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# ANOMALOUS LIMB DARKENING IN THE ATMOSPHERES OF COMPONENTS OF CLOSE BINARY SYSTEMS WITH A STRONG REFLECTION EFFECT

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*It is shown that in close binary systems of the Algol type, the companions of which are of spectral type K0-K2 and later, there is a temperature inversion in the surface layers of the photosphere and, as a consequence, the angular distribution of the intensity of escaping radiation for individual small areas proves to differ considerably from the limb-darkening law in the spherically symmetrical case.*

## 1. Introduction

It is generally known that in the classical theory for determining the orbital elements of eclipsing binaries one uses models based on the achievements of the theory of radiative transfer in stellar atmospheres. As the classical methods of analysis of light curves came to be replaced by methods of their synthesis in which physically unfounded assumptions of the traditional methods of the theory of eclipsing variables (spherical or ellipsoidal figures of the components, a blackbody energy distribution in the spectra of the components, the hypothesis that the matter is gray, etc.) are rejected and the achievements of the methods of model atmospheres are successively incorporated, it became possible to allow in more detail for effects of the radiative and gravitational interaction of the components. For example, Rubashevskii [1] made a very careful investigation of the limb-darkening effect with allowance for the latest results in modeling stellar atmospheres and evaluated the possibilities of the appearance of effects of nonlinearity of limb darkening by analyzing light curves of various close binary systems (CBSs). Pustyl'nik and Tomasson [2] estimated the influence of departures from local thermodynamic equilibrium (LTE) on the physical conditions in a CBS atmosphere caused by anisotropy and dilution of the companion's radiation. The use of methods of computer analysis of light curves resulted in the fact that in methods of synthesis of light curves one now takes into account a very extensive set of physical effects capable of affecting in one way or another the observed luminosity of a CBS, starting with allowance for effects of the ellipsoidality of the orbit, the influence of a third body, and ending with the calculation of the distorting influence of circumstellar gas (in the form of accretion disks, gas streams, common translucent shells, etc.). In the analysis of light curves up to now, however, a nonrigorous assumption, not yet subjected to quantitative study, has been made, that the angular distribution $I(\theta)$ of the intensity of escaping radiation (more precisely, the intensity of escaping radiation as a function of the angle between the direction toward the observer and the normal) is identical to the so-called limb-darkening law $I(\mu)$ of a CBS component. Methods of synthesis of light curves contain an implicit and, as far as we know, not previously discussed assumption that the angular intensity distribution of escaping radiation is identical for all elementary areas into which the surface of a CBS component is divided in a model calculation of light curves. Even the fullest investigation of the limb-darkening effect (see [1] and the very complete references therein) contains no mention of the fact that the aforementioned assumption that $I(\theta)$ and $I(\mu)$ are identical actually holds only in the spherically symmetrical case. Since in the vast majority of actual CBSs the assumption that the components have a spherical figure is not satisfied, at least for one of the components, this problem merits a quantitative examination. Even in those cases in which a component is spherical, the aforementioned angular intensity distribution $I(\theta)$ of escaping

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radiation and the limb darkening $I(\mu)$ should differ, in principle, if there is a pronounced reflection effect and (or) a gas stream from the companion. It seems obvious from general considerations that the presence of these effects must inevitably result in a fairly complicated brightness distribution over the stellar disk. The presence of humps on light curves, most likely associated with hot spots, observed not only in nova-like objects but also, for example, in serpentids (see, e.g., [3]), is the clearest manifestation of the effect described here, which should occur in virtually all CBSs, generally speaking.

In the present paper we calculate the angular distribution $I(\theta)$ of the intensity of escaping radiation for the secondary components of CBSs of the Algol type, and for one particular case (the total phase of the eclipse and $i = 90°$) it is compared with the limb darkening $I(\mu)$. We shall show that $I(\theta)$ and $I(\mu)$ differ strongly. We thereby demonstrate that the classical procedure for rectifying a light curve is fundamentally nonrigorous, since it does not allow for the influence of the reflection effect on the brightness distribution over the disk.

## 2. Calculation Method

We consider a binary system consisting of spherical stars with radii $r_1$ and $r_2$ in units of the semiaxis of the relative orbit, which are in circular orbits. Let the two components radiate as absolutely black bodies with effective temperatures $T_1$ and $T_2$. We consider the case of $T_1 >> T_2$ and $r_2 > r_1$, which occurs in CBSs of the Algol type, and we calculate the angular distribution of the intensity of escaping radiation for the companion in the presence of a pronounced reflection effect and under the following initial assumptions: a) the atmosphere of the companion, with an effective temperature $T_1$ and a radius $r_1$, is in LTE; b) only negative hydrogen ions $H^-$ are considered as the source of opacity (which is a good approximation to reality for the temperature range $T \approx$ 4000-6000 K under consideration); c) the actual radiation flux reaching the surface of the companion from outside is replaced by a parallel beam incident at an angle $\arccos \mu_0$.

Under these assumptions, the intensity of radiation escaping from the photosphere of the companion can be represented in the form

$$I_\lambda = \int_0^\infty B_\lambda e^{-\frac{k_\lambda \tau}{\nu}} \frac{k_\lambda dt}{\nu}, \tag{1}$$

where $B_\lambda(T)$ is the Planck source function for the wavelength $\lambda$, $\tau$ is the radial optical depth in the photosphere, $\nu$ is the cosine of the angle between the normal and the given direction, and $k_\lambda$ is the ratio of the monochromatic coefficient of absorption by $H^-$ ions to the average coefficient. The temperature distribution $T(t)$ can be calculated using the results of Sobieski's work [4], devoted to the reflection effect, or that of Pustyl'nik [5]. The temperature distribution in the companion's photosphere, according to [5], can be represented in the form

$$T(t) = T_{eff\,2} f(t), \quad f(t) = f_1(t) + f_2(t),$$

$$f_1(t) = c_1\left(c_2 e^{-k_1 t} + t + D\right)^{\frac{1}{4}}, \quad c_2 = \frac{(1-\mu_1 k_1)(1-\mu_2 k_1)}{k_1}, \quad D = \mu_1 + \mu_2 - k_1^{-1},$$

$$f_2(t) = \left\{\frac{1}{4}\gamma(\mu_0)\left(\frac{T_{eff\,1}}{T_{eff\,2}}\right)^4 \frac{r_1^2}{1-2r_2\mu + r_2^2}\left[e^{-\frac{t}{\mu_0}} - \frac{c_2\mu_0 e^{-k_1 t}}{(\mu_0-\mu_2)(\mu_0-\mu_1)} + \frac{\mu_0(1-\mu_0 k_1)}{(\mu_0-\mu_1)(\mu_0-\mu_2)k_1}\right]\right\}^{\frac{1}{4}},$$

$$c_1 = 0.9306, \quad \gamma(\mu_0) = \left(1 - \frac{1}{2}\sum_{j=1}^{2}\frac{a_j}{1+\frac{\mu_j}{\mu_0}}\right)^{-1}. \tag{2}$$

The temperature distribution (2) was obtained in [5] by solving the equation of radiative transfer by Chandrasekhar's method of discrete ordinates. In Eq. (2) $a_j$ and $\mu_j$ ($j = 1, 2$) are the weights and nodes of the Gauss quadrature formula, respectively, and $k_1$ is a root of the Chandrasekhar characteristic equation. The temperature distribution (2) can be refined with allowance for the fact that the matter is not gray (see [5] for more details). The correction for nongrayness is insignificant for the problem being considered here, however.

Sobieski [4] obtained the following expression for the temperature distribution within the framework of Eddington's approximation:

$$T(t)=\left\{\frac{3}{4}\left(t+\frac{2}{3}\right)T_{eff\,2}^{4}+3W\left[\mu_0^2+\frac{2}{3}\mu_0-\left(\mu_0^2-\frac{1}{3}\right)e^{-\frac{t}{\mu_0}}\right]\left(\frac{T_{eff\,1}}{T_{eff\,2}}\right)^4\right\}^{\frac{1}{4}}, \tag{3}$$

where $W$ is the dilution factor for the radiation of the hot component,

$$W=\frac{1}{2}\left[1-\sqrt{1-r_2^2}\right].$$

We calculate the coefficient of absorption $k_\lambda$ by $H^-$ ions as

$$k_\lambda=\left(4.158\cdot 10^{-10}\,k_\lambda\,\theta^{\frac{5}{2}}\cdot 10^{0.754\theta}+k_\lambda^2\right),$$

$$\log k_\lambda=\sum_{i=0}^{3}a_i\left(\lambda-8500\right),$$

$$\log k_\lambda^2=\sum_{i=0}^{2}\sum_{j=1}^{3}b_j\left(\log\lambda\right)^j\log^i\theta,\quad \theta=\frac{5040}{T}, \tag{4}$$

where the coefficients $a_i$ and $b_i$ were taken from Gray's monograph [6] while $T(t)$ was found from Eq. (2) or (3). The angular distribution $I(\theta)$ was calculated from Eqs. (1)-(4) for different values of $\lambda$ (1500-16,500 Å) for values of the parameters $T_{1eff}$, $T_{2eff}$, $r_1$, and $r_2$ typical for CBSs of the Algol type. In calculating the integral (1), the upper limit was taken to be $t_{max} = 20$, the integrand was approximated using cubic splines, and the number of integration points over $t$ varied in the range of 20-40. The accuracy of the calculations was estimated by doubling the number of the quadrature formula and is about 0.001 in relative units.

### 3. Results

From general considerations and the results of calculations in earlier research [4, 5], it follows that in the presence of a pronounced flux of radiation incident from outside, the temperature gradient (under the assumption of LTE in the photosphere) decreases considerably in comparison to that of a single star of the same effective temperature. Because of this, the limb-darkening effect should decrease considerably. As follows from the above calculations, for $T_{1eff} \approx 12{,}000$ K, $T_{2eff} \approx 4000$ K, $r_1 = 0.2$, and $r_2 = 0.3$ there is an inversion of the tempeerature distribution, and instead of a decrease in the radiation intensity $I(\theta)$ with increasing angle between the normal to the surface of the given small area and the given direction, one observes an increase in $I(\theta)$. This is illustrated in Figs. 1 and 2 for the two wavelengths $\lambda = 3750$ Å and $\lambda = 15{,}000$ Å. These values correspond approximately to the parameters of the CBSs RW Tau, U Cep, and WW Cyg, which we took from Svechnikov's catalog of binary stars [7].

This effect is typical for the companions of CBSs of the Algol type later than K0-K2. At temperatures on the order of $T_{1eff} \approx 6000$ K, i.e., for spectral type G and earlier, it disappears, although even in this case the limb-darkening effect is considerably smaller than for a single star.

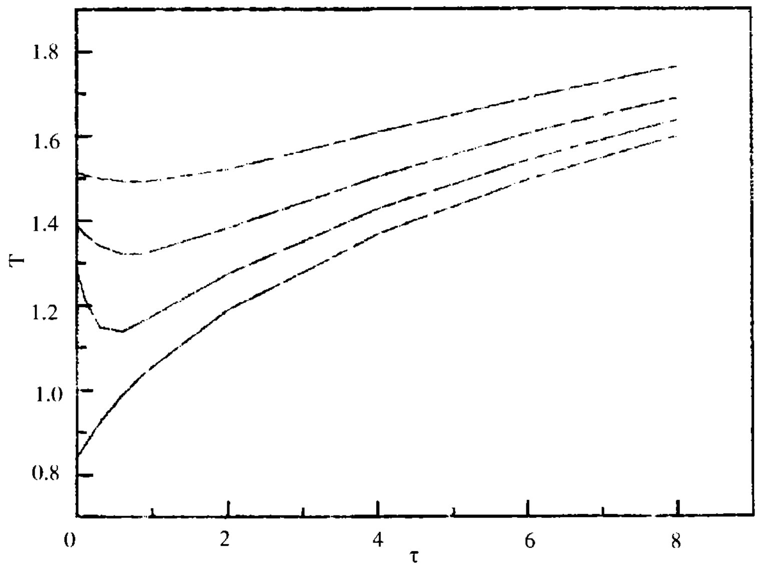

Fig. 1. Emission temperature as a function of optical depth $\tau$. For the lower curve $W = 0$, while for the other curves $W = 0.02$ and $\mu_0 = 0.2$, 0.5, and 0.97 from bottom to top.

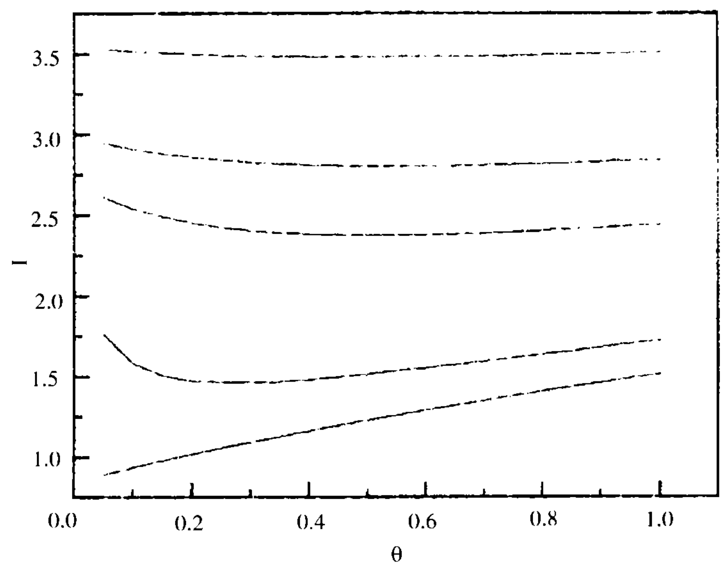

Fig. 2. Intensity of escaping radiation as a function of $\theta$. For the lower curve $W = 0$, while for the other curves $W = 0.02$ and $\mu_0 = 0.05$, 0.3, 0.5, and 0.95 from bottom to top. $\lambda = 15{,}000$ Å.

In the particular case of $i = 90°$ and at times of conjunctions ($\varphi = 0$ and $\varphi = 180°$), we can easily change from the angular dependence $I(\theta)$ of the intensity of escaping radiation to the limb-darkening law $I(\mu)$ using the simple equation

$$\mu_0 = \frac{1 - r_2 \mu}{\sqrt{1 - 2r_2 \mu + r_2^2}}, \tag{5}$$

where $\mu$ is the cosine of the angle between the direction of the line of centers of the CBS and the radial direction from the center of the companion.

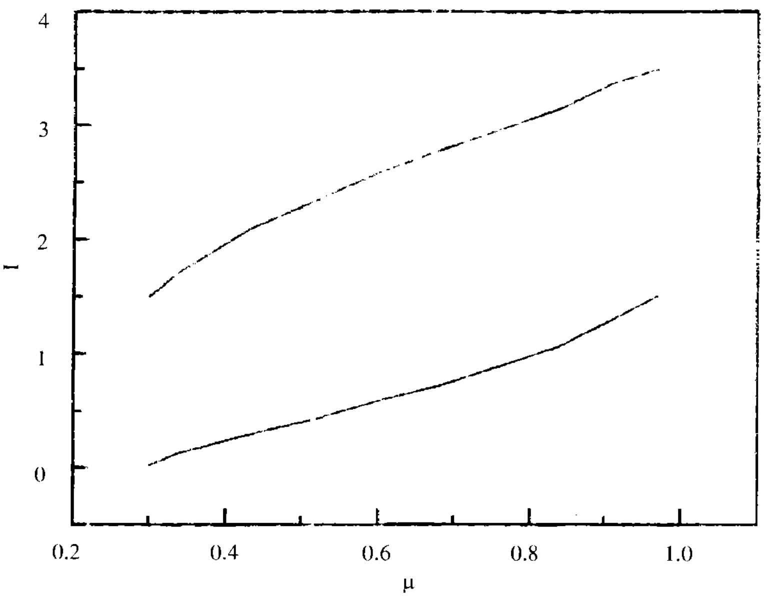


Fig. 3. Intensity of escaping radiation as a function of μ. $W = 0.02$ and $\lambda = 3750$ Å for the lower curve and $\lambda = 15{,}000$ Å for the upper one.

As seen from Figs. 1-3, the limb-darkening law proves to be nonlinear, the limb darkening is far stronger than in the case of a single star, and, which is no less important, the functions $I(\theta)$ and $I(\mu)$ differ strikingly. Since Eq. (5) does not apply to small areas on which only part of the flux from the hot component falls, it is not possible to determine the darkening effect to the very limb of the disk.

**4. Conclusion**

Thus, as the results of our calculations show, for CBSs of the Algol type, for which the effective temperatures of the components differ by a factor of 2.5-3, the reflection effect results in the need to correctly allow for anomalous limb darkening and its difference from the angular dependence of the intensity of escaping radiation for individual points on the companion's surface. In other words, it was shown that in this case the procedure for rectifying a light curve is incorrect, since it is unable to allow for the influence of irradiation on the limb-darkening law.

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