An Experimental Investigation of the Cross-flow of Power Law Liquids Past a Bundle of Cylinders and in a Bed of Stacked Screens

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The steady flow of power-law polymer solutions normal to arrays of cylinders and in a bed of screens has been investigated experimentally. Extensive pressure drop measurements have been made for three different test cells (two tube bundles and one bed made of screens) of different voidages for a series of inelastic carboxymethyl cellulose and sodium alginate aqueous solutions. The resulting values of friction factor correlate well with a modified definition of the Reynolds number based on the capillary bundle representation of the bed. The new data extend considerably the range of voidage values from ~0.6 to ~0.87. Extensive comparisons have also been made between the present experimental data and our previous calculations based on the use of simple cell models. The close correspondence between the two demonstrates the utility of such idealized analyses. All in all, the present results embrace the following ranges of physical and kinematic conditions: Reynolds number: 0.01 to ~1200; power-law flow behaviour index: 0.38 to 1; and three values of voidage, namely, 0.74, 0.78 and 0.87.

On a étudié de façon expérimentale l'écoulement en régime permanent perpendiculairement à des arrangements de cylindres et dans un lit d'écrans dans le cas de solutions de polymères obéissant à une loi de puissance. Des mesures de pertes de charge extensives ont été effectuées pour trois cellules d'essai différentes (deux faisceaux tubulaires et un lit composé d'écrans) pour différents degrés de vide pour une série de solution aqueuses de carboxyméthyle cellulose et d'alginate de sodium inélastiques. Les valeurs de facteurs de friction obtenues montrent une bonne corrélation avec une définition modifiée du nombre de Reynolds reposant sur la représentation en faisceaux capillaires du lit. Les nouvelles données étendent considérablement la gamme des valeurs de vide de ~0,6 à ~0,87. Des comparaisons extensives ont également été effectuées entre les présentes données expérimentales et nos calculs antérieurs s'appuyant sur l'utilisation de modèles de cellules simples. L'étroite correspondance entre les deux démontre l'utilité de telles analyses idéalisées. Globalement, les présents résultats couvrent les gammes de conditions physiques et cinématiques suivantes: nombre de Reynolds: 0,01 à ~1200; indice de comportement d'écoulement de loi de puissance: 0,38 à 1; et trois valeurs de degré de vide, soient 0,74, 0,78 et 0,87.

Keywords: friction factor, power-law fluids, rod bundles, screens, arrays of cylinders.

The steady cross-flow of non-Newtonian fluids past an array of cylinders represents an idealisation of many industrially important processes encountered in chemical, polymer and processing industries. For instance, this flow configuration has often been used to model the flow in the shells of the tubular heat exchangers (Adams and Bell, 1968; Prakash et al., 1987; Zukauskas and Ulinskas, 1988). Flow through screens that are used extensively for the filtration of polymer melts prior to shaping and moulding operations has often been approximated by that normal to a collection of randomly oriented cylinders (Chhabra and Richardson, 1985; Kilajanski and Dziubinski, 1996). Similarly, the autoclave process of manufacturing fibre-reinforced composites entails the important step of the flow of polymer resin through a bed of fibres (Williams et al., 1974; Skartsis et al., 1992). Further related applications include the coating of textile, paper and fibrous mats (Kyan et al., 1970; Mauret and Renaud, 1997) and in aerosol deposition applications involving hairy root cultures (Wyslouzil et al., 1997). Finally, the flow of fluids past an array of long cylinders has also been used to elucidate the flow behaviour of fluids in two-dimensional and/or fibrous porous media (Dybbs and Edwards, 1984; Drummond and Tahir, 1984; Rahli et al., 1996; Mauret and Renaud, 1997; Satheesh et al., 1999). In addition, this flow geometry has also been employed widely to delineate the role of fluid viscoelasticity in determining the frictional pressure drop in fixed beds of particles. While it is readily agreed that the actual flow field in most of the above-mentioned examples is quite complex, it can be argued that the flow can be approximated by a weighted average of the cross- and parallel flow behaviours in arrays of long cylinders. In spite of such overwhelming theoretical and practical significance, even the flow of the simplest type of non-Newtonian fluid behaviour, namely, purely shear thinning fluids, has received very little attention, though the analogous flow of Newtonian fluids has been studied extensively (Kyan et al., 1970; Drummond and Tahir, 1984; Rahli et al., 1996; Mauret and Renaud, 1997; Zukauskas, 1972). It is readily acknowledged that the parameter of central interest in all these applications is the estimation of the frictional pressure drop through arrays of cylinders of known size, the voidage of the array and as a function of the kinematic variables. The present work reports new extensive experimental results on the flow of power-law polymer solutions through arrays of cylinders and in a bed of stacked screens. However, prior to the presentation of the new results, it is instructive and useful to briefly recount the previous pertinent studies available in the literature.

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Previous Work

As mentioned previously, little theoretical and experimental studies relating to the cross-flow of power-law fluids over a bank of cylinders has been reported in the literature. From a theoretical standpoint, owing to the nonlinear constitutive viscosity model, analytical solutions are not possible for power-law fluids even when the inertial terms are neglected in the governing equations. Furthermore, in addition to the field equations, a mathematical description of the inter-cylinder interactions is needed. One approach which has been quite successful in yielding satisfactory predictions of the gross flow parameters, namely, the flow rate–pressure drop relationship, is the so-called concentric cylinders cell model (Happel, 1959; Kuwabara, 1959). Thus, based on the free surface cell model (Happel, 1959), Tripathi and Chhabra (1992, 1996) obtained approximate upper and lower bounds on pressure drop for the flow of power-law and Carreau model fluids respectively. While Tripathi and Chhabra (1992) analyzed both flow configurations, namely, cross- and axial flow of power-law fluids, their subsequent work for the Carreau model fluids was limited to the cross-flow configuration only. However, both these studies are limited to the creeping flow regime only. More recently, numerical solutions have been sought which extend the range of these predictions based on the cell models up to Reynolds number of 10 or so (Vijaysri et al., 1999; Dhotkar et al., 2000). These predictions are, however, limited to only two values of array voidage, namely, 0.5 and 0.9. Qualitatively, the drag on cylinders in shear thinning power-law fluids is reduced below the corresponding value for the flow of Newtonian fluids, though the extent of decrease in pressure drop varies from one cell model to another (Vijaysri et al., 1999; Dhotkar et al., 2000). The other approach commonly used to model such flows is to consider periodic arrays of cylinders arranged in different geometrical configurations. Thus, Skartsis et al. (1992) and Bruschke and Advani (1993) considered the creeping flow of power-law fluids over in-line and staggered arrays of cylinders. These results are restricted to two values of array voidage and to the conditions when the inertial effects are negligible. In particular, Bruschke and Advani (1993) considered different approximations based on the value of the voidage. For instance, they invoked the so-called lubrication flow approximation at low values of voidage and preferred the use of the zero vorticity cell model (Kuwabara, 1959) at high voidages. The transition zone in between these two conditions was approximated as a weighted average of these two limiting behaviours. Aside from these analytical and/or numerical results, Kozicki et al. (1966) introduced the concept of geometric parameters which are postulated to be independent of fluid behaviour. Tiu (1985) has provided an excellent appraisal of this approach and has contrasted the predictions of pressure drop for the laminar flow of purely viscous fluids through a range of geometrical configurations including cross-flow past a bundle of cylinders.

Likewise, there have been only a few experimental studies on the flow of power-law fluids over bundles of cylinders. Adams and Bell (1968) reported experimental results on pressure drop for the flow of carboxymethyl cellulose solutions normal to arrays of cylinders (voidage ~ 0.55-0.6) and they put forward an empirical correlation in terms of a friction factor and Reynolds number. Subsequently, Prakash et al. (1987) have experimentally studied the flow of carboxymethyl cellulose solutions and employed a modification of the capillary bundle approach of a porous medium. In particular, Prakash et al. (1987) were able to reconcile most of the literature data on power-law fluid flow over bundles of rods with an equation of the form of the Ergun equation (Ergun, 1952). Admittedly, combined together these results encompass Reynolds number up to about 1000, the values of array voidage covered are limited to about ~0.45~0.60 only. More recently, Sadiq et al. (1995) have reported limited results (in creeping flow regime only) in the form of a dimensionless Kozeny constant. It is somewhat surprising that, despite the availability of theoretical predictions and scant experimental results, no effort seems to have been made to perform a direct comparison between the predictions and observations (Chhabra 1993a, b; Ghosh et al., 1994). While some investigators (Vossoughi and Seyer, 1974; Barboza et al., 1979; Chmielewski et al., 1990, for instance) have employed this flow configuration to merely elucidate qualitatively the role of fluid viscoelasticity. However, in none of these studies was an attempt made to put forward predictive expressions for the frictional pressure drop.

All in all, it is thus abundantly clear from the aforementioned discussion that relatively little is known about the frictional pressure drop for the flow of power-law liquids over bundles of circular cylinders. The present study aims to alleviate this situation. In particular, extensive results on pressure drop for the flow of a range of polymer solutions through three test modules are reported herein. The paper is concluded by presenting detailed comparisons between various predictions and the present as well as the literature data.

Experimental

Materials

A series of aqueous solutions of high viscosity grade of carboxymethyl cellulose (Robert Johnson, Bombay, India) and sodium alginate (Loba Chemie, Bombay) were used as model power-law test fluids. Tap water was used as the solvent and the polymer solutions were prepared by adding small quantities of dry polymer powder accompanied by gentle stirring. Trace amounts of formalene were added to these polymer solutions to prevent their biodegradation due to bacterial growth. The approximate (% wt) concentrations used herein include 0.3% to 2% for carboxymethyl cellulose and 0.3% to 1.6% for sodium alginate. Owing to relatively low molecular weight of the polymers used herein, even the 2% CMC solution is expected to be inelastic as ascertained in our previous studies, e.g., see Srinivas and Chhabra (1992). In addition, tap water was used as a Newtonian fluid to calibrate and gauge the overall accuracy of the results gleaned in this study. The density of each solution was measured using a constant volume density bottle and their steady shear stress–shear rate behaviour was inferred by using a capillary (pipeline) viscometer.

Test Sections

Two tube banks having different layouts, namely, one with equilateral triangular and the other staggered square were used in this work, as shown schematically in Figure 1. Copper tubes of 3.17 mm outside diameter were used to fabricate the test modules. In the first case, a circular tube of 56 mm internal diameter was used to arrange the tubes (test module T1). Evidently, in this case the flow area (and thus the voidage) varies across the cross-section of the tube. The mean voidage was estimated from a knowledge of the number and size of copper tubes, and the dimensions of the pipe. This value was within

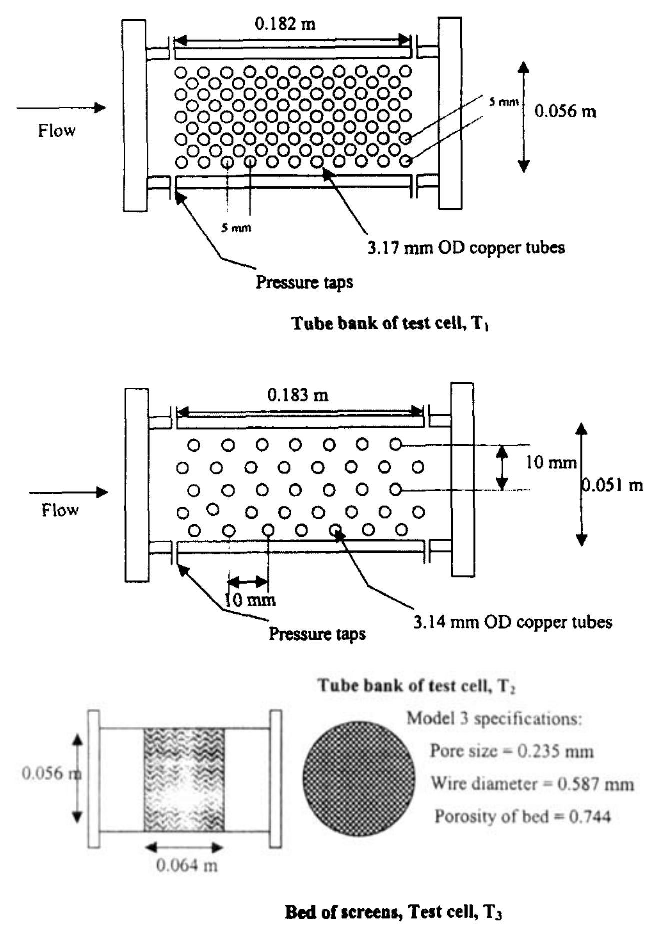

Figure 1. Schematics of test cells.

3% of the value of porosity measured by the volume of water required to fill the test cell. Thus, a mean value of $\varepsilon = 0.78$ was used for this module. The second module (T_2) was made out of a square cross-section (51 mm × 51 mm) using copper tubes of 3.14 mm OD. In this case, the flow area is uniformily distributed across the cross-section of the tube in contrast to that in T_1. This module was found to have a voidage of 0.87. Finally, a third module was prepared by packing a circular tube (diameter = 56 mm) with disks cut out of a 40 mesh screen, having an aperture of 587 μm and made of wire of diameter of 235 μ m. The overall mean voidage of this module (T_3) was found to be 0.774. Clearly, all the three modules have values of voidage which are much larger than the values used by previous investigators (Adams and Bell, 1968; Prakash et al., 1987).

Flow Loop and Capillary Viscometer

The experimental setup used in this work is shown in Figure 2. It primarily consisted of a tank to which a centrifugal pump was attached for pumping the test fluid through the flow loop including the test module as well as through a 12.7 mm inside diameter 3 m long tube used as the pipeline viscometer. The flow rate was measured using a magnetic flow meter; the calibration was checked by collecting a batch of liquid at the discharge point. The two values seldom differed by more than 0.75%. The pressure drop was measured using the simple U-tube manometers containing mercury or carbon tetra chloride beneath water as the manometric fluids. Thus, a series of measurements of pressure drop as a function of flow rate through the test module were used to develop the friction factors–Reynolds number relationship for flow across tube bundles. Similarly, the flow rate–pressure drop data in the straight section of 12.7 mm inside diameter tube was used to infer the rheological constants of polymer solutions by plotting wall shear stress as a function of the nominal shear rate at the wall.

The tank was fitted with a helical copper coil through which cooling water was circulated to remove the heat of pumping. Thus, the temperature of the test fluid did not vary by more than ±1K during the course of a test. Furthermore, the shear stress-shear rate behaviour was checked again at the end of each test to check for the possible mechanical degradation of polymer solutions due to pumping. No measurable change in shear stress-shear rate data was observed for any of the test liquids used in this study.

Results and Discussion

Rheological Behaviour of Fluids

Steady shear stress-shear rate data were obtained using the pipeline viscometer. Figure 3 shows typical results shown in the form of wall shear stress [$\tau = \{(\Delta p/L)(D/4)\}$] versus nominal shear rate at the wall ($8V/D$) on logarithmic coordinates. Included in these plots are the repeat data obtained after a test. There is no evidence of any measurable mechanical degradation and/or the effect of small temperature rise due to pumping. Furthermore, the constant value of slope, $n' = \{d \log \tau_w / d \log(8V/D)\}$ suggests the power-law type of behaviour over the shear rate range of interest here. Thus, the apparent power-law constants m' and n' are linked to the true power-law parameters m and n as:

$$m' = m\left(\frac{3n+1}{4n}\right)^n \tag{1}$$

$$n' = n \tag{2}$$

In this work, the values of n ranged from unity (for water) to 0.37 and of m from 9 mPa·s^n to 14 Pa·s^n for carboxymethyl cellulose solutions and these are summarized in Table 1. Similarly, the values of m and n for the aqueous solutions of sodium alginate are also listed in this table and these are seen to display relatively mild shear-thinning characteristics. Furthermore, the range of shear rates likely to be encountered in the flow experiments were ascertained by using an expression consistent with the capillary model (Ghosh et al., 1994) and it ranged from ~7 to 380 s^{-1} which is well within the range of tube viscometer data in each case. The density of each test fluid was measured using the constant volume density bottle and none was found to deviate from that of water by more than 0.7% to 0.8% and hence a constant value of 1000 kg/m^3 was used.

Calibration Results for Water

For each of the three test modules, water was pumped through the flow loop and in each of the test module at different flow rates and the corresponding pressure drop was measured using the simple U-tube manometers. Owing to a limitation on the pump coupled with the resulting low values of pressure drop, it was not possible to achieve very low flow rates. These results are compared with the following equation proposed by Prakash et al. (1987) and Ghosh et al. (1994):

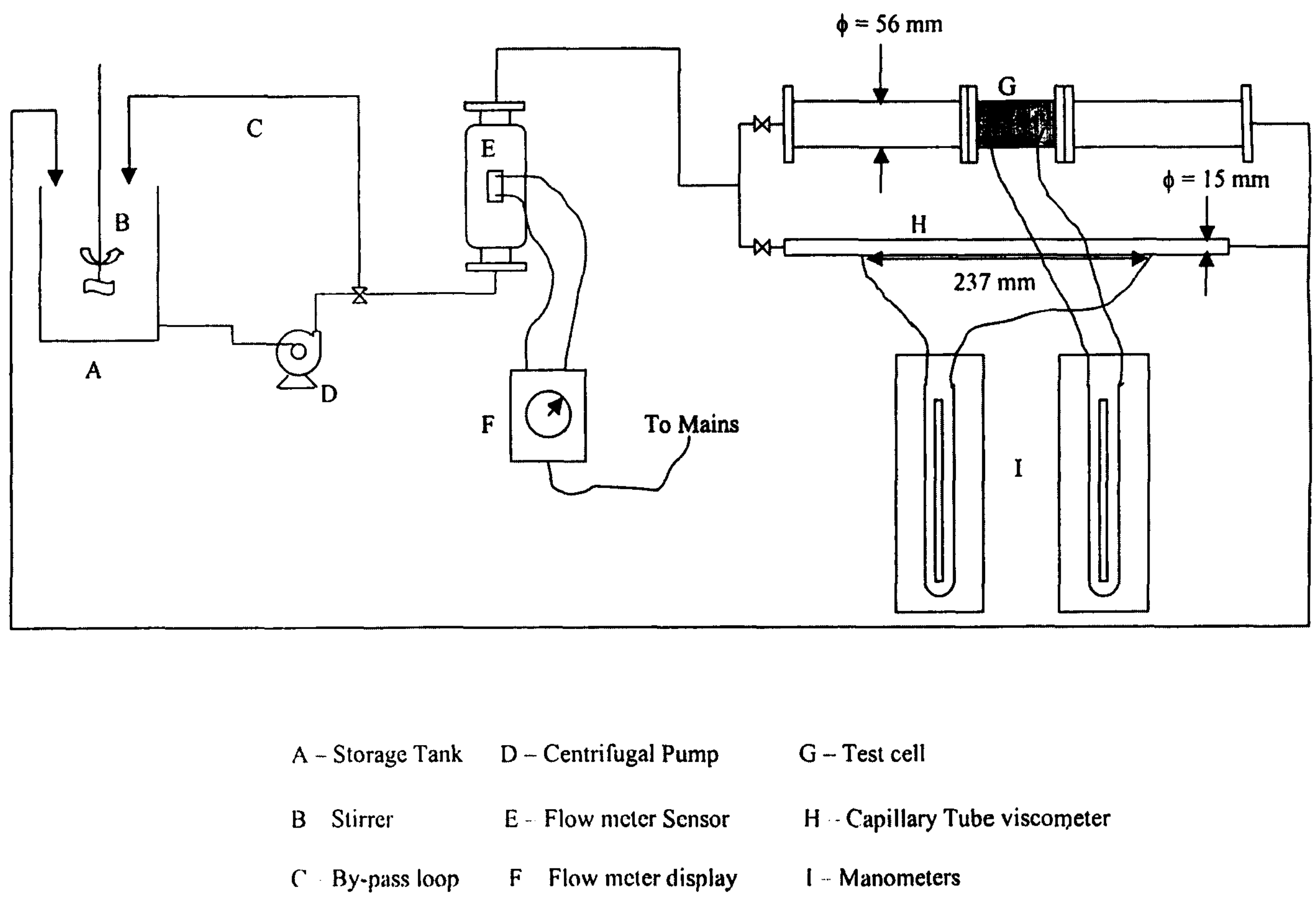

Figure 2. Schematics of flow loop.

Table 1. Summary of rheological constants of test fluids.

Polymer	Concentration (wt/wt)	Range of m' (Pa·s$^{n'}$)	Range of n'
Carboxymethyl cellulose	0.3% to 2%	0.008-14	0.37-1
Sodium alginate	0.3% to 1.6%	0.0032-0.081	0.73-0.99

$$f = \frac{64}{Re} + 0.45 \tag{3}$$

where the friction factor, f, is defined as:

$$f = \frac{(-\Delta p) d \varepsilon^3}{2 \rho V_o^2 L (1-\varepsilon)} \tag{4}$$

and the corresponding Reynolds number, Re, is given as follows:

$$Re = \frac{\rho V_o d}{\mu (1-\varepsilon)} \tag{5}$$

Figure 4 contrasts the present results with the predictions of Equation (3) where good correspondence is seen to exist

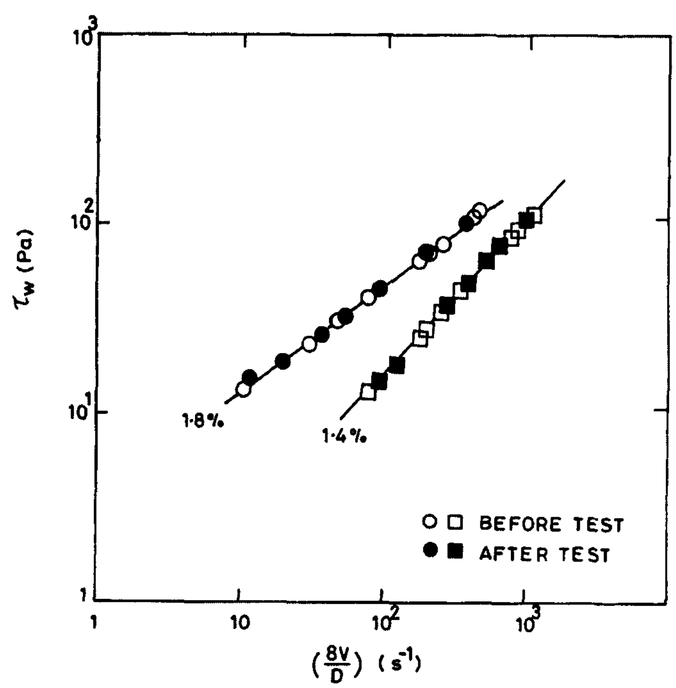

Figure 3. Representative rheograms for two test fluids (carboxymethyl cellulose).

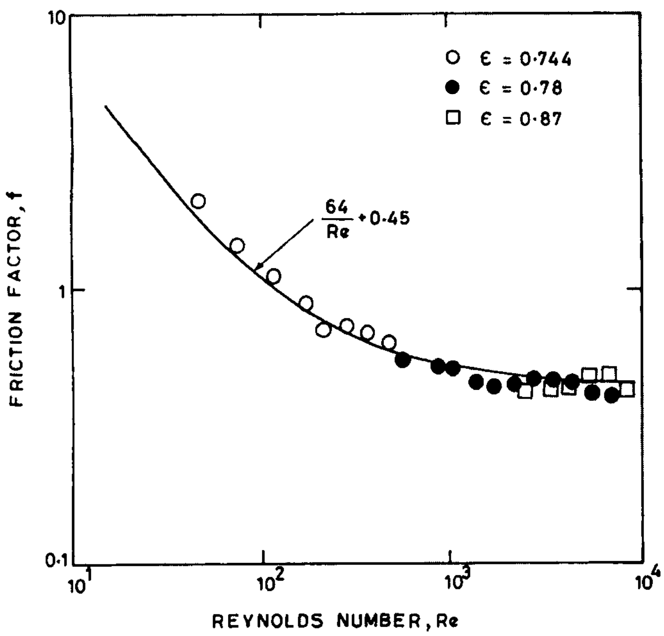

Figure 4. Calibration with water data.

between the two; the maximum error being of the order of 15%, which is well within the accuracy of Equation (3). Furthermore, keeping in mind the fact that Equation (5) is based on data relating to values of ε in the range of 0.4 to 0.6 only, the comparison shown in Figure 4 attests to its applicability to high values of ε such as that used in this work. Based on this comparison, it is perhaps reasonable to infer that the new results obtained for power-law polymer solutions entail experimental errors of the order of 15% or so.

Results for Power Law Fluids

At the outset, it is appropriate to convert the pressure drop–flow rate results into dimensionless friction factor–Reynolds number, defined appropriately, for power-law fluids. While the friction factor, f, can still be defined by Equation (4), the extension of the capillary model to include power-law fluids yields the following definition for the corresponding Reynolds number, Re_{NN} (Prakash et al., 1987; Ghosh et al., 1994):

$$Re_{NN} = \frac{\rho V_0^{2-n} d^n}{m'(8)^{n-1}\left\{\frac{(1-\varepsilon)}{\varepsilon^2}\right\}^{n-1}(1-\varepsilon)} \tag{6}$$

Evidently, in the limit of $n = 1$, Equation (6) reduces correctly to Equation (5). By assuming that the overall resistance to flow is made up of two-components, i.e., at low Reynolds number $\Delta p \propto V_0^n$ and at very high Reynolds numbers, $\Delta p \propto V_0^2$, and that these two effects are additive, Prakash et al. (1987) and Ghosh et al. (1994) put forward the following correlation for friction factor:

$$f = \frac{64}{Re_{NN}} + 0.45 \tag{7}$$

Despite the fact that the present results relate to the values of voidage which are beyond the range of applicability of Equation (7), at the outset, it is appropriate to contrast the present results with the predictions of Equation (7). Such a comparison is shown in Figures 5(a) to 5(c) separately for each value of voidage in order to look for the effect of ε, if any. Indeed, there does not seem to be any additional effect of voidage on the values of the constants appearing in Equation (7). Further detailed statistical analysis of data showed that at 95% confidence level, the constants appearing in Equation (7) were independent of porosity. This was also cross-checked by plotting the results in the form of fRe_{NN} versus Re_{NN}. The present results are seen to be in excellent agreement with the

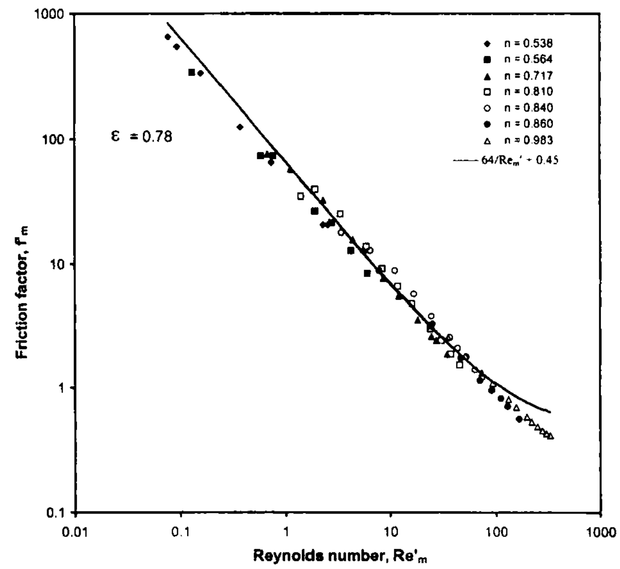

Figure 5a. Friction factor–Reynolds number relationship for test cell T_1

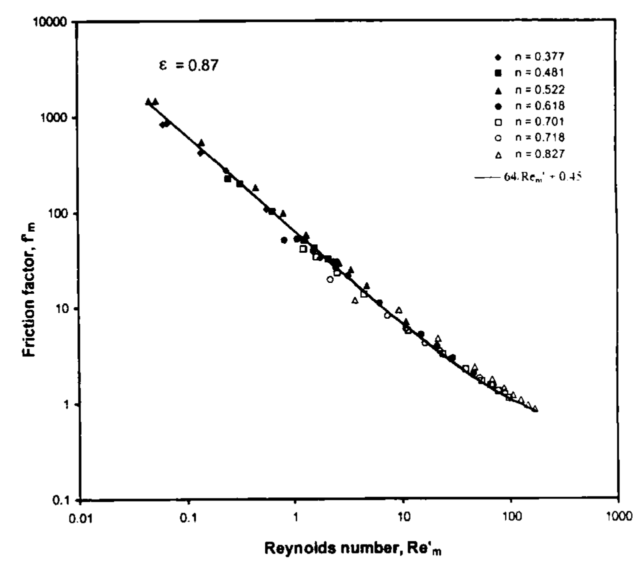

Figure 5b. Friction factor–Reynolds number relationship for test cell T_2

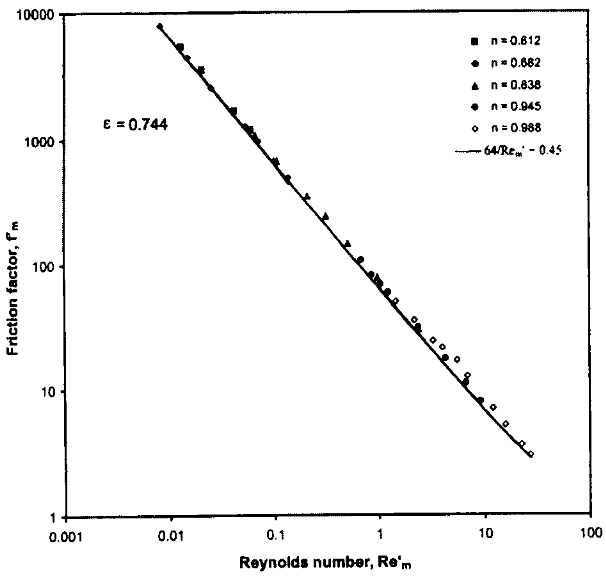

Figure 5c. Friction factor–Reynolds number relationship for test cell T_3.

predictions of Equation (7) without any discernable trends with respect to the value of the voidage or the type of flow cell. However, if regression is performed on the entire data set, the best values of the constants are found to be 69.3 and 0.59 instead of 64 and 0.45, respectively. The resulting mean and maximum errors respectively are 16% and 31% which are quite acceptable in this type of work. However, if the original values of 64 and 0.45 are used, the resulting average and maximum errors respectively are 19% and 37% which are marginally higher than that obtained with the new values. The comparisons shown in Figure 5 also seem to suggest that the pressure drop is relatively insensitive to the actual structure of the bed, arrangement of rods, etc in the range of conditions covered herein, for the results for all three test cells are equally well correlated by the same equation without any discernable trends. However, under fully turbulent conditions the friction factor (predominantly due to form drag) is expected to attain a constant value which is likely to be strongly dependent on the actual cylinder arrangement. Therefore, it is not yet possible to establish the value of the inertial term in Equation (7) with a great degree of reliability. Finally, it is likely that at such high values of porosity, complex circulation patterns are present in the system but no attempt was made to quantify this effect and this contribution is implicitly reflected in the numerical constants of Equation (7).

Comparison with Theoretical Predictions

As mentioned earlier, some theoretical results are available for the creeping flow of power-law liquids normal to an array of long cylinders. Some of these results are based on the use of the free surface cell model (Tripathi and Chhabra, 1992; Dhotkar et al., 2000) or on the zero vorticity cell model (Vijaysri et al., 1999) or periodic arrays (Bruschke and Advani; 1993; Skartsis et al., 1992). It is thus appropriate to compare the present experimental results with some of these predictions. Most of the aforementioned investigators have reported their results in terms of a drag coefficient C_D which is defined as:

$$C_D = \frac{F_D/l}{\left(\frac{1}{2}\rho V_o^2\right)(\pi d)} \tag{8}$$

The scaling of the governing equations and the boundary conditions leads to the following definition of the Reynolds number, Re_p:

$$Re_p = \frac{\rho V_o^2 d^n}{m} \tag{9}$$

Detailed analysis of theoretical results and of the computed flowfields suggests that Re_p~1 marks the limit of the so-called creeping flow (Vijaysri et al., 1999; Dhotkar et al., 2000). Though the creeping flow was seen to occur up to about Re_p~5-10 in concentrated systems (low values of voidage), $Re_p = 1$ was used in this work as the limiting value. By calculating the rate of energy dissipation per unit volume, it can easily be shown that the friction factor f and the drag coefficient C_D are inter-related as:

$$C_D = \frac{f}{\varepsilon^3} \tag{10}$$

Furthermore, since in the creeping flow regime, $C_D \propto Re_p^{-1}$, it is convenient to contrast the present experimental results with theoretical predictions using a loss coefficient defined as $\Lambda = (C_D \cdot Re_p)$. Such a comparison is shown in Table 2. Altogether there are 82 points which satisfy the criterion of the creeping flow ascertained as $Re_p < 1$. An examination of this table clearly shows that the present results for the two tube bundles used in

Table 2. Comparison between the present experimental and theoretical results.

ε	n	Value of Λ: Present Experiments	Dhotkar et al. (2000)	Vijaysri et al. (1999)
0.78	0.54	19.54	13.00	15.8
	0.56	20.02	13.50	16.8
	0.72	26.66	17.20	23.4
	0.81	29.60	19.90	27.5
	0.84	33.00	20.90	28.9
0.87	0.38	12.37	~8.50	-
	0.48	14.03	~9.40	-
	0.52	14.44	9.70	11.30
	0.62	14.84	10.60	13.25
	0.70	15.88	11.40	14.83
	0.72	17.28	11.60	15.15
0.744	0.61	36.27	16.60	21.30
	0.68	34.30	18.8	25.10
	0.84	41.95	24.7	34.2
	0.945	43.68	29.8	41.4
	0.99	47.20	32.0	44.0

this study are much closer to the predictions of the zero vorticity cell model than that based on the free surface cell model (Tripathi and Chhabra, 1992; Dhotkar et al., 2000). Though qualitatively similar observations can also be made for the bed made of screens, the deviations between experimental and predicted values are much larger in this case than that in the case of rod bundles, albeit the correspondence between the two improves as the fluid becomes less shear-thinning, i.e., the increasing value of the flow behaviour index. This is perhaps due to the additional drag arising from the crossing of wires in screens which is not accounted for in theoretical predictions. All in all, the maximum discrepancy between the predictions and the experimental results is of the order of 40% for $n = 0.61$ and $\varepsilon = 0.744$. In all other cases, the divergence is of the order of 20% to 25%. Furthermore, theoretical analyses invariably assume the cylinders to be infinitely long whereas in the present experimental studies, it is of the order of ~20. The experimental results involve a further contribution from the wall effects which would yield higher values of drag than the predictions, a trend borne out by present results. Keeping in mind all the aforementioned factors, the correspondence between the experimental results and the predictions of the zero vorticity cell model (Vijaysri et al., 1999) is regarded to be reasonable and acceptable. Similar correspondence between the scant results of Sadiq et al. (1996) and these predictions has been reported by Vijaysri et al. (1999). Unfortunately, the present results can not be compared with the predictions of Skartsis et al. (1992) and Bruschke and Advani (1993) simply because their theoretical results are not available for the values of ε encountered in this study.

Conclusions

In this work, extensive experimental results on the cross-flow of power-law liquids across bundles of rods and in a packed bed of screens are reported. In particular, the dependence of the frictional pressure drop on the volumetric flow rate has been elucidated for three values of bed voidage and for scores of shear thinning polymer solutions. The ranges of rheological parameters coupled with the flow rate are such that the Reynolds number varied by about six orders of magnitude ($0.01 \le Re_{NN} \le 1200$), thereby embracing the viscous and transitional regions. Based on the application of a capillary model, the present experimental results for high voidage systems (0.74 to 0.87) are in excellent agreement with an equation available in the literature which was initially based on data relating to $\sim 0.45 \le \varepsilon \le \sim 0.60$. This demonstrates the utility of such a simple approach in correlating friction factor results for rod bundles and screens, etc. Thus, the following expression correlates the limited prior as well as the present experimental results with the mean and maximum errors of 19% and 37%, respectively:

$$f = \frac{64}{Re_{NN}} + 0.45$$

Marginal improvement in predictability can be achieved by modifying the values of the constants as 69.3 and 0.59, respectively; however, sufficient data is not available in the fully turbulent region to ascertain the value of the constant term in this equation with a great degree of reliability.

Detailed comparisons with the predictions of the cell models are also encouraging, though these are limited only to the so-called creeping flow region only. In almost all cases, the cell models underpredict the value of the loss coefficient which can be safely attributed to the finite values of the length-to-diameter ratio for tube bundles and to crosses present in screens as opposed to the assumption of the infinitely long cylinders inherent in all theoretical analyses.

Nomenclature

C_D	drag coefficient
d	cylinder diameter, (m)
D	pipe internal diameter, (m)
f	friction factor
F_D	drag force, (N)
l	Length of cylinder in transverse direction, (m)
L	length of test cell in the axial direction, (m)
m	power-law consistency coefficient, ($Pa \cdot s^n$)
m'	apparent power-law consistency coefficient, ($Pa \cdot s^{n'}$)
n	power-law flow behaviour index
n'	apparent power-law flow behaviour index
Δp	pressure drop, (Pa)
Re	Reynolds number
Re_{NN}	Reynolds number for a power-law fluid
Re_p	modified Reynolds number
V_o	superficial velocity, ($m \cdot s^{-1}$)
V	mean velocity of liquid, ($m \cdot s^{-1}$)

Greek Symbols

ε	voidage or porosity
Λ	loss coefficient
ρ	fluid density, (kg/m^3)
μ	Newtonian viscosity, ($Pa \cdot s$)
τ	wall shear-stress, (Pa)

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