities in turbulent flows have been confined for the most part to large particles which produce stratified flows. Although coarse particles, with median diameters greater than approximately 0.5 mm, form slurries of some industrial importance it is true that finer particles constitute the majority of mineral slurries for which pipelines must be designed.

Many of the correlations which have been presented in the past have considered data obtained in academic investigations. Although wide ranges of particle and fluid properties have been used, the pipe sizes have often been significantly smaller than modern industrial practice finds appropriate.

Even when test pipelines of industrial scale are available for use to generate design data, the quantity of solids available for use in the slurry flow tests is often limited because the mine is not in production. For this reason scale-up of deposition velocities to higher solids concentrations and/or larger pipes is often necessary.

To improve the correlations and to provide a guide for use in scale-up of laboratory test data, new and existing data obtained in the Saskatchewan Research Council laboratory using a range of pipe sizes have been re-examined and compared with previous work. This data has been obtained under isothermal flow conditions and the viscosity of the carrier fluid (water + fines) has been measured.

Deposition Velocity Correlations

The form of the correlations was established by Durand (1953) and may be stated as:

$$V_c = F_L \sqrt{2\, g\, D(S_s - 1)} \tag{1}$$

In the correlation, F_L was presented in graphical form as a function of particle diameter and solids concentration. To generalize the effect of particle diameter, Wilson and Judge (1976) used the available experimental data to obtain an equation which included the particle diameter, pipe diameter and the particle drag coefficient:

$$F_\Delta = 2.0 + 0.3 \log_{10} \Delta \tag{2}$$

where $\Delta = (d/D\, C_D)$.

Equation (2) was considered to be applicable to slurries of particles with median diameters less than about 0.5 mm. A method for predicting velocities for coarser particles was deduced from Wilson's two-layer model and the combined results were presented conveniently in nomographic form (Wilson, 1979) for particles larger than 0.15 mm in pipes with diameters greater than 100 mm. The inherent lack of precision of the nomogram is realistic because deposition velocities are difficult to determine with precision and the effects of fluid viscosity and solids concentration are sometimes significant.

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A notable feature of the nomogram and of the analysis which was made for coarse particles is that the deposition velocity is predicted to decrease as the particle diameter increases above 0.5 mm. The original Durand correlation showed this type of decrease and experimental verification of this prediction has been obtained. A qualitative explanation of the decrease is that as the particle diameter increases the frictional resistance to motion eventually becomes independent of particle diameter but the axial impelling force for a given bulk velocity continues to increase.

For particles with diameters near 0.5 mm, i.e., near the peak values of deposition velocity, Wilson et al. (1992) have suggested that the (Fanning) fluid friction factor f_f should be used in the correlation, i.e.:

$$F_L = (0.0045/f_f)^{0.13} \tag{3}$$

An alternative empirical correlation for deposition velocities of aqueous slurries with in-situ solids concentrations in the range $0.10 < C_r < 0.35$ was presented in terms of particle drag coefficient and fluid kinematic viscosity by Gillies and Shook (1991). In making calculations for industrial slurries which contain significant quantities of fines which increase the fluid viscosity, Gillies and Shook suggested that the viscosity and density of the (fines + fluid) fraction should be measured and used with the correlation. The "fines" fraction was defined arbitrarily as that below 0.074 mm. Since the correlation of Gillies and Shook used data for particles of median diameter greater than 0.18 mm, predictions for particles finer than 0.15 mm can only be made using Equations (1) and (2). Because many industrial slurries have median diameters finer than 0.15 mm, further experiments are desirable.

A.D. Thomas (1979) extended Wilson's approach to predicting deposition velocities for particles of diameter smaller than the thickness of the viscous sublayer δ_L. δ_L was calculated from the bulk velocity V, the fluid density and viscosity and the Fanning friction factor f_f using the criterion:

$$\delta_L = 5\frac{\mu}{\rho V\sqrt{\frac{f_f}{2}}}$$

For these very fine particles Thomas proposed a limiting equation for solids concentrations less than about 20% by volume:

$$V_{c\delta}\sqrt{\frac{f_f}{2}} = 1.1\left[\frac{g\mu(S_s-1)}{\rho}\right]^{1/3} \tag{4}$$

In contrast with the behaviour of slurries of large particles, for which the deposition velocity should vary almost as $\sqrt{D}$ according to Equations (1) and (2), Equation (4) predicts that for fine particles the deposition velocity should be almost independent of pipe diameter.

In the transition region, for particles with $(d/\delta) > 0.3$, Thomas suggested as a practical method:

$$V_c^2 = V_{c\delta}^2 + V_{c\Delta}^2 \tag{5}$$

In Equation (5), $V_{c\Delta}$ is calculated from Equation (1) using F_Δ as F_L.

Although these theoretical approaches have proved useful in explaining the form of the Durand correlation and providing estimates and qualitative explanations of the effects of particle, fluid and pipe parameters, the situation is less satisfactory for the effect of solids concentration.

Wilson and co-workers had considered the effect of solids concentration on deposition velocity to be significant and Wilson denoted the predictions of V_c in the 1979 nomogram as the maximum values which would be observed for a given set of particle, fluid and pipe parameters at any solids concentration. The effect of solids concentration on V_c was examined by Wilson in a subsequent publication (1986).

For solids concentrations greater than 20% by volume Thomas (1979) suggested that $V_{c\delta}$ would eventually increase because the slurry viscosity increases more rapidly with increasing solids concentration than its density. Although Thomas provided evidence showing that increases in deposition velocity sometimes occur with increasing solids concentration, this is not always the case. For example experiments conducted with 45 µm iron ore particles (Schriek et al., 1973), using pipes of diameter between 0.105 m and 0.315 m, showed substantial decreases in V_c with increasing concentration. The only cases in which V_c increased with increasing concentration occurred with the smallest pipe and the highest slurry concentrations. In these cases it is possible that deposition occurred because the flow became laminar. The friction losses measured for the flows tend to support this suggestion.

A comparison of the experimental results, for the iron ore particles at $C_r = 0.15$, with Thomas' (1979) predictions showed that the values from Equation (4) were satisfactory for a pipe of diameter 0.105 m. In this case the value of Δ was approximately 4×10^{-6}. For larger pipes, the predictions were unsatisfactory.

To explain the effect of solids concentration on deposition velocity for the 45 micrometre iron ore particles, Gillies et al. (1997) considered the equilibrium of forces for the incipient stationary deposit. This approach differs from that of A.D. Thomas (1979) in the following respects:

a) Deposition is assumed to be associated with an incipient deposit of finite thickness δ which is not equal to the thickness of the viscous sublayer.
b) The force on the incipient deposit resulting from the axial pressure gradient can be neglected in comparison to kinetic frictional drag.
c) Turbulence may be assumed to provide sufficient suspension for the slurry to exert a buoyant effect on the particles which is proportional to its density.

In common with earlier derivations, Gillies et al. (1997) assumed that the only particles in the flow which contribute Coulomb or sliding friction are those in the incipient deposit. This assumption is strictly true only at very high values of the ratio (V/v_∞), i.e., for fine particles. With larger particles, the axial stress on the upper surface of the incipient deposit cannot be assumed to be purely kinetic and lack of understanding of the dynamics of these flows complicates mechanistic analysis.

For fine particles Gillies et al. (1997) derived an equation in terms of the mean in-situ concentration of the slurry, C_r, the concentration C_{lim} in the incipient deposit, the coefficient of particle–wall friction η_s and the friction factor at the surface of the deposit f_{12}:

$$V_c^2 = 1.33\left(\frac{\delta\eta_s}{f_{12}}\right)g(S_s-1)\Phi(C_r, C_{lim}) \tag{6}$$

where $\Phi(C_r, C_{lim}) = \dfrac{(C_{lim}-C_r)(1-C_{lim})}{(1-C_{lim}-C_r)}[1+C_r(S_s-1)]$

Since δ, the thickness of the incipient deposit, is independent of pipe diameter, Equation (6) resembles Equation (4) in that it does not predict a pipe diameter dependence of V_c. In practice, the quantity $(\delta\ \eta_s/f_{12})$ would have to be regarded as a correlating parameter to be established by experiments.

The preceding discussion has been restricted to turbulent flows. Empirical observations suggest that high pressure gradients are required to avoid deposition in laminar slurry flows and a theoretical explanation for this observation has been offered (Gillies et al., 1999). In the absence of these high pressure gradients, deposition often is observed as the laminar flow condition is approached, as in the case of the experiments of Schriek et al. (1973) mentioned above. Where deposition is not observed in laminar flow and the pressure gradient is not high, D.G. Thomas (1979) has remarked that particles may settle so slowly that very long test pipelines may be required to detect deposition.

Experimental Measurements

Several experimental investigations have been conducted since the correlation of Gillies and Shook (1991) was presented and these have extended the range of particle diameters for which data are available for correlation. Several test loops, with pipes of diameter ranging between 52 and 495 mm have been employed. The common characteristics of these loops are important. These include:

1. Heat exchangers were included to ensure isothermal operation.
2. Long radius pipe bends were used wherever flow direction changes occurred.
3. The pipe diameter in each test loop was constant. In combination with item 2, this ensures that the solids concentration does not vary in the axial direction.
4. The viscosity of the fluid phase, consisting of water and fines, was measured in each experiment.

Item 4 is very important when the viscosity plays an important role because small quantities of fines often flocculate and increase the viscosity of the (fluid + fines) mixture substantially.

Deposition was observed visually in a section of transparent pipe, whenever this was possible. With high fines concentrations and large pipes, visual observations become difficult and it was necessary to use another method. In these cases, the beam of a gamma ray densitometer was located so that it spanned the pipe on a horizontal chord close to the bottom of the pipe. The mean concentration of the solids in this chord was then measured as a function of bulk velocity. As the bulk velocity decreases the chord concentration increases, slowly at first and then rapidly when deposition occurs. A typical set of measurements is shown in Figure 1. The vertical line at a bulk velocity of 2.1 m/s indicates where deposition was observed visually. The chord concentrations begin to rise abruptly at a slightly lower velocity when the stationary deposit becomes thick enough to affect the gamma ray beam of the densitometer.

New Experimental Results

Since modern industrial practice favours higher solids concentrations and larger pipes, it is the effects of these two independent variables which is of greatest interest. If the effect of fines on solids concentration is quantified, the residual effect of solids concentration on deposition velocity seems to be most important at very low concentrations and very high concentrations. Typical results are shown in Figure 2 where a significant increase in deposition velocity at low concentrations can be seen. It should be noted that the independent variable in this graph is the mean in-situ volumetric concentration of coarse particles, i.e., the fines in the slurry have been excluded.

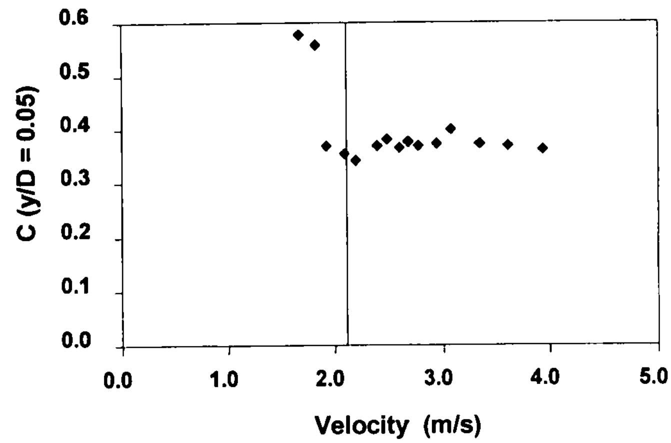

Figure 1. Chord-average solids concentration close to the bottom of the pipe as a function of bulk velocity. The visual deposition velocity was 2.1 m/s.

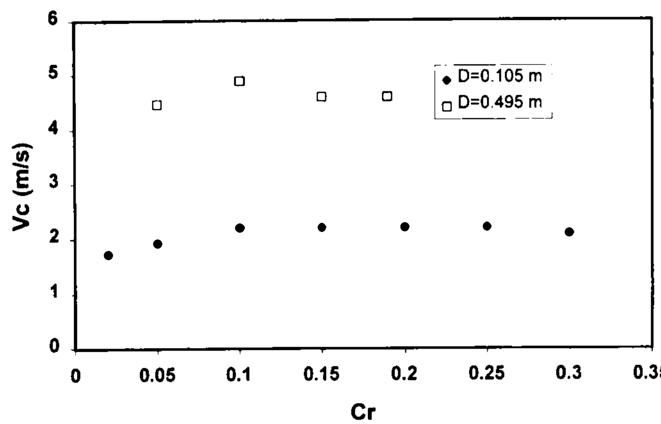

Figure 2. Effect of solids concentration on deposition velocity for 0.62 mm sand particles.

A decrease in deposition velocity is usually observed at concentrations greater than 35% by volume but for narrow particle size distributions this effect is smaller than predicted by Wilson (1986), at least for C_r values less than about 45%. At very high concentrations, shearing of the slurry becomes very difficult so that plug flow can occur. In this case Wilson's sliding bed model can be used to show that deposition velocities decrease significantly.

An indication of the effect of solids concentrations in the range $0.3 < C_r < 0.45$ is given in Figure 3 which compares the extreme values of deposition velocities, in the form of F_L values, for a sand of diameter 0.42 mm. The solids concentrations in these experiments ranged between 30% and 45% by volume and the effect upon deposition Froude number is evidently small and only slightly greater than the uncertainty in the individual measurements.

The interesting observation to be drawn from Figure 3 is that F_L is virtually independent of pipe diameter. This seems to be characteristic of "coarse-particle" slurries in turbulent flow.

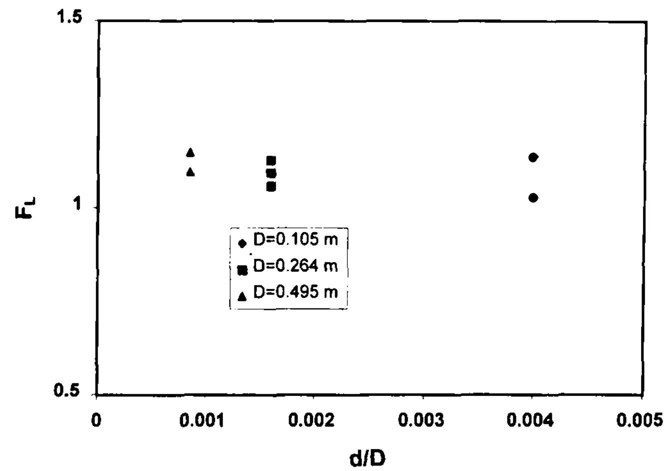

Figure 3. Effect of pipe diameter on deposition Froude Number for 0.42 mm sand particles: $0.3 < C_r < 0.45$.

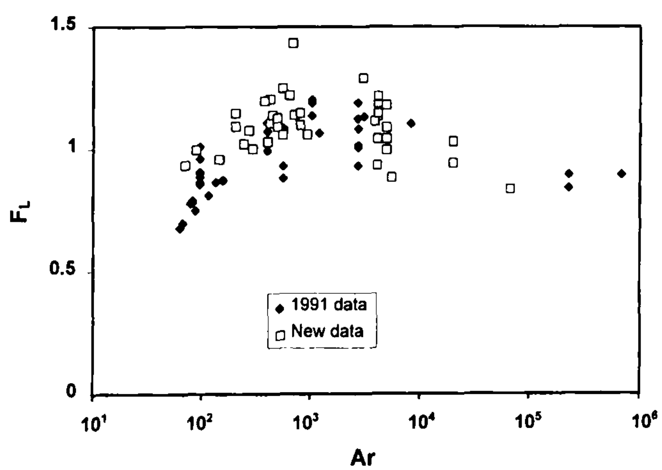

Figure 4. Deposition Froude number as a function of particle Archimedes number.

Figure 4 presents a comparison of the data used in the 1991 correlation of Gillies and Shook with that which has been obtained subsequently. The particle Archimedes number is defined as:

$$Ar = \left(\frac{4}{3}\right)[g d^3 \rho_L (\rho_s - \rho)/\mu^2] \tag{7}$$

For Archimedes numbers greater than about 80, there is evidently little difference between the experimental F_L values in the 1991 data set and the subsequent experiments. This correspondence occurs despite the effects of solids concentration and of parameters such as particle shape which may affect deposition and which do not appear in Ar. An important deviation was observed at lower Ar values, however.

This deviation is of the type predicted by the correlation of Wilson and Judge. Figure 5 shows a comparison of their correlation with the experimental data which was included in Figure 4.

In addition to the data shown in Figure 4, data obtained recently by Schaan (1999) are included in Figure 5. The Archimedes numbers for these tests were close to 10 and the data did not agree with the high Archimedes number correlation shown in Figure 4. These points provide the lowest abscissa values shown in Figure 5 and the agreement with the correlation of Wilson and Judge is encouraging. It is therefore suggested that the equation of Wilson and Judge should be used for slurries with Archimedes numbers below 80. It will be recalled that the lower limit of Δ for which this equation was found was about 4×10^{-6}.

Since particle diameter and pipe diameter are the parameters of greatest interest in deposition velocity predictions it is of interest to indicate the scope of the data which have been obtained in the controlled experiments summarized in Figures 4 and 5. Figure 6 presents these aspects of the data set and it is evident that few data have been collected for large pipes in the important region of particle diameter below 0.15 mm. It should also be noted that the viscosity of the carrier fluid, i.e., the fines + water fraction, ranged between 0.5 and 5 mPa·s in these experiments.

For particles with diameters less than 0.1 mm, the phenomena which lead to the correlations in Equations (2), (4) and (5) must eventually converge but in the present state of knowledge it is not yet clear how this occurs.

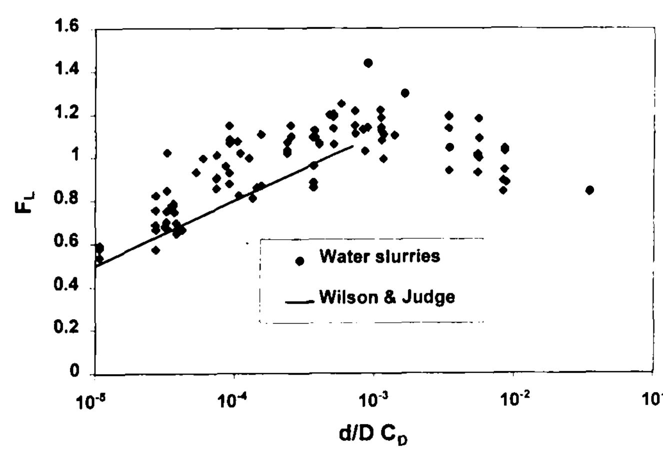

Figure 5. Comparison of experimental deposition Froude numbers with the correlation of Wilson and Judge (1976).

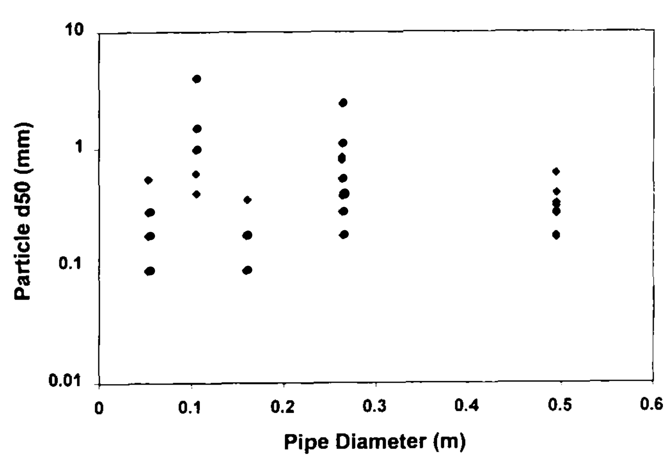

Figure 6. Scope of experimental measurements included in Figures 4 and 5.

Conclusions

1. The existing experimental evidence suggests that estimates of deposition velocity can be made using F_L values from Figure 4, provided $Ar > 80$. The scatter in the correlation is due primarily to the effect of solids concentration, with lower deposition velocities being observed at very low and very high concentrations.
2. The equation of Wilson and Judge is recommended for $Ar <$ 80 and $\Delta > 4 \times 10^{-6}$.
3. Further systematic investigation of deposition velocities is desirable for particles with diameters below 100 μm in as wide a range of pipe diameters as possible.

Nomenclature

Ar Archimedes number for particles settling in the fluid, defined by Equation (7)
C volume fraction solids (chord-average)
C_{lim} volume fraction solids in an incipient stationary deposit
C_r mean in-situ slurry concentration (volume fraction)
C_D particle drag coefficient for sedimentation in the carrier fluid
d particle median diameter, (m)
D pipe diameter, (m)
f_f Fanning friction factor for fluid, defined by Equation (3)
f_{12} Fanning friction factor at upper surface of incipient deposit
F_L Froude number at deposition
g gravitational acceleration, (m/s^2)
S_s ratio, solids density/fluid density
V bulk velocity, (m/s)
V_c deposition velocity, (m/s)
$V_{c\delta}$ deposition velocity of particles with diameters less than the viscous sublayer, (m/s)
v_∞ terminal velocity for particles settling in the carrier fluid, (m/s)

Greek Symbols

δ thickness of first detectable deposit layer, (m)
δ_L viscous sublayer thickness, (m)
Δ correlation parameter, $d/D\ C_D$
η_s coefficient of kinematic friction between incipient deposit layer and pipe wall
μ viscosity of carrier fluid, including fines, (Pa·s)
ρ density of carrier fluid, including fines, (kg/m^3)
ρ_s density of solid particles, (kg/m^3)

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