A NON-ITERATIVE DIRECT DISPLACEMENT-BASED DESIGN PROCEDURE FOR SDOF STEEL COLUMNS: USING SUBSTITUTE STRUCTURE

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ABSTRACT

Traditionally, the yield displacement of a nonlinear structure was calculated by using the direct displacement-based seismic design method which usually requires a repeatedly iterative procedure no matter whether the substitute structure or inelastic design spectra has been adopted in the procedure. This will sometimes result in inefficiency if too many iterative cycles need to be produced in a design case for convergency. To avoid this disadvantage, this paper presents a non-iterative direct displacement-based design procedure for SDOF steel columns using the substitute structure approach. By combining the yielding property with the stiffness property of the designed steel columns, the procedure can immediately obtain the column's cross-section via the chosen target displacement and ductility ratio.

Keywords : Direct displacement-based design, Substitute structure, Equivalent damping.

1. INTRODUCTION

Emphases in performance-based seismic design of structures are that the structural responses can be reliably met for a selected performance objective. To correspond with this purpose, it is necessary to develop a simple and effective method in order to design and analyze the structures. Based on the needs, several methods which included the simplified nonlinear static analysis procedure, capacity spectrum method and coefficient method, have been included in ATC-40 [1] and FEMA-273 [2] for evaluation and rehabilitation of buildings. Differently, the direct displacement-based seismic design was developed to design new constructions and only a static linear analysis was needed when employing this method. In addition, the seismic design of structures using the direct displacement-based procedure can be carried out from a specified target displacement. It should be noted that the strength and stiffness are not the design variables in the procedure but the end results.

The fountainhead of the direct displacement-based seismic design methodology can be traced back to three decades ago. In the 1970s, Gulkan and Sozen [3] proposed an approach of substitute structure using the equivalent linear systems associated with equivalent stiffness and equivalent viscous damping to predict the responses of nonlinear structures. Recently, the concept has been adopted [4~10] to design the single degree-of-freedom (SDOF) and multi-degree of freedom (MDOF) bridges or buildings. Wallace [11]; Sasani and Anderson [12]; Bachman and Dazio [13] focus on buildings with wall systems. In addition to the substitute structure approach, Court and Kowalsky [14] presented an equal displacement-based design procedure for buildings with longer periods based on the relations of displacement responses between elastic and inelastic systems. Further, Chopra and Goel [15]; Fajfar [16] modified the direct displacement-based design procedure using the inelastic design spectra.

In order to obtain the yield displacement of a designed nonlinear structure, the above literatures usually requires a repeatedly iterative procedure no matter whether the substitute structure or the inelastic design spectra was used in the direct displacement-based seismic design method. This will result in inefficiency if too many iterative cycles are required. To simplify and improve the efficiency of the procedure, this paper presents a non-iterative direct displacement-based design procedure for SDOF steel columns using the substitute structure approach and elastic design spectrum. The procedure merges the properties of yielding and elastic stiffness of the designed columns to directly obtain its cross-section and yield displacement from the chosen target displacement and ductility ratio. Design examples are implemented to illustrate the proposed method. And, the static nonlinear analyses and dynamic nonlinear time-history analyses for the

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designed examples are also carried out to assess the accuracy of the proposed non-iterative procedure.

2. THE SUBSTITUTE STRUCTURE APPROACH

The force-displacement relationship of a bilinear lumped-mass SDOF system can be illustrated in Fig. 1. Where K, α, V_y, V_u, Δ_y, Δ_u and μ are the elastic stiffness, post yield stiffness ratio, yield force, maximum force, yield displacement, target (maximum) displacement and ductility ratio ($\mu = \Delta_u/\Delta_y$), respectively. The basic concept of the substitute structure approach (Gulkan and Sozen [3]; Shibata and Sozen [17]) is to model an inelastic system by using an equivalent linear system. This implies that the maximum force and maximum displacement response of a nonlinear system can be approximately estimated by an equivalent linear system associated with equivalent secant stiffness K_{eq}, natural period T_{eq} and equivalent viscous damping ξ_{eq} [18~20].

$$T_{eq} = T_n \sqrt{\frac{\mu}{1+\alpha\,\mu-\alpha}} \tag{1}$$

$$\xi_{eq} = \xi_I + \xi_h\ ; \qquad \xi_h = \frac{1}{\pi}\left[1-\left(\frac{1-\alpha}{\mu}+\alpha\right)\right] \tag{2}$$

where T_n = elastic natural vibration period of the bilinear system; ξ_I = inherent damping of the bilinear system vibrating within its linearly elastic range; ξ_h = equivalent hysteretic damping. Notice that the equivalent viscous damping can be derived by considering the effect of ductility on damping and its value was related to the hysteretic energy absorbed. The expression of Eq. (2) used in this paper was based on the Takeda hysteretic model [4] without considering the effect of stiffness degrading.

Since the properties of the substitute structure are elastic and linear, the elastic displacement response spectra with various damping can then be used for design. Therefore, the substitute structure approach allows an inelastic system to be designed by using static linear analysis and elastic displacement response spectra.

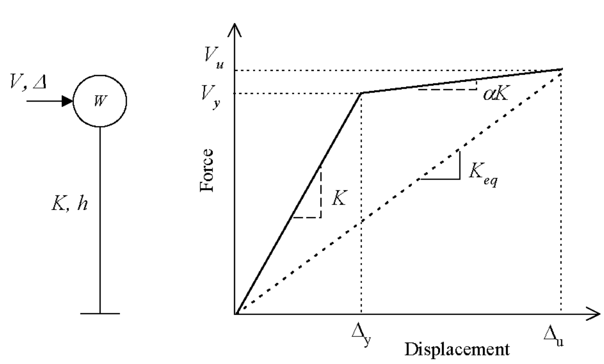

Fig. 1 Force-displacement relation of bilinear SDOF systems

3. FUNDAMENTALS OF THE NON-ITERATIVE DESIGN PROCEDURE

For the given lumped-mass SDOF steel system of Fig. 1(a), the yield force (V_y) and the yield moment (M_y) can be represented as

$$V_y = K\,\Delta_y = \frac{3EI}{h^3}\Delta_y \tag{3}$$

and

$$M_y = V_y\,h = S\,F_y \tag{4}$$

where K and Δ_y are the elastic lateral stiffness and yield displacement of the structural system, respectively. E and F_y are the elastic modulus and yield stress of steel, respectively. Moreover, I and S are the moment of inertial and section modulus of the steel cross-section, respectively. h is the height of the column.
Substitute V_y in Eq. (3) into Eq. (4) and then Eq. (4) can be rearranged as

$$\frac{I}{S} = \frac{F_y h^2}{3E\,\Delta_y} \tag{5}$$

According to the existing relationship of $S = \dfrac{I}{b/2}$ between the moment of inertial and the section modulus, I/S can be replaced with $b/2$ in Eq. (5). Rearrange Eq. (5) and then the width (b) of the designed section can be obtained as

$$b = \frac{2F_y h^2}{3E\,\Delta_y} \tag{6}$$

According to Eq. (6), the width (b) of the designed section can be determined. It shows from Eq. (6) that the section width of the designed members depends only on Δ_y and h when the elastic modulus (E) and the yield stress (F_y) of steels are pre-determined. Moreover, the yield displacement (Δ_y) of a system is related to the target displacement (Δ_u) and ductility (μ), i.e., $\mu = \Delta_u/\Delta_y$.

If a square box section is used, as shown in Fig. 2, the moment of inertia can be represented as I = $1/12[b^4 - (b - 2t)^4]$. The thickness (t) of the square box section can be determined according to Eq. (4), as shown as

$$t = \frac{1}{2}\left[b - \sqrt[4]{b^4 - \frac{6M_y\,b}{F_y}}\right] \tag{7}$$

If a circular hollow section is used (Fig. 2), the moment of inertia is $I = \pi/64\,[D^4 - (D - 2t)^4]$. The thickness of the circular hollow section can be determined from Eq. (4) as

$$t = \frac{1}{2}\left[D - \sqrt[4]{D^4 - \frac{32M_y\,D}{F_y}}\right] \tag{8}$$

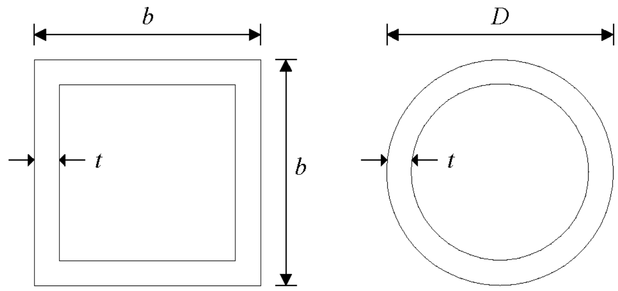

Fig. 2 The square box section and circular hollow section

It can be seen from Eqs. (7) and (8) that the thickness of the designed members depends on the width and the yield moment of the design section for a chosen yield stress (F_y).

4. STEP-BY-STEP PROCEDURE

A non-iterative direct displacement design procedure for lumped-mass SDOF steel columns using substitute structure and elastic design spectrum is shown as following steps (Fig. 3):

1. Choose a target displacement (Δ_u) and a ductility ratio (μ) for the designed structure. It should be noted that, unlike the studies carried out by Kowalsky, *et al.* [4], Chopra and Goel [15], the ductility ratio (μ) of the design structure can be pre-determined in this proposed non-iterative method.
2. The yield displacement (Δ_y) then can be calculated as $\Delta_y = \Delta_u/\mu$.
3. Estimate the equivalent viscous damping (ξ_{eq}) based on the design ductility from Eq. (2), i.e., $\xi_{eq} = \xi_I + \xi_h$.
4. Enter the elastic displacement design spectrum with the known values of Δ_u and ξ_{eq} to read T_{eq} as shown in Fig. 4. Then, the equivalent stiffness (K_{eq}) of the substitute structure can be determined according to the relationship between mass and stiffness.

$$K_{eq} = M\left(\frac{2\pi}{T_{eq}}\right)^2 \tag{9}$$

where M is the mass of the system.

5. Obtain the ultimate force (V_u), design yield force (V_y) and design yield moment (M_y).

Since the substitute structure is elastic, V_u can be calculated referring to Fig. 1(b), as shown as

$$V_u = K_{eq} \times \Delta_u \tag{10}$$

Based on the bilinear force-displacement model of Fig. 1(b), the design yield force (V_y) of the nonlinear structure can also be obtained as

$$V_y = \frac{V_u}{1+\alpha\,(\mu-1)} \tag{11}$$

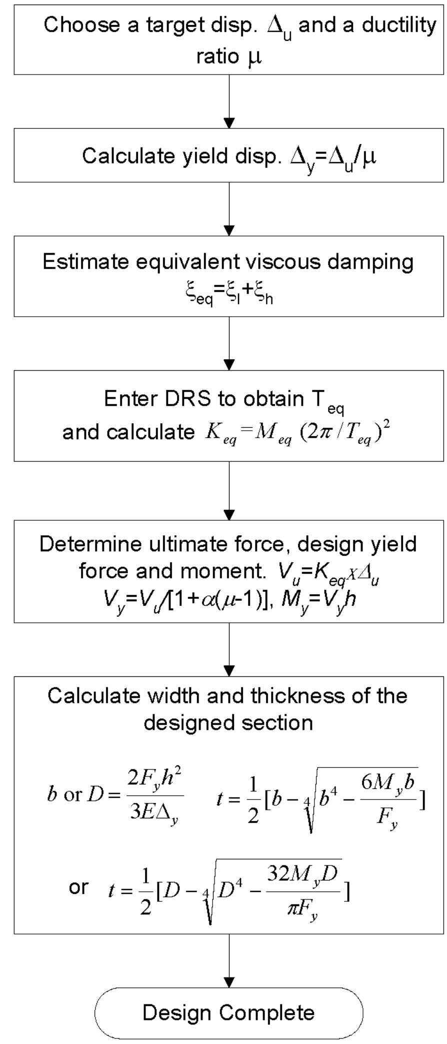

Fig. 3 Flowchart of the non-iterative direct displacement-based design procedure using substitute structure

And, the design yield moment (M_y) of the bilinear structure can be calculated as

$$M_y = V_y\,h \tag{12}$$

6. Design the structure.

According to Eq. (6), the width (b) or diameter (D) of the designed cross-section can be represented as

$$b \text{ or } D = \frac{2F_y\,h^2}{3E\,\Delta_y} \tag{6}$$

If a square box section is used, the thickness (t) of the cross-section can be obtained from Eq. (7).

$$t = \frac{1}{2}\left[b - \sqrt[4]{b^4 - \frac{6M_y\,b}{F_y}}\right] \tag{7}$$

Furthermore, if a circular hollow section is used, the thickness (t) of the cross-section can be calculated from Eq. (8).

$$t = \frac{1}{2}\left[D - \sqrt[4]{D^4 - \frac{32 M_y D}{\pi F_y}}\right] \quad (8)$$

5. DESIGN EXAMPLES

The proposed non-iterative procedure of direct displacement-based design for lumped-mass SDOF steel columns using substitute structure and elastic design spectra can be further illustrated by the following two examples. These examples are similar to those proposed by Chopra and Goel [15]. The elastic fundamental period of the first example falls in the velocity-sensitive region of the design spectrum, and that of the second example falls in the acceleration-sensitive region.

5.1 Example 1

The geometrical dimension of this case is a portion of a steel bridge. The total weigh of the superstructure (190kN/m) is supported on identical bents 9m high, uniformly spaced at 39.6m. Each bent consists of a single circular hollow column. For the transverse ground motion, the viaduct can be idealized as a SDOF system (Fig. 5) with the mass of $M = W/g$ = 190kN/m × 39.6m/9.81 = 767ton which is lumped at the top of this column. The design spectrum is shown in Fig. 4 which is the displacement response spectrum derived from an artificial earthquake (Fig. 6(a)) consisted with the Taiwan design spectrum for Soil Type II (Fig. 6(b), dot line) with the peak ground acceleration (PGA) of 0.33g. Yield stress of steel material (F_y) is 250,000kN/m^2. The modulus of elasticity of steel (E) is 2.0×10^8kN/m^2. The design procedure is illustrated as follows.

1. In this example, a drift ratio of 3% and a ductility ratio of 4 are chosen, then $\Delta_u = 3\% \times 9\text{m} = 0.27\text{m}$.
2. The yield displacement can be calculated as $\Delta_y = \Delta_u/\mu = 0.27/4 = 0.0675\text{m}$.
3. For α = 5% and μ = 4, Eq. (2) gives ξ_h = 22.68%. The equivalent viscous damping for the substitute structure is $\xi_{eq} = \xi_I + \xi_h$ = 2% + 22.68% = 24.68%.
4. Entering the displacement design spectrum (Fig. 7) of elastic systems with Δ_u = 0.27m and ξ_{eq} = 24.68%, gives T_{eq} = 2.462s. Then, the equivalent stiffness (K_{eq}) of the substitute structure is $K_{eq} = M(2\pi/T_{eq})^2 = 767(2\pi/2.462)^2$ = 4991kN/m.
5. The ultimate force, design yield force and design yield moment are shown as follows

$$V_u = K_{eq} \times \Delta_u = 4991 \times 0.27 = 1350\text{kN} \quad (13)$$

$$V_y = \frac{V_u}{1+\alpha(\mu-1)} = \frac{1350}{1+0.05(4-1)} = 1170\text{kN} \quad (14)$$

$$M_y = V_y h = 1170\text{kN} \times 9\text{m} = 10550\text{kN - m} \quad (15)$$

6. Using Eq. (6) and Eq. (8), the diameter (D) and the thickness (t) of the circular hollow column are respectively obtained as

$$D = \frac{2F_y h^2}{3E\,\Delta_y} = \frac{2 \times 250000 \times 9^2}{3 \times 2.0 \times 10^8 \times 0.0675} = 1.0\text{m} \quad (16)$$

and

$$t = \frac{1}{2}\left[D - \sqrt[4]{D^4 - \frac{32 M_y D}{\pi F_y}}\right] = \frac{1}{2}\left[1.0 - \sqrt[4]{1.0^4 - \frac{32 \times 10550 \times 1.0}{\pi \times 250000}}\right] = 0.065\text{m} \quad (17)$$

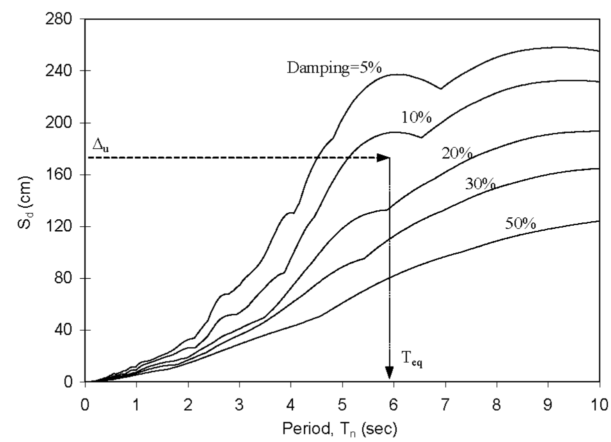

Fig. 4 Elastic disp. response spectra for soil type II of TWA building code, PGA = 0.33g

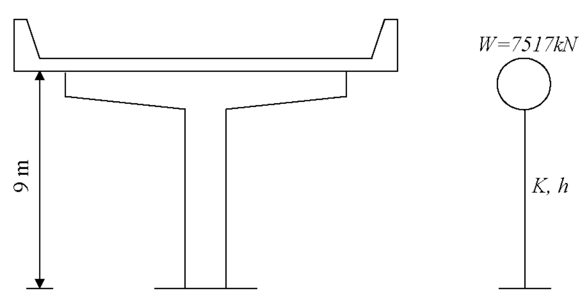

Fig. 5 Bridge example and idealized SDOF system

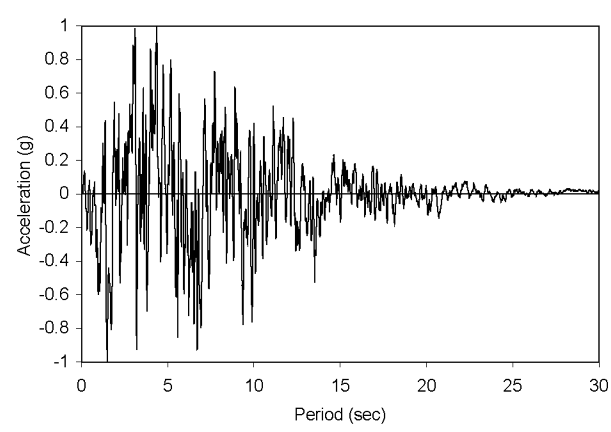

Fig. 6(a) Artificial earthquake for soil type II of TWA building code

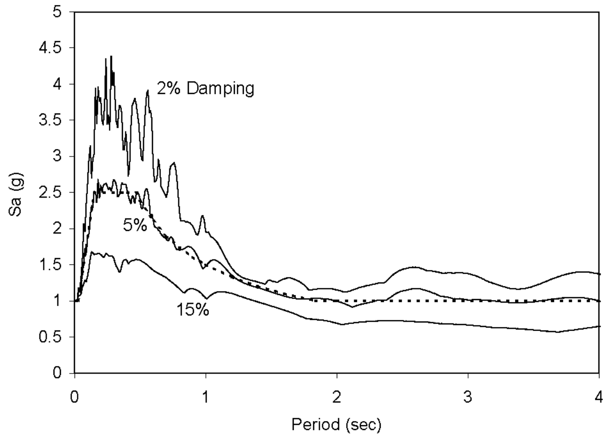

Fig. 6(b) Elastic acc. response spectra for soil type II of TWA building code

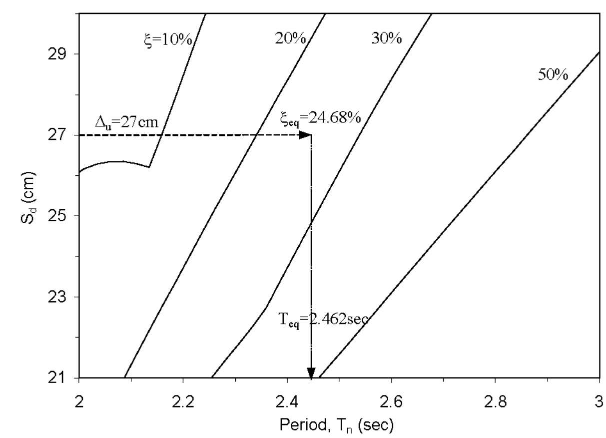

Fig. 7 Elastic disp. response spectrum for soil type II of TWA building code, PGA = 0.33g

From the above illustration, it is clear that the proposed direct displacement-based design procedure does not need any iteration. Once the target displacement and the ductility ratio were chosen, the cross-section dimensions of the designed example can be easily determined.

5.2 Example 2

The second example is the same as Example 1 except that the height of the bents is 4m. The initial elastic fundamental period of this system falls in the acceleration-sensitive region of the design spectrum. For this system, a drift ratio of 3% and a ductility ratio of 6 are chosen. Following the proposed non-iterative procedure, it is obtained that Δ_u = 0.12m, Δ_y = 0.02m, ξ_{eq} = 27.2%, T_{eq} = 1.413s, K_{eq} = 15,150kN/m, V_u = 1,820kN, V_y = 1,450kN, M_y = 5,820kN-m. The diameter (D) and the thickness (t) of the circular hollow column are 0.67m and 0.11m, respectively. Again, the proposed method does not need any iteration.

6. VERIFICATIONS USING NONLINEAR ANALYSIS

6.1 Static Nonlinear Analysis

In order to assess the accuracy of the proposed non-iterative procedure for direct displacement-based design, the static nonlinear (pushover) analysis and the dynamic inelastic time-history analysis were carried out in this section. Using the program of Drain-2D+ [21], a static and dynamic analysis program of inelastic 2D structures, Figs. 8(a) and 8(b) show the pushover curve for both Example 1 and Example 2 when the ultimate forces (V_u) were applied gradually to the top of these columns. These figures showed that the top displacements of designed structures under the static nonlinear analyses are very close to the designed values obtained from the proposed design procedure.

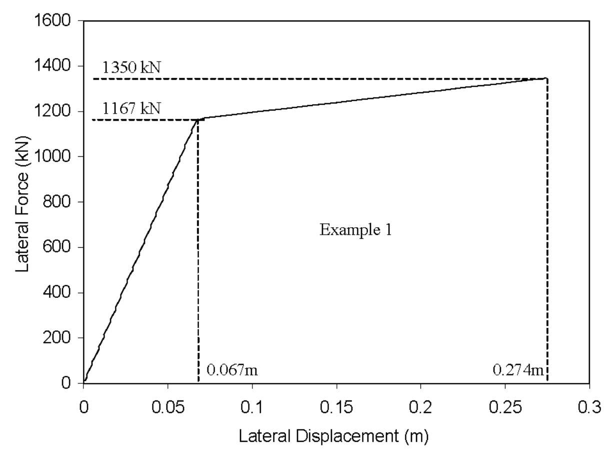

Fig. 8(a) Verification using static nonlinear, pushover, analysis for Example 1

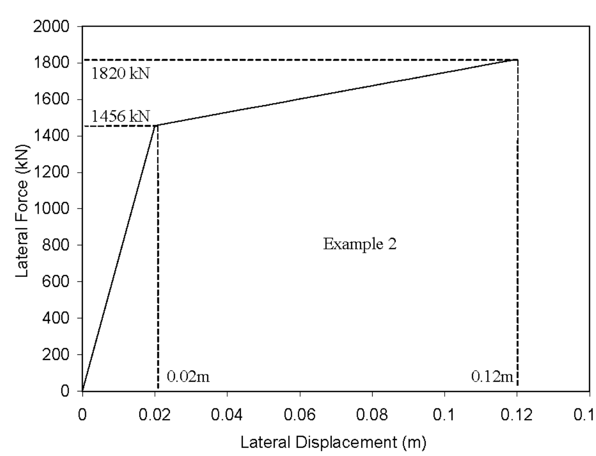

Fig. 8(b) Verification using static nonlinear, pushover, analysis for Example 2

6.2 Dynamic Nonlinear Time-History Analysis

The dynamic inelastic time-history analyses were also carried out by using the Drain-2D+ program [21]. An inherent damping ratio of 2% and a strain hardening ratio of 5% are used in the analysis. For the previous two design examples, a list of comparisons of the target displacements and yield displacements under the artificial earthquake of Fig. 6(a) with the peak ground acceleration (PGA) of 0.33g are tabulated in Table 1. The time-history responses are shown in Figs. 9(a) ~ 9(d). It can be seen that the objectives, target displacement (Figs. 9(a) and 9(c)) and yield displacement (Figs. 9(b) and 9(d)), of the designed examples can be reliably achieved by the proposed non-iterative procedure. It is noted that the yield PGA of Figs. 9(b) and 9(d) are obtained by gradually increasing the magnitude of the earthquake until the column yields.

7. CONCLUSION

For performance-based seismic engineering, the direct displacement-based design is one of the important methods to achieve the design objectives. Differing from the traditional force-based design, it eliminates the use of a force reduction factor, directly addresses the inelastic nature of a structure during earthquakes, and can provide a reliable indication of damage potential. The only initial design parameters of the method are the target displacement and the ductility ratio of the designed structures. Strength and stiffness are the results of the design procedure and are dependent on the chosen design parameters.

Table. 1 Verifications of non-iterative direct displacement-based design using dynamic nonlinear time-history analysis

		Design Value	Dynamic Nonlinear Analysis (ξ_I = 2%)	
Example 1	Δ_u (m)	0.27	0.276	0.33g
	Δ_y (m)	0.0675	0.067	0.124g
Example 2	Δ_u (m)	0.12	0.116	0.33g
	Δ_y (m)	0.02	0.02	0.068g

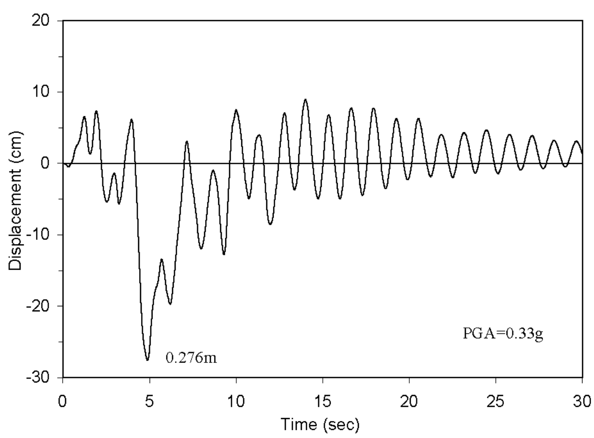

Fig. 9(a) Displacement history at the top of Example 1, PGA = 0.33g

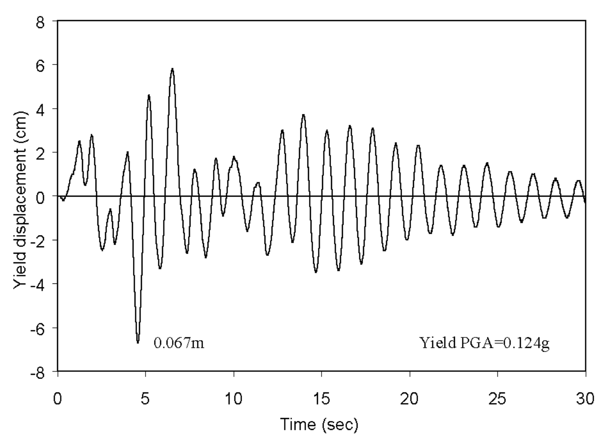

Fig. 9(b) History of yield displacement at the top of Example 1, PGA = 0.124g

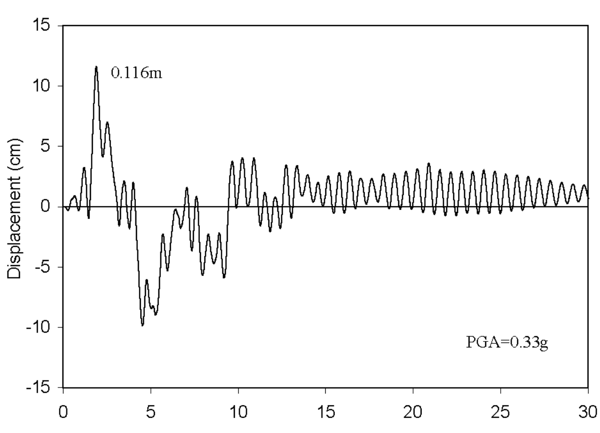

Fig. 9(c) Displacement history at the top of Example 2, PGA = 0.33g

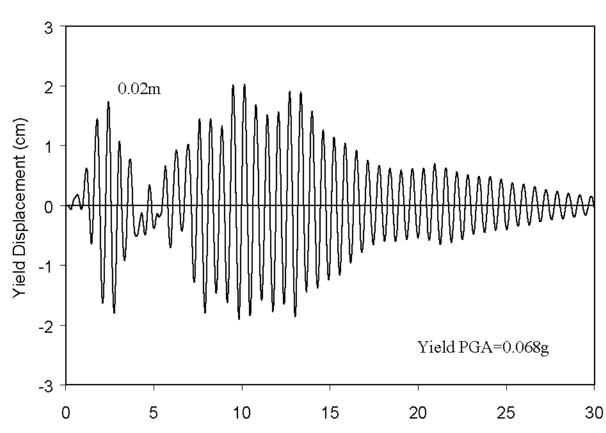

Fig. 9(d) History of yield displacement at the top of Example 2, PGA = 0.068g

However, in order to obtain the cross-sections and the yield displacement of the designed nonlinear structures, the direct displacement-based seismic design method proposed by several researchers in recent years always requires a repeatedly iterative procedure. In order to simplify and improve the efficiency of the method, a non-iterative direct displacement-based design procedure for SDOF steel columns using the substitute structure approach and elastic design spectrum has been presented in this paper. The procedure combines the elastic stiffness with the yielding properties of a designed column to obtain its cross-section and achieve the design objectives. Examples were employed to illustrate the proposed procedure in detail. In addition, the static nonlinear analyses and dynamic nonlinear time-history analyses for the designed examples were also carried out to verify the accuracy of the presented method. Results show that the objectives of the designed structure can be easily achieved via the proposed non-iterative procedure.

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