Derivation of Uniform PO Diffraction Coefficients Based on Field Equivalence Principle

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SUMMARY

A novel approach for asymptotic reduction of physical optics (PO) integration is proposed for two-dimensional line source diffraction from a half-sheet. The field equivalence principle provides alternative integration surfaces not on the original half-sheet but on the geometrical shadow (SB) and reflection (RB) boundaries, where analytical integration leads to the well-known Fresnel-type uniform PO diffraction coefficient of UTD type. The superiority of the uniform diffraction coefficient to those of other types is explained in terms of the location of the integration surfaces and is demonstrated numerically. © 2001 Scripta Technica, Electron Comm Jpn Pt 2, 84(2): 54–62, 2001

Key words: PO; PO diffraction coefficient; field equivalence principle.

1. Introduction

Physical optics (PO) [1] is a high-frequency technique in which the total induced currents **J** are approximated in the sense of geometrical optics (GO). The PO currents \mathbf{J}_{PO} thus defined are then integrated over the surface to give finite fields everywhere, including geometrical boundaries and caustics in focusing systems. PO has been widely applied to the pattern analysis of reflector antennas.

In PO, the scattering fields are obtained by evaluating the surface radiation integrals of J_{PO} , which is performed numerically in general. The asymptotic evaluation of these surface integrals [2, 3] leads us to line integral representations or closed-form expressions of fields, which greatly contributes not only to reducing the computation time but also to mechanism extraction of PO [4]. In general, the asymptotic reductions of PO surface integrals such as the geometrical theory of diffraction (GTD) become infinite at geometrical boundaries and caustics. To eliminate these difficulties, several uniform expressions have been proposed. In Ufimtsev's physical theory of diffraction (PTD) [5–7], PO currents are improved by adding another component called fringe wave currents J_{FW} . Many works about the evaluation of surface integrals for diffraction from a half-sheet are developed in the spectral domain [8–11].

Efforts to achieve surface to line integral reduction have included both an exact approach based on the Helmholtz–Huygheng principle [12–15] and asymptotic approaches such as the high-frequency approximation [16–21]. The asymptotic and local expressions are quite different from the exact and global ones and sometimes have the advantage that the former is applicable to a much wider class of scatterers based on local features of the diffraction phenomena.

In two-dimensional (2D) problems of half-sheet diffraction illuminated by a line source, the edge contribution of the PO integral is asymptotically expressed in terms of PO diffraction coefficients. PO diffraction coefficients of the classical Keller type are nonuniform at geometrical boundaries, such as shadow and reflection boundaries (SB/RB) [16–18]. Two types of uniform expressions to cope with these difficulties are available. The coefficients of the first kind were derived by directly applying the uniform asymptotic evaluation to integration on a half-sheet [3, 19, 22]. Those of the second kind have the symmetry analogous to those based on uniform theory of diffraction (UTD) [8, 10, 23–26] that have been proposed in the spectral domain [8, 10]. Though numerical comparison indicated that the second kind is superior to the first kind, comparison of the two in terms of accuracy or applicability is insufficient since the direct derivation of the second kind in the spatial domain has not been accomplished. For a full understanding of the accuracy and the applicability of the PO diffraction coefficient of UTD type, its spatial domain derivation is indispensable.

This paper presents the mathematical derivation of the uniform PO diffraction coefficient in the spatial domain for the first time. In our derivation based on the field equivalence principle [27, 28], the original PO integration surface on the half-sheet is transferred into two surfaces coinciding with GO-SB and GO-RB, on which the integrals can be evaluated analytically [29–31]. We obtain some important results by analyzing the problem in the spatial domain as follows:

- The diffraction coefficients are uniformly valid for arbitrary combinations of the angular positions of the source and the observer, provided that the distance between the source and the edge is large.
- The distance between the source and the edge is always larger than the parameter in the conventional derivation on the original half-sheet, that is, the distance between the source and the halfsheet. Therefore, the superiority of the uniform PO diffraction coefficient of UTD type is clearly identified.
- The difference between the uniform PO diffraction coefficient of UTD type and the conventional coefficient increases as the angular position of the source moves to the infinite plane including the half-sheet.

The above results are confirmed numerically.

2. Physical Optics and PO Diffraction Coefficients

We consider the 2D problem of cylindrical wave diffraction by the half-infinite conducting sheet C_0 shown in Fig. 1. In the PO approach, the induced currents (PO currents) on the surface of the sheet are given by $\mathbf{I} = 2\mathbf{\hat{n}} \times \mathbf{H}^i$, $\mathbf{M} = 0$ and hence, the total field is

$$\mathbf{E}(P) = \mathbf{E}^{i}(P) - j\omega\mu \int_{C_{0}} 2\hat{\mathbf{n}} \times \mathbf{H}^{i} G \, dx \qquad (1)$$



Fig. 1. The scattering problem of cylindrical waves by the half-sheet C_0 .

where ω is the angular frequency, μ is the permeability, $\mathbf{E}^{i}(P)$ is the incident field from the line source \mathbf{I}_{0} at the observation point P, $\mathbf{\hat{n}}$ is the unit vector normal to the half-sheet C_{0} , and G is the 2D Green's function with wave number k for the far field:

$$G = \frac{1}{4j} \sqrt{\frac{2}{\pi \, k\rho}} e^{-jk\,\rho+j\frac{\pi}{4}} \tag{2}$$

 \mathbf{H}^{i} is the magnetic field incident on C_{0} and is given as follows with the amplitude of the line current \mathbf{I}_{0} assumed to be unity:

$$\mathbf{H}^{i} = \frac{k}{4j} H_{1}^{(2)}(ks)\hat{\phi}$$
(3)

where $H_{1,\lambda}^{(2)}$ is the first-order Hankel function of the second kind, and ϕ is the unit vector in the direction of ϕ . If the distance d_{\min} between the current and the half-sheet satisfies the relation

$$kd_{\min} = kd\sin\phi_i >> 1 \tag{4}$$

Equation (3) can be approximated as

$$\mathbf{H}^{i} \approx \frac{k}{4} \sqrt{\frac{2}{\pi \, k \, s}} e^{-jks+j\frac{\pi}{4}} \hat{\boldsymbol{\phi}} \tag{5}$$

By substituting Eqs. (2) and (5) into Eq. (1), we get the PO integral to be evaluated asymptotically,

$$\mathbf{E}^{S} = -j\omega\mu\hat{\mathbf{z}}\int_{0}^{\infty} \frac{2}{\eta} \left(\frac{d}{s}\right)^{3/2} \sin\phi_{i} e^{jk(d-s)+jkx\cos\phi} dx$$
(6)

where η denotes the intrinsic impedance of free space and \hat{z} is the unit vector pointing into the positive direction along

the *z*-axis. Direct application of asymptotic theory [2] to the integral leads us to the diffracted wave

$$\mathbf{E}^{d}(P) = j\hat{\mathbf{z}}\sqrt{\frac{1}{2\pi k \rho_{0}}} e^{-jk\rho_{0}+j\frac{\pi}{4}} E_{z}^{i} D(d,\phi,\phi_{i}) \quad (7)$$

where E_z^i is the z-component of the incident electric field at the edge of half-sheet and D is the PO conventional diffraction coefficient, expressed as

$$2D = \tan\frac{\phi - \phi_i}{2} - \tan\frac{\phi + \phi_i}{2} \tag{8}$$

This coefficient diverges at GO-SB($\phi = \phi_i + \pi$) and GO-RB($\phi = \phi_i - \pi$), as the GTD diffraction coefficient of Keller does. On the other hand, the use of the uniform asymptotic method [22, 23] gives a slightly different uniform diffraction coefficient:

$$D_{direct}^{U} = -F\left(\frac{kd}{2}\left(\frac{\cos\phi + \cos\phi_{i}}{\sin\phi_{i}}\right)^{2}\right)\frac{\sin\phi_{i}}{\cos\phi + \cos\phi_{i}} \quad (9)$$

This coefficient gives finite fields even at GO-SB and GO-RB [19, 22]. F(x) is the modified Fresnel function

$$F(x) = 2j\sqrt{x} e^{jkx} \int_{\sqrt{x}}^{\infty} e^{-jt^2} dt$$
(10)

We denote the numerical integration in Eq. (6) by POapprox.inc. It is noted that in the uniform asymptotic manner [3, 19, 22] we must evaluate the integration over the extended range $(-\infty, \infty)$. In this case we have the condition $kd_{\min}^{\infty} >> 1$ instead of Eq. (4), d_{\min}^{∞} being the distance from the source to the infinite plane containing the half-sheet C_0 . This condition means that the accuracy of Eq. (9) degrades when $\phi_i \rightarrow 0^\circ$ and 180°. It proves that the diffraction coefficient of UTD type presented in this paper maintains high accuracy for such locations.

3. Transformation of Integration Plane by Field Equivalence Principle [30, 31, 33–36]

We first decompose the PO currents on the half-sheet into two parts <1> and <2>, as shown in Fig. 2. The location of the source as well as the definition $\hat{\mathbf{n}}$ in <2> is then changed to that of the image in Fig. 3, where $\mathbf{H}^{i'}$ and $\mathbf{E}^{i'}$ denote the incident fields from this rotated current \mathbf{I}_{o}^{\prime} ; these fields are related to the original incidence as $\hat{\mathbf{n}}' \times \mathbf{H}^{i'} = \hat{\mathbf{n}} \times \mathbf{H}^{i}$ and $\mathbf{E}^{i_{\prime}} = \mathbf{E}^{i}$. Now the surface currents on the two surfaces consist of the special combination of $\{\mathbf{I} = \hat{\mathbf{n}} \times \mathbf{H}^{i} \text{ and } \mathbf{M} = \mathbf{E}^{i} \times \hat{\mathbf{n}}\}$ and $\{\mathbf{I} = \hat{\mathbf{n}}' \times \mathbf{H}^{i'} \text{ and} \mathbf{M} = \mathbf{E}^{i'} \times \hat{\mathbf{n}}'\}$ for <1> and <2>, respectively, which appear in the field equivalence principle [27–29, 35]. After the



Fig. 2. The decomposition of PO currents.

decomposition in Fig. 2, we can rewrite the PO field in Eq. (1) as

$$\mathbf{E}^{\prime \circ}(p) = \mathbf{E}^{\prime}(P)$$

$$-\int_{C_{0}}^{C} \{j\omega\mu(\hat{\mathbf{n}}\times\mathbf{H}^{i})G(\mathbf{r}_{p},\mathbf{r}_{l}) + (\mathbf{E}^{i}\times\hat{\mathbf{n}})\times\nabla G(\mathbf{r}_{p},\mathbf{r}_{l})\}dx$$

$$-\int_{C_{0}}^{C} \{j\omega\mu(\hat{\mathbf{n}}^{\prime}\times\mathbf{H}^{i^{\prime}})G(\mathbf{r}_{p},\mathbf{r}_{l}) + (\mathbf{E}^{i^{\prime}}\times\hat{\mathbf{n}}^{\prime})\times\nabla G(\mathbf{r}_{p},\mathbf{r}_{l})\}dx$$
(11)

Note that the original field in Eq. (1) remains unchanged through these manipulations into Eq. (11).

In order to change the integration plane to the above decomposed problem, we define the closed curves $C \equiv C_0 \cup C_1 \cup C_\infty$ for <1>, which consist of a half-line C_0 on the half-sheet, C_1 on GO-SB, and the circular arc C_∞ with



Fig. 3. Closed curves for the field equivalence principle.

infinite radius as in Fig. 3. The closed curve *C* divides the whole space into subspaces *V* and \overline{V} . In a similar way, $C' \equiv C_0 \cup C'_1 \cup C'_{\infty}$, *V*', and \overline{V}' are defined for <2>. Note that $\mathbf{\hat{n}} \times \mathbf{H}^i$ and $\mathbf{\hat{n}}' \times \mathbf{H}^{i'}$ vanish on C_1 and C'_1 , respectively. Now the relation between integrals on C_0 and C_1 is derived based on the field equivalence principle so that the PO integrations on GO-SB (C_1) and GO-RB (C'_1). In terms of geometrical optics, there are three kinds of position of observer *P*; in the shadow region ($P \in V$), the reflection region ($P \in V'$), and the rest ($P \in \overline{V} \cap \overline{V}'$).

Applying the field equivalence principle to the closed curves *C* and *C'*, we obtain the relation between integrals as follows, under the conditions that $\mathbf{\hat{n}} \times \mathbf{H}^{i} = 0$ on *C* and $\mathbf{\hat{n}}' \times \mathbf{H}^{i'} = 0$ on *C'*:

$$\int_{C_0} \{j \omega \mu (\hat{\mathbf{n}} \times \mathbf{H}^i) G + (\mathbf{E}^i \times \hat{\mathbf{n}}) \times \nabla G \} dx$$
$$+ \int_{C_1 \cup C_{\infty}} (\mathbf{E}^i \times \hat{\mathbf{n}}) \times \nabla G \, dl = \begin{cases} \mathbf{E}^i (P) & P \in V \\ 0 & others \end{cases}$$
(12)

where $\mathbf{\hat{n}} \cdot \mathbf{E}^{i} = 0$ on *C*. In a similar way, we have

$$\int_{C_0} \left\{ j \omega \mu(\hat{\mathbf{n}}' \times \mathbf{H}') G + (\mathbf{E}' \times \hat{\mathbf{n}}') \times \nabla G \right\} dl$$
$$+ \int_{C_1' \cup C_{\infty}'} (\mathbf{E}' \times \hat{\mathbf{n}}') \times \nabla G \, dl = \left\{ \begin{array}{c} \mathbf{E}' (P) & P \in V' \\ 0 & others \end{array} \right.$$
(13)

From Eqs. (12) and (13), the PO total field in Eq. (11) is expressed in terms of integrals on C_1 and C'_1 instead of that on C_0 as

$$\mathbf{E}^{PO}(p) = \int_{C_1} (\mathbf{E}^i \times \hat{\mathbf{n}}) \times \nabla G \, dl + \begin{cases} 0 & P \in V \\ \mathbf{E}^i(P) & P \in \overline{V} \cap \overline{V}' \\ \mathbf{E}^i(P) - \mathbf{E}^i(P) & P \in V \end{cases}$$
(14)

where $-\mathbf{E}^{i}(P)$ is the incident field from the image source, that is, the reflection field at *P*. Thus, the last term in this equation indicates the geometrical optics contribution in the reflective region; we finally reach the following formula for the original diffracted field from the half-sheet.

$$\mathbf{E}^{d}(p) \equiv \mathbf{E}^{PO}(P) - \mathbf{E}^{GO}(P) = \int_{C_{1}} (\mathbf{E}^{i} \times \hat{\mathbf{n}}) \times \nabla G \, dl + \int_{C_{1}} (\mathbf{E}^{i'} \times \hat{\mathbf{n}}') \times \nabla G \, dl$$
(15)

Thus, the integral over the original semi-infinite sheet C_0 for the PO total field in Eq. (1) is now transformed into

integrals over two semi-infinite lines on shadow boundaries C_1 and C'_1 for the PO diffracted field. The above transformation is exact and its significance is that the minimum distance between the surface C_1 (and C'_1) and the source \mathbf{I}_0 (and \mathbf{I}'_0) is *d* and is always larger than d_{\min} . This loosens the restriction in Eq. (4) and enhances the accuracy of asymptotic reduction of integrals.

4. Uniform PO Diffraction Coefficient

We focus on evaluation of the integral $\int_{C_1} (\mathbf{E}^i \times \mathbf{\hat{n}}) \times \nabla G \, dl$ in the far field region, referring to Fig. 4. Let the incident electric field at Q on C_1 be

$$\mathbf{E}' = -\hat{\mathbf{z}} \frac{k\eta}{4} \mathbf{H}_{0}^{(2)}(k(d+l)) \approx -\hat{\mathbf{z}} \frac{k\eta}{4} \sqrt{\frac{2}{\pi k(d+l)}} e^{-jk(d+l)+j\frac{\pi}{4}}$$
(16)

where d and l are defined in Fig. 4. The following loose condition is assumed:

$$kd >>1 \tag{17}$$

which is satisfied more easily than condition in Eq. (4). Furthermore, since the integrals in Eq. (15) contain only the diffracted component, an important contribution to the integral comes from small *l* near the edge. Thus, the distance ρ from *Q* to the observer and that ρ_0 from the edge of the sheet to the observer are related by

$$\rho = \sqrt{\rho_0^2 + l^2 + 2\rho_0 l\cos(\phi - \phi_i)}$$
$$\approx \rho_0 + l\cos(\phi - \phi_i) \tag{18}$$



Fig. 4. The coordinate system for evaluating the integral on C_1 .

$$(\nabla G)_{\varphi} = \frac{1}{l} \frac{\partial G}{\partial \phi_{i}} = -\frac{k\rho_{0}}{j4\rho} \sin(\phi - \phi_{i}) \operatorname{H}_{1}^{(2)}(k\rho) \approx$$
$$-\frac{k\rho_{o}}{j4\rho} \sin(\phi - \phi_{i}) \sqrt{\frac{2}{\pi k \rho_{0}}} e^{-jk\rho_{0} - jkl\cos(\phi - \phi_{i}) + j\frac{3}{4}\pi}$$
(19)

Then the integral for C_1 become

$$\int_{C_1} (\mathbf{E}^{t} \times \hat{\mathbf{n}}) \times \nabla G \, dl \cong -\hat{\mathbf{z}} \frac{k^2 \eta}{j8} e^{-jk\rho_o + jkd\cos(\phi - \phi_i)}$$
$$\frac{\sin(\phi - \phi_i)}{\pi k \sqrt{\rho_0}} \int_{d}^{\infty} \frac{1}{\sqrt{u}} e^{-jku - jku\cos(\phi - \phi_i)} du \tag{20}$$

where $\rho_0/\rho \approx 1$ is used implicitly. The integral in Eq. (20) is identical to the definition of the modified Fresnel function F(x) in Eq. (10). We then get

$$\int_{C_1} (\mathbf{E}^i \times \hat{\mathbf{n}}) \times \nabla G \, dl \cong$$

$$- j \, \hat{\mathbf{z}} \frac{k\eta}{16} \sqrt{\frac{2}{\pi \, kd}} e^{-jkd + j\frac{\pi}{4}} \sqrt{\frac{2}{\pi \, k \, \rho_0}} e^{-jk \, \rho_0 + j\frac{\pi}{4}}$$

$$\cdot \mathbf{F} (2kd \cos^2 \frac{\phi - \phi_i}{2}) \tan \frac{\phi - \phi_i}{2} \qquad (21)$$

The integral on C'_1 can be obtained by simply replacing ϕ_i with $2\pi - \phi_i$ in Eq. (21), taking account of the direction of $\hat{\mathbf{n}}'$, as

$$\int_{C_{1}} (\mathbf{E}^{\prime} \times \hat{\mathbf{n}}^{\prime}) \times \nabla G \, dl \cong$$

$$j \, \hat{\mathbf{z}} \frac{k\eta}{16} \sqrt{\frac{2}{\pi \, kd}} e^{-jk \, d+j\frac{\pi}{4}} \sqrt{\frac{2}{\pi \, k \, \rho_{0}}} e^{-jk \, \rho_{0}+j\frac{\pi}{4}} \qquad (22)$$

$$\cdot \mathbf{F}(2kd \cos^{2}\frac{\phi+\phi_{i}}{2}) \tan\frac{\phi+\phi_{i}}{2}$$

We finally obtain the uniform PO diffraction coefficient of UTD type $D^{U}(d, \phi, \phi_i)$ as

$$2D^{U}(d,\phi,\phi_{i}) =$$

$$F(2kd\cos^{2}\frac{\phi-\phi_{i}}{2})\tan\frac{\phi-\phi_{i}}{2}$$

$$-F(2kd\cos^{2}\frac{\phi+\phi_{i}}{2})\tan\frac{\phi+\phi_{i}}{2}$$
(23)

This gives finite fields even at GO-SB and GO-RB, and is similar to the uniform expression in UTD [24].

5. Numerical Discussion

The superiority of the derivation is now demonstrated numerically. The necessary conditions for the new and conventional derivation in approximating the PO surface integration are (17) and (4), respectively. We compare various expressions for PO diffraction coefficients with the original PO integration (1) with exact incidence (3). They are "Uniform" defined by this paper in Eq. (23), "Uniform (Direct)" in Eq. (9), "POapprox.inc." in Eq. (1) with approximation (5), and "Nonuniform" in Eq. (8). We predict the general accuracy of various expressions as in Table 1. The key parameters used in the comparison are illustrated in Fig. 1. First, the half-sheet problems are discussed for decreasing values of d with $\phi_i = 45^\circ$ in Figs. 5(a), 5(b), and 5(c). Figures 6(a) and 6(b), 7(a), and 7(b) compare the total field for $d = 1\lambda$, 3λ , and 0.2λ , respectively. From these figures, uniform expression (23) derived for the alternative surfaces on GO-SB is the most accurate and gives results almost identical to PO for small values of d down to 0.2λ . On the other hand, the uniform expression "Uniform(Direct)" derived for theoriginal half-sheet suffers from error in Fig. 5(b) for $\phi_i = 45^\circ$ and $d = 1\lambda$ ($d_{\min} \approx$ 0.7λ). "POapprox.inc." has large errors, while "Nonuniform" has singularities at GO-SB and GO-RB. Thus, the prediction in Table 1 is confirmed.

6. Conclusion

A new derivation for the uniform diffraction coefficient for the half-sheet is provided, based on the field equivalence principle. The PO integral on the original halfsheet is transformed into semi-infinite integrals on the geometrical shadow boundaries, expressed by using the

Table 1. Prediction of accuracy (O, high accuracy; ×, low accuracy) for various integral evaluations for different source angles

	parameters	\$\$\$ \$\$ \$\$ \$\$ \$\$ \$\$ \$ \$ 	φ _i = 90•	φ _i > 90°
Uniform direct Eq. (9)	d [∞] _{min}	×	0	×
UTD form Eq. (23)	d	0	0	0
Keller form Eq. (8)		×	×	×
POapprox. inc. Eq. (6)	d _{min}	×	0	0



Fresnel function. The coefficient is valid when the distance between the source and the edge of the half-sheet is larger than about 0.2 wavelength. The superiority of the uniform diffraction coefficient to other types is explained for the first time in terms of the location of the integration surfaces and is demonstrated numerically.



Fig. 5. Accuracy of diffraction coefficients as functions of distance *d* to the edge of the half-sheet ($\phi_i = 45^\circ$).

Fig. 6. Degradation of conventional uniform diffraction coefficients for small distance $d_{\min} \approx 0.2\lambda$. (a) $\phi_i = 5^\circ$, $d = 1\lambda$; (b) $\phi_i = 175^\circ$, $d = 1\lambda$.



Fig. 7. Accuracy of diffraction coefficients for $\phi_i = 90^\circ$. (a) $a = 3\lambda$; (b) $d = 0.2\lambda$.

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Fig. 8. Accuracy of diffraction coefficients for $\phi_i = 135^\circ$. (a) $d = 3\lambda$; (b) $d = 0.2\lambda$.

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