On Jumps and ARCH Effects in Natural Resource Prices: An Application to Pacific Northwest Stumpage Prices

Jean-Daniel Saphores, Lynda Khalaf, and Denis Pelletier

Continuous-time models of natural resource prices usually preclude the possibility of large changes (jumps) resulting from unexpected events. To test for the presence of jumps and/or ARCH effects, we combine bounds and the Monte Carlo test technique to obtain finite-sample, level-exact *p*-values. We apply this methodology to stumpage prices from the Pacific Northwest and find evidence of jumps and ARCH effects. To assess the impact of neglecting jumps on the decision to harvest old-growth timber, we develop an autonomous, infinite-horizon stopping model for which we provide a new method of resolution. Our numerical results show the importance of modeling jumps explicitly.

Key words: conditional heteroscedasticity, jump diffusions, Monte Carlo tests, real options, stumpage prices.

With the development of real options theory and its applications, there has been an increasing interest in modeling natural resource prices with continuous-time stochastic processes, especially the geometric Brownian motion (GBM), to deal with decisions that have both uncertain and irreversible characteristics. Examples of papers where the price of a natural resource follows a GBM include Pindyck, Brennan and Schwartz, or Lund (1992). In forestry, the GBM is often used to model timber prices, as in Morck, Schwartz, and Stangeland; Clarke and Reed (1989, 1990); Zinkhan; Thomson; Reed; Conrad and Ludwig; or Yin and Newman. While the usefulness of the geometric Brownian motion as a theoretical tool is well established, its adequacy for deriving practical decision rules for natural resources has been questioned on both theoretical and empirical grounds.

First, Lund (1993) argues that the GBM is unlikely to be an equilibrium price process for exhaustible resources produced by heterogeneous firms. Moreover, if markets are sufficiently competitive, we might expect natural resource prices to exhibit mean-reversion (Schwartz). In fact, the theoretical nature of the data generating process for prices is still an open question given all the factors contributing to their formation. Here, although our focus is on the GBM (following unit root tests), the methodology we propose is easily adaptable to mean-reverting processes.

A second line of inquiry about the adequacy of the GBM (and of pure diffusion processes in general) for modeling prices comes from the underlying assumption of smooth changes. As explained by Merton, a diffusion process precludes the possibility of large changes, or jumps, which may result from the sudden arrival of information. In forestry, these jumps can be caused, for example, by political decisions (a ban on imported timber), court rulings (logging restrictions to protect endangered species), or natural events (fires, diseases, or storms).

If the data generating process of a price time series is a jump-GBM process, the distribution of the increments of the logarithm of prices has tails heavier than the normal distribution (the so-called fat tails). This common feature has been invoked to explain discrepancies between the actual pricing of financial options and theoretical predictions.

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Fat tails may, however, also result from other processes. A tractable and popular alternative is a process with time varying parameters, such as ARCH (Engle). Another possibility is a GBM with both jumps and ARCH errors. Since the implications of these models may differ, it is important to have a statistical methodology for assessing the significance of jumps and/or ARCH effects on price volatility that is reliable with small samples (a common occurrence in natural resource economics.) To date, however, no finite-sample level-exact test for ARCH in the presence of jumps and for jumps in the presence of ARCH seems available.[1]

In this context, this article makes several contributions. First, building on earlier work by Khalaf, Saphores, and Bilodeau, we propose a methodology based on the Monte Carlo (MC) test technique (Dufour 1995) to obtain finite-sample, level-exact tests for ARCH effects in the presence of jumps, and for jumps in the presence of ARCH effects.[2] To deal with nuisance parameters, we derive exact *bound* cutoff points to make sure rejections (which provide evidence in favor of ARCH and/or jumps) are conclusive.

We then apply this methodology to four quarterly stumpage price time series from public forests in the Pacific Northwest region. Our testing strategy is particularly relevant here because stumpage price data sets are usually fairly small. To assess the adequacy of the GBM, we conduct the well-known Perron (1989) unit root test and find that we cannot reject the unit root model.[3] Since all four of our time series exhibit fat tails, we investigate ARCH and mixture distributions as probable causes and find evidence of jumps and ARCH effects.

To investigate the empirical impact of jumps, we reconsider the classical tree-cutting problem when stumpage prices for old-growth forest follow a GBM with jumps. We formulate this autonomous, infinite-horizon stopping problem in continuous time using the theory of real options (Dixit and Pindyck). The resulting Bellman equation is complicated because it contains both an integral and derivatives of an unknown function. To solve it, we propose a new approach, based on an extension of a Galerkin procedure (Delves and Mohamed), which simply requires solving a system of linear equations. We find that ignoring jumps when they are present can lead to significantly suboptimal decisions. The study of the impact of ARCH effects is left for future work because in continuous time they lead to complex stochastic variability models (Duan).

This article is organized as follows. In the next section, we present our econometric framework. We then describe the data and report our empirical results. Following the next section, we develop a simple stopping problem to assess the impact of neglecting jumps on the decision to harvest timber. The last section presents our conclusions.

Models and Tests for Jumps and Arch Effects

Let the random variable S_t denote a price (e.g., a stumpage price) at time t. If S_t follows a GBM with trend γ and variance parameter σ,

$$(1) \qquad dS_t = \gamma S_t\,dt + \sigma S_t\,dz$$

then $x_t \equiv \mathrm{Ln}(S_{t+1}) - \mathrm{Ln}(S_t)$ is normally distributed with variance σ^2 and mean $\mu = \gamma - 0.5\sigma^2$.

[1] Deriving valid p-values for no-jump likelihood ratio tests, with or without ARCH, is an econometric challenge often overlooked in empirical work for at least two reasons. First, the rate of arrival of jumps is on the boundary of its domain, and second, there are unidentified nuisance parameters under the null hypothesis. The former may cause the limiting distribution of the LR test statistic to be discontinuous (Brorsen and Yang), and the latter may cause it to be nonstandard. These problems are compounded in small samples. In addition, standard ARCH tests (such as Engle's 1982 test) may not be appropriate in the presence of jumps. Indeed, the jump process parameters (which intervene under the null and the alternative hypotheses) are identifiable if the rate of arrival of jumps is restricted to be strictly positive. This implies, however, that the nuisance parameters' space includes a locally almost unidentified region (Dufour 1997), which may seriously distort test sizes. In practice, this means that for the tests of interest here, one must seriously guard against spurious rejections.

[2] To define an exact test (or p-value), consider a test problem pertaining to a parametric model (i.e., the data generating process is determined up to a finite number of unknown real parameters $\omega \in \Omega$). Let Ω_0 refer to the subspace of Ω compatible with the null hypothesis H_0, which we suppose (without loss of generality) corresponds to a test statistic with critical region $S \geq c$. To have an α-level test, c must be chosen so that $\sup P_\omega(S \geq c \mid \omega \in \Omega_0) \leq \alpha$. This test has size α if and only if $\sup P_\omega(S \geq c \mid \omega \in \Omega_0) = \alpha$. Size control is usually very difficult to achieve.

[3] Note that the performance of unit root tests (including Perron's test) in the presence of jumps has not been formally assessed. In addition, it is well known that ARCH and/or breaks-in-variance can lead to serious under- or over-rejections in these tests (Kim and Schmidt). Since the processes we consider involve heteroscedasticity with related characteristics; this question deserves further consideration; yet it is beyond the scope of this article.

To allow for discontinuities in S_t, consider the mixed jump-diffusion process,

$$(2) \quad dS_t = \gamma S_t\,dt + \sigma S_t\,dz + S_t\,dq$$

where $\gamma\,dt$ is the expected change in S_t during dt when there is no jump, dz is an increment of a standard Wiener process, and dq is a discrete increment in S_t due to a jump. We assume that dq and dz are independent. If a jump occurs at τ, then $dq = Y_\tau - 1$, where Y_τ is the percentage change in S_τ (Merton), and 0 otherwise. More formally, Y_τ is the ratio of S_τ after a jump divided by S_τ before a jump: $Y_\tau \equiv S_{\tau+}/S_{\tau-}$ with

$$S_{\tau+} = \lim_{t\to\tau,\,t>\tau} S_t \quad \text{and} \quad S_{\tau-} = \lim_{t\to\tau,\,t<\tau} S_t.$$

Following Ball and Torous, we assume that the arrival of jumps has a Bernoulli distribution with arrival rate λ and identically independently distributed lognormal jump sizes ($\ln(Y_t) \sim N(\theta, \delta^2)$); θ is thus the mean of the logarithm of jump sizes and δ^2 is the corresponding variance. With the jump process,

$$(3) \quad x_t = \mu + \sigma\varepsilon_t + \ln(Y_t)n_t.$$

In the above, $\varepsilon_t \sim N(0,1)$ and n_t is a Bernoulli random variable. It equals one when a jump occurs (with probability λ) in the interval $[t-1;\, t]$, and zero otherwise.

An alternative to the GBM with jumps that can also produce fat tails is a GBM with ARCH(1) errors, with or without jumps. With the notation above, it can be written

$$(4) \quad x_t = \mu + \sqrt{h_t}e_t + \ln(Y_t)n_t.$$

In equation (4) $e_t \sim N(0,1)$ and the conditional variance, h_t, is defined by

$$(5) \quad h_t = \beta_0 + \beta_1(x_{t-1} - \mu)^2.$$

Duan shows that the discrete model described by (4) and (5) converges to a stochastic volatility model as the frequency of observations goes to infinity.

We estimate the parameters of these models by numerical maximization of the likelihood function of the parameter vector given the observations $x_t \equiv \mathrm{Ln}(S_{t+1}) - \mathrm{Ln}(S_t)$, $t = 1, \ldots, T$, after taking out seasonal effects and the impacts of a definition change (see below). Let ϕ designate the density of the standard normal distribution. Then the log-likelihood functions for the GBM, jump-GBM, ARCH, and jump-ARCH models are given respectively by

$$(6) \quad L_{\mathrm{GBM}} = \sum_{t=1}^{T} \ln\left[\frac{1}{\sigma}\phi\left(\frac{x_t - \mu}{\sigma}\right)\right]$$

$$(7) \quad L_{\text{Jump-GBM}} = \sum_{t=1}^{T} \ln\left[\frac{1-\lambda}{\sigma}\phi\left(\frac{x_t - \mu}{\sigma}\right) + \frac{\lambda}{\sqrt{\sigma^2+\delta^2}}\phi\left(\frac{x_t-\mu-\theta}{\sqrt{\sigma^2+\delta^2}}\right)\right]$$

$$(8) \quad L_{\mathrm{ARCH}} = \sum_{t=1}^{T} \ln\left[\frac{1}{\sqrt{h_t}}\phi\left(\frac{x_t - \mu}{\sqrt{h_t}}\right)\right]$$

$$(9) \quad L_{\text{Jump-ARCH}} = \sum_{t=1}^{T} \ln\left[\frac{1-\lambda}{\sqrt{h_t}}\phi\left(\frac{x_t - \mu}{\sqrt{h_t}}\right) + \frac{\lambda}{\sqrt{h_t+\delta^2}}\phi\left(\frac{x_t-\mu-\theta}{\sqrt{h_t+\delta^2}}\right)\right].$$

For both L_{ARCH} and $L_{\text{Jump-ARCH}}$, h_t is defined by equation (5).

To obtain evidence on ARCH/jump effects, we conduct four likelihood ratio-based (LR) tests. For ease of exposition, we introduce the notation H_{ij}, where index i refers to the absence (when $i = \overline{A}, \beta_1 = 0$) or presence (when $i = A, \beta_1 > 0$) of ARCH effects, and index j refers to the absence (when $j = \overline{J}$, $\lambda = 0$) or presence (when $j = J, \lambda > 0$) of jumps. If $LR(H_{ij}, H_{kl})$ denotes the LR statistic for testing the null, H_{ij}, against the alternative H_{kl}, hypothesis, then

$$(10) \quad LR(H_{ij}, H_{kl}) = 2\left[\widehat{L}_{H_{kl}} - \widehat{L}_{H_{ij}}\right]$$

where $\widehat{L}_{H_{ij}}$ and $\widehat{L}_{H_{kl}}$ are respectively the maximum of the log-likelihood function under the null and the alternative hypotheses. We test for jumps in the GBM($LR(H_{\overline{A}\overline{J}}, H_{\overline{A}J})$) and in the jump-ARCH ($LR(H_{A\overline{J}}, H_{AJ})$) models. We also test for ARCH in the jump-ARCH model ($LR(H_{\overline{A}J}, H_{AJ})$), and we apply Engle's no-ARCH test, denoted by $LM(H_{\overline{A}\overline{J}}, H_{A\overline{J}})$, to (4) and (5). $LM(H_{\overline{A}\overline{J}}; H_{A\overline{J}})$ is the Lagrange multiplier (LM) test for ARCH(1) effects over a GBM. It equals TR^2 from the regression of x_t^2 on a constant and x_{t-1}^2, where R^2 is the coefficient of determination and T is the sample size in terms of x_t. The asymptotic distribution of $LM(H_{\overline{A}\overline{J}}; H_{A\overline{J}})$ is $\chi^2(1)$.

It is well known that Engle's LM test tends to under-reject if $\chi^2(1)$ critical points are used (see Dufour et al. and references therein). The no-jump tests $LR(H_{A\overline{J}}, H_{AJ})$ and $LR(H_{\overline{A}\overline{J}}, H_{\overline{A}J})$ may suffer from even more serious problems: as observed in Brorsen and Yang, when the no-jump hypothesis is imposed, λ (the rate of arrival of jumps) lies on the boundary of the parameter space and the nuisance parameters θ

and δ (respectively the mean and the standard deviation of the logarithm of jumps) are not identified. Therefore, the standard χ^2-approximation to the null distribution of LR does not obtain, even asymptotically, and the statistic's limiting null distribution is nonstandard (Davies 1977, 1987; Hansen). Finally, when testing for ARCH in the presence of jumps, the nuisance parameters λ, θ, and δ are "estimable" under the null and the alternative hypothesis if the restriction $\lambda > 0$ is imposed. Although this justifies the use of standard asymptotic cutoff points, it may conceal important distributional problems because the relevant nuisance parameter space includes a locally almost unidentified (LAU) region as λ approaches the zero boundary. As demonstrated in Dufour (1997), severe size distortions may then occur with standard critical points, even if identifying restrictions are imposed. The challenge for all tests considered is thus how best to approximate the statistics' finite sample distribution under the null hypotheses.

To circumvent the unidentified nuisance parameter problem and obtain improved p-values in finite samples, we combine bounds and the MC test technique (Dufour 1995), which is closely related to the parametric bootstrap. Whereas a standard parametric bootstrap does not, in principle, guarantee size or level control for finite T (sample size) or N (replications), the MC technique provides a randomized version of a test that controls its size provided this test's null distribution can be simulated.

Let us first describe how the MC test technique may be implemented for a right-tailed LR test when there are no nuisance parameters under H_0:

1. Using the observed sample, calculate the LR statistic LR_0.
2. Using draws from the null data generating process (DGP), generate N simulated samples.
3. For simulated sample $n, 1 \le n \le N$, compute the LR statistic LR_n.
4. In $LR_0, \ldots, LR_N$, find the rank $\widehat{R}_N(LR_0)$ of the observed statistic LR_0.
5. Reject the null hypothesis at level α if $\widehat{R}_N(LR_0) \ge (N+1)(1-\alpha)+1$. A MC p-value may be obtained from:

$$\hat{p}_N(LR_0) = 1 - \frac{\widehat{R}_N(LR_0) - 1}{N+1}.$$

Note that

$$1 - \frac{\widehat{R}_N(LR_0) - 1}{N}$$

would often be used in a standard bootstrap as it relies only on asymptotic arguments.

Two of the test criteria we use, ($LR(H_{\overline{AJ}}, H_{\bar{A}J})$ and $LM(H_{\overline{AJ}}, H_{A\bar{J}})$), are pivotal (their distribution under H_0 does not depend on nuisance parameters). For $LR(H_{\overline{AJ}}, H_{\bar{A}J})$, the null hypothesis sets λ (the rate of arrival of jumps) to zero so neither θ nor δ (the mean and standard deviation of the logarithm of jumps) is identifiable. However, since the MC p-value calculated as described above by drawing from the no-jump DGP depends neither on θ nor on δ, the null distribution of $LR(H_{\overline{AJ}}, H_{\bar{A}J})$ is nuisance parameter-free. The same argument holds for $LM(H_{\overline{AJ}}, H_{A\bar{J}})$ and for $LR(H_{\overline{AJ}}, H_{AJ})$, a test statistic used below. Applying the MC test procedure to $LR(H_{\overline{AJ}}, H_{\bar{A}J})$ and $LM(H_{\overline{AJ}}, H_{A\bar{J}})$ thus yields exact size p-values.[4] To emphasize the pivotal test property, the associated MC p-value is labeled PMC ("P" stands for pivotal).

However, our other two test statistics ($LR(H_{A\bar{J}}, H_{AJ})$ and $LR(H_{\bar{A}J}, H_{AJ})$) are not pivotal. The ARCH parameter β_1 for the former and jump parameters λ, θ, and δ for the latter intervene as (identified) nuisance parameters. A standard parametric bootstrap would rely on point estimates of the nuisance parameters to generate a p-value. It would be unlikely to yield reliable results because of potential convergence failure due to boundary problems for $LR(H_{A\bar{J}}, H_{AJ})$ (λ is on the frontier of its parameter space under H_0), and to the presence of the LAU nuisance parameter λ for $LR(H_{\bar{A}J}, H_{AJ})$, which may cause spurious rejections in finite samples.

By contrast, the MC method described above can be modified to still guarantee level control: in this case, the MC p-value is defined as the largest simulated p-value over the relevant nuisance parameter space. For details on the validity and the implementation of the MC method in the presence of nuisance parameters, see Dufour (1995). However, this approach is likely to be computationally demanding. This difficulty may be avoided, however, if we can

[4] $LR(H_{\overline{AJ}}, H_{\bar{A}J})$, $LR(H_{\overline{AJ}}, H_{AJ})$, and $LM(H_{\overline{AJ}}, H_{A\bar{J}})$ can also be seen to be pivotal because under H_0, x_t follows a normal distribution for which there is invariance to location and scale (μ and σ).

find a pivotal statistic that bounds our LR test statistic. This is the approach we follow here. Indeed, both the null distributions of $LR(H_{\overline{A}J}, H_{AJ})$ and $LR(H_{A\overline{J}}, H_{AJ})$ are bounded by the null distribution of the pivotal statistic $LR(H_{\overline{A}\overline{J}}, H_{AJ})$: since $H_{\overline{A}\overline{J}} \subseteq H_{\overline{A}J}$ and $H_{\overline{A}\overline{J}} \subseteq H_{A\overline{J}}$, both $LR(H_{\overline{A}J}, H_{AJ})$ and $LR(H_{A\overline{J}}, H_{AJ})$ are smaller than or equal to $LR(H_{\overline{A}\overline{J}}, H_{AJ})$. Thus, if we use the cutoff points associated with $LR(H_{\overline{A}\overline{J}}, H_{AJ})$, we are sure that rejections are conclusive.[5] These bounding cutoff points must be obtained by simulation here since the null distribution of $LR(H_{\overline{A}\overline{J}}; H_{AJ})$ is nonstandard. The p-values thus obtained are labeled BMC (for bounds MC) while the parametric bootstrap p-value obtained in the same context are referred to as *local* MC (LMC) p-values.

To illustrate the implementation of this procedure, consider the case of $LR(H_{A\overline{J}}, H_{AJ})$:

1. Using the observed sample, estimate (4) and (5) with and without jumps to get H_{AJ} and $H_{A\overline{J}}$ respectively, then calculate the likelihood ratio $LR_0 = 2[\widehat{L}_{H_{AJ}} - \widehat{L}_{H_{A\overline{J}}}]$.
2. Generate N simulated samples drawing from the null DGP ((4) and (5) without jumps).
3. For simulated sample $n, 1 \leq n \leq N$, estimate (4) and (5) first without ARCH nor jumps and then with ARCH and jumps; compute the likelihood ratio $LR_n = 2[\widehat{L}_{H_{AJ}} - \widehat{L}_{H_{\overline{A}\overline{J}}}]$.
4. In $LR_0, \ldots, LR_N$, find the rank $\widehat{R}_N(LR_0)$ of LR_0.
5. The BMC p-value is

$$1 - \frac{\widehat{R}_N(LR_0) - 1}{N + 1}.$$

The application to $LR(H_{\overline{A}J}, H_{AJ})$ is straightforward. This conservative approach prevents spurious rejections of ARCH effects in the presence of jumps ($LR(H_{\overline{A}J}, H_{AJ})$) and of jumps in the presence of ARCH effects ($LR(H_{A\overline{J}}; H_{AJ})$), even with small samples.

We also conduct commonly used random walk diagnostic tests: (i) Perron's (1989, 1993) unit root test,[6] (ii) the Jarque–Bera (skewness and kurtosis) tests, (iii) the Ljung–Box tests, and (iv) the Lo and McKinlay variance ratio tests (Campbell, Lo, and McKinlay, chapter 2).

Application to Forestry Prices

Stumpage Data

Quarterly stumpage "cut prices" from 1973 to the first quarter of 1997 for Douglas Fir, Ponderosa and Jeffrey Pines, Western Hemlock, and True Firs were provided by the USDA Pacific Northwest Research Station, in Portland, Oregon. The "cut price" is the high-bid price adjusted for rates actually paid for timber, when the logs are scaled after harvest; it thus represents the current value of harvested timber in the marketplace. We deflate these data using the wholesale price deflators from the Bureau of Economic Analysis. The National Forest Service also publishes "sold-stumpage prices," which are three-month averages of high-bid prices for the right to harvest timber, but they are available only as an average for all tree species.

Because of the increased difficulty of logging during the winter months and the cost of storing logs that could be harvested during the summer months, there are seasonal variations in stumpage prices (Sohngen and Haynes). Fall and winter stumpage prices tend to be higher, while summer stumpage prices tend to be lower than the yearly average. In addition, since 1984, stumpage prices have included estimated purchaser credit for road construction (Haynes and Warren). To account for these seasonal effects and for the 1984 definitional change, we follow Davidson and MacKinnon (1993, chapter 19). We perform a preliminary maximum likelihood estimation (MLE) for all maintained models using a definition change dummy and three seasonal dummies. The residuals from the preliminary MLE yield a seasonally adjusted series upon which we perform our diagnostic tests and to which we fit our models. This is numerically equivalent to adding the dummies to the models analyzed.

Results

Table 1 gives a summary of diagnostic statistics for the logarithm of stumpage prices $\mathrm{Ln}(S_t)$ (Perron's test) and their change $\mathrm{Ln}(S_{t+1}) - \mathrm{Ln}(S_t)$ (Jarque–Bera, Ljung–Box,

[5] This is the basic reasoning behind the Durbin–Watson autocorrelation bounds test.

[6] We use Perron's test instead of more "standard" tests like the Dickey–Fuller unit root test to account for a 1984 definition change that may create a structural break in our data (see below and notes pertaining to table 1).

Table 1. Summary Statistics

Statistic	5% Critical Points	Douglas Fir	Ponderosa and Jeffrey Pines	Western Hemlock	True Firs
Perron (1989)	−3.93	−3.55	−2.44	−4.15	−3.79
Skewness	±0.479	0.728	−0.006	−1.276	−0.684
Kurtosis	3.52	8.907	4.698	10.686	4.710
Jarque–Bera	5.99	148.052	11.527	262.357	19.195
Autocorrelations					
Lag 1		−0.255	−0.119	−0.420	−0.341
Lag 2		0.018	−0.007	0.049	−0.056
Lag 3		−0.105	0.058	−0.062	0.016
Lag 4		−0.040	−0.190	0.111	−0.184
Ljung–Box	9.49	7.75	5.44	19.36	15.32
Variance ratios					
$Z^*(2)$	±1.96	−1.302	−0.793	−2.021	−2.126
$Z^*(4)$	±1.96	−1.232	−0.618	−1.814	−1.972

Notes: Each stumpage price sample ($S_1, \ldots, S_T$) has 97 observations. Except for Perron's test, diagnostic tests are applied to changes in the logarithm of stumpage prices ($\text{Ln}(S_{1+1}) - \text{Ln}(S_t)$) after removing seasonal effects and the impacts of a definition change.

Since standard unit root tests (e.g., Dickey–Fuller) are not reliable in the presence of structural breaks, we apply the unit root test proposed by Perron (1989, Model B) when there is an exogenous break point occurring at a known date t_B. We first de-trend each series by regressing $\text{Ln}(S_t)$ on a time trend and the structural change dummy variable $DT_t^* = t - T_B$. Let e_t^* denote the residuals from this regression. We then regress ($e_t^* - e_{t-1}^*$) on a constant, the seasonal dummies, and e_{t-1}^*. The standard t statistic associated with e_{t-1}^* yields a valid unit root test criterion, provided cutoff points form Perron (1993, table I) are used. Perron shows that these cutoff points are typically farther in the tails than the corresponding Dickey–Fuller tests critical points. In connection, see Perron (1989, p. 1378).

Critical points for the skewness and kurtosis tests are taken from D'Agostino and Stephens (1986, table 9.3, p. 379 and table 9.5, p. 385). Under the null hypothesis of normality and relevant regularity conditions, the asymptotic distribution of the Jarque–Bera is $\chi^2(2)$.

and variance ratios) after taking out seasonal effects and the impacts of a definition change. First, except for Western Hemlock, Perron's test fails to reject the presence of a unit root at the 5% level. Note, however, that in this case, Perron's test is not significant at 2.5%. This result gives some support to our choice of GBM-based models, but it must be qualified given that the performance of unit root tests in the presence of jumps has not been formally assessed. Second, we observe that for three of the series, logarithmic changes are skewed at 5% (the exception is Ponderosa and Jeffrey Pines), and we see evidence of high kurtosis (especially for Douglas Fir and Western Hemlock). This is confirmed by the Jarque–Bera statistic, which is significant at 5% for all times series, a clear sign of fat-tailed distributions. Third, the Ljung–Box autocorrelation test is significant at 5% for Western Hemlock and True Firs, but not for Douglas Fir or Ponderosa and Jeffrey Pines. This effect, however, may be due to the presence of heteroscedasticity that distorts the test's size in smaller samples (see Jorion, p. 432). Fourth, the variance ratio tests also reject the random walk null for Western Hemlock and True Firs. As is well known, caution must be exercised in interpreting decisions from a battery of diagnostic tests. Yet on the whole, we can conclude that the GBM hypothesis seems soundly rejected.[7]

Table 2 shows the parameters estimated by fitting a GBM, a jump-GBM, an ARCH, and a jump-ARCH to the four series considered. These parameters were obtained by numerical maximization of the corresponding log-likelihood functions (see equations (6) to (9)), using GAUSS. Looking first at the continuous components of our models, we note that μ, the trend parameter for $\text{Ln}(S_t)$, appears to be close to 0, with the possible exception of the jump-ARCH for Ponderosa and Jeffrey Pines, and both jump models for True Firs. As expected, the inclusion of a jump process reduces the variance of the continuous process. Looking at the jump parameters, we observe that the rate of arrival of jumps (λ) varies from ~0.2 (i.e., one jump every 5 quarters on average) for Douglas Fir to between ~0.4 and ~0.5 (one jump every 2 to 2.5 quarters on average) for True Firs.[8] We also note that θ

[7] In a discrete time framework, Haight and Holmes find that quarterly (average) stumpage prices of Loblolly Pine follow a GBM, but that nonaveraged quarterly or monthly stumpage prices follow stationary autoregressive models. They do not, however, consider the presence of seasonal effects or of jumps in their data.

[8] λ is smaller for Western Hemlock and somewhat larger for Ponderosa and Jeffrey Pines but we will see below that jumps are not statistically significant for these two series.

Table 2. Maximum Likelihood Parameters

Parameter	μ	σ	β_0	β_1	λ	θ	δ
			Douglas Fir				
GBM	−0.014	0.370					
	(0.038)	(0.027)					
Jump-GBM	0.003	0.191			0.219	−0.068	0.670
	(0.029)	(0.032)			(0.106)	(0.170)	(0.164)
ARCH	−0.047		0.097	0.274			
	(0.033)		(0.017)	(0.154)			
Jump-ARCH	−0.014		0.023	0.258	0.199	−0.161	0.590
	(0.023)		(0.007)	(0.114)	(0.085)	(0.163)	(0.141)
			Ponderosa and Jeffrey Pines				
GBM	0.003	0.320					
	(0.033)	(0.023)					
Jump-GBM	0.060	0.160			0.514	−0.113	0.376
	(0.042)	(0.061)			(0.241)	(0.088)	(0.063)
ARCH	0.051		0.044	0.614			
	(0.024)		(0.010)	(0.227)			
Jump-ARCH	0.116		0.002	0.724	0.630	−0.151	0.208
	(0.013)		(0.002)	(0.250)	(0.153)	(0.048)	(0.037)
			Western Hemlock				
GBM	−0.025	0.519					
	(0.053)	(0.038)					
Jump-GBM	−0.005	0.407			0.042	−0.485	1.472
	(0.047)	(0.052)			(0.062)	(1.303)	(0.807)
ARCH	0.013		0.096	0.714			
	(0.036)		(0.019)	(0.214)			
Jump-ARCH	0.031		0.028	0.718	0.380	−0.018	0.415
	(0.035)		(0.017)	(0.230)	(0.223)	(0.118)	(0.113)
			True Firs				
GBM	−0.025	0.478					
	(0.049)	(0.035)					
Jump-GBM	0.106	0.228			0.503	−0.259	0.559
	(0.052)	(0.054)			(0.156)	(0.136)	(0.082)
ARCH	−0.040		0.185	0.171			
	(0.044)		(0.032)	(0.108)			
Jump-ARCH	0.105		0.060	0.123	0.368	−0.419	0.484
	(0.061)		(0.027)	(0.073)	(0.176)	(0.211)	(0.107)

Note: Let $x_t = \text{Ln}(S_{t+1}) - \text{Ln}(S_t)$. For the GBM, $x_t \sim N(\mu, \sigma^2)$. For ARCH, $x_t \sim N(\mu, h_t)$, $h_t = \beta_0 + \beta_1(x_{t-1} - \mu)^2$. Jump process: λ is the arrival rate; $\ln Y \sim N(\theta, \delta^2)$ gives the jump size. The standard error of a parameter is below it in parentheses.

(the mean size of the logarithm of jumps) appears to be negative, so jumps tend to decrease stumpage prices. The precision of the parameter estimates is, of course, affected by the small size of our samples. Even though we report them, asymptotic standard errors (and related t-tests) should not be used to assess the precision of estimates because the theoretical econometric literature cited above shows that it is impossible to control the size of these tests with the models considered. This is why we use LR tests and control the level of all tests conducted.

Table 3 presents the statistics for the four tests described above calculated with $N = 99$ replications. For $LM(H_{\overline{AJ}}, H_{A\overline{J}})$ and $LR(H_{\overline{AJ}}, H_{\overline{A}J})$, we report the PMC (size-exact) p-value while for $LR(H_{\overline{A}J}, H_{AJ})$ and $LR(H_{A\overline{J}}, H_{AJ})$, we give the [LMC, BMC] p-values which are respectively the local bootstrap and the bounds level-exact p-values. Looking first at $LM(H_{\overline{AJ}}, H_{A\overline{J}})$, we find ARCH(1) effects over the GBM significant at the 2% level for Douglas Fir, Western Hemlock, and True Firs, and at 1% for Ponderosa and Jeffrey Pines. The data do, however, also support the presence of jumps over a simple GBM ($LR(H_{\overline{AJ}}, H_{\overline{A}J})$), except for Ponderosa and Jeffrey Pines: we find significant jumps at the 1% level for Douglas

Table 3. Tests for ARCH and Jumps

	$LM(H_{\overline{A}\overline{J}}, H_{A\overline{J}})$	$LR(H_{\overline{A}J}, H_{AJ})$	$LR(H_{\overline{A}\overline{J}}, H_{\overline{A}J})$	$LR(H_{A\overline{J}}, H_{AJ})$
Douglas Fir	8.30	11.97	34.57	31.31
	[0.02]	[0.01, 0.13]	[0.01]	[0.01, 0.01]
Ponderosa and Jeffrey Pines	24.10	25.36	9.60	12.72
	[0.01]	[0.01, 0.01]	[0.13]	[0.05, 0.11]
Western Hemlock	8.34	15.53	23.89	4.92
	[0.02]	[0.01, 0.02]	[0.01]	[0.29, 0.69]
True Firs	8.97	6.10	12.07	11.34
	[0.02]	[0.06, 0.54]	[0.05]	[0.09, 0.14]

Notes: $LM(H_{\overline{A}\overline{J}}; H_{A\overline{J}})$ is Engle's Lagrange multiplier test for ARCH(1) effects over a GBM. With reference to (4) and (5), it tests $H_0 : \beta_1 = 0$ versus $H_A : \beta_1 > 0$, when $\lambda = 0$.
$LR(H_{ij}; H_{kl}) \equiv 2[\widehat{L}_{H_{kl}} - \widehat{L}_{H_{ij}}]$ is the likelihood ratio statistic for testing the null, H_{ij}, against the alternative hypothesis H_{kl}. In H_{ij}, index i refers to the absence (when $i = \overline{A}, \beta_1 = 0$) or presence (when $i = A$, $\beta_1 > 0$) of ARCH effects, and index j refers to the absence (when $j = \overline{J}, \lambda = 0$) or presence (when $j = J, \lambda > 0$) of jumps. $\widehat{L}_{H_{ij}}$ and $\widehat{L}_{H_{kl}}$ are respectively the maximum of the log-likelihood function under the null and the alternative hypotheses. With reference to (4) and (5), $LR(H_{\overline{A}J}, H_{AJ})$ thus tests $H_0 : \beta_1 = 0$ versus $H_A : \beta_1 > 0$ with $\lambda \geq 0$, $LR(H_{\overline{A}\overline{J}}, H_{\overline{A}J})$ tests $H_0 : \lambda = 0$ versus $H_A : \lambda > 0$ when $\beta_1 = 0$, and $LR(H_{A\overline{J}}, H_{AJ})$ tests $H_0 : \lambda = 0$ versus $H_A : \lambda > 0$ with $\beta_1 \geq 0$.
This table reports the values of the test statistics $LM(H_{\overline{A}\overline{J}}, H_{A\overline{J}})$, $LR(H_{\overline{A}J}, H_{AJ})$, $LR(H_{\overline{A}\overline{J}}, H_{\overline{A}J})$, and $LR(H_{A\overline{J}}, H_{AJ})$ as well as MC p-values in brackets. MC p-values are calculated with $N = 99$ replications. For $LM(H_{\overline{A}\overline{J}}, H_{A\overline{J}})$ and $LR(H_{\overline{A}\overline{J}}, H_{\overline{A}J})$, we report the PMC (exact-size) p-value. For $LR(H_{\overline{A}J}, H_{AJ})$ and $LR(H_{A\overline{J}}, H_{AJ})$, we report the [LMC, BMC] p-values which are respectively the local bootstrap and the bound level-exact p-values (see section on models and tests).

Fir and Western Hemlock and at 5% for True Firs. As mentioned above, both ARCH effects and jumps could produce the observed fat tails. Since both tests are exact, rejections are statistically sound.

To sort out the contributions of ARCH and jumps, we test for ARCH effects in the presence of jumps ($LR(H_{\overline{A}J}, H_{AJ})$) and for jumps in the presence of ARCH effects ($LR(H_{A\overline{J}}, H_{AJ})$). We find that we cannot exclude the presence of ARCH over a jump-GBM model, at the 1% level for Ponderosa and Jeffrey Pines, and at 2% for Western Hemlock. If ARCH effects over a jump-GBM model may possibly be ruled out for True Firs, our tests do not uphold a definite (nonspurious) answer to this problem for Douglas Fir (the bootstrap and MC bounds p-value are 0.01 and 0.13, respectively).[9] On the other hand, there is strong evidence of jumps over an ARCH model for Douglas Fir (at 1%) and to a lesser degree for Ponderosa and Jeffrey Pines (between 5% and 11%), and True Firs (between 9% and 14%). We fail to reject the no-jumps null hypothesis in the presence of ARCH for Western Hemlock.

To summarize, we find strong evidence of jumps with a possibility of ARCH effects for Douglas Fir, strong evidence of ARCH with a possibility of jumps for Ponderosa and Jeffrey Pines, strong evidence of ARCH without jumps for Western Hemlock, and jumps with a possibility of ARCH for True Firs. Such differences between the dynamic characteristics of these series may seem surprising. One possible explanation is that they may have been affected differently first by speculative bubbles in stumpage prices that burst during the 1980s, and later by the Spotted Owl controversy that resulted in harvest restrictions in some areas.[10] Alternatively, our normality assumption for the logarithm of jumps may not be adequate and a distribution with fatter tails (such as a t-distribution) or an asymmetric distribution may be needed to capture better what happened during this period of extreme price changes.

We also note that the LM and the LR tests for ARCH yield different results for True Firs: the former is significant whereas the latter fails to reject the no-ARCH null in the presence of jumps (at 5%). Of course, the LM test does not take jumps into consideration. Yet there seems to be evidence in favor of jumps (with or without ARCH). Alternatively, if ARCH effects (which seem present) are not accounted for, jumps are falsely detected in the case of Western Hemlock.

More generally, our results show that it may be difficult to distinguish between jumps

[9] Recall that we rely on bounds when testing for ARCH in the presence of jumps or for jumps in the presence of ARCH. If the bounds' p-value $\leq \alpha$, we can conclude that the test is significant, while it is not significant if the bootstrap p-value $> \alpha$. Indeed, the bootstrap provides an empirical p-value corresponding to a point estimate for the nuisance parameter. If this estimated p-value is larger than α, then *a fortiori* the largest p-value over the nuisance parameter space exceeds α. However, there is no clear decision when bootstrap p-value $< \alpha <$ bounds' p-value.

[10] For more details on these speculative bubbles and on the management of Federal Forests, see Ando.

and ARCH effects in small data sets. Nonnested tests may be performed, although no reliable procedure is available for such problems (recall the inference problems documented above).

Implications for Harvesting Old-Growth Timber

From results reported in the finance literature (e.g., see Bakshi, Cao, and Chen, and the references therein), we anticipate that jumps and ARCH effects in the price of a natural resource can have important consequences for its management. In the case of stumpage prices, they could, for example, impact the decision to cut a public forest.

Off-the-shelf models for financial assets are not always adequate for natural resources problems, however. In this section, we thus focus on the impact of neglecting jumps, when they are statistically significant, in the context of an infinite-horizon, continuous-time stochastic dynamic model based on the GBM. We leave the investigation of the impact of ARCH effects in this framework for future work because when we move to continuous time, ARCH effects translate into complex stochastic volatility models (Duan).

We revisit the classic problem of the optimal timing of cutting a stand of old-growth forest (e.g., see Reed or Conrad and Ludwig) so timber volume is assumed to be constant and equal to unity. This stand generates a constant amenity A per time period, net of maintenance costs. Cutting this stand at time t would provide net revenues S_t from timber sales, where S_t varies stochastically, plus the present value of the flow of land rents (L per time period, assumed constant). We consider a single rotation and denote by r the social discount rate.

We compare two models, which we apply to the same data: in the first model, S_t follows a GBM with jumps, and in the second one, it follows a simple GBM. For each model, we want to find S^*, which separates the values of S_t where the stand should be preserved (the continuation region: $S_t \leq S^*$), from the values of S_t where the trees should be cut (the stopping region: $S_t \geq S^*$). We use the theory of real options to solve these two optimal stopping problems (Dixit and Pindyck).

First, let us suppose that net stumpage price follows a GBM with jumps, as given by equation (2), and that the arrival of jumps follows a Poisson process with arrival rate λ. We know from option theory that there exists an implied value function $V(S_t)$ that verifies the following optimality condition when the stand of old-growth forest should be preserved:

$$(11) \quad rV(S_t) = A + \frac{1}{dt}E_t[dV(S_t)].$$

$E_t[\cdot]$ is the expectation operator at t and $dV(S_t)$ is the differential of the unknown value function. This asset equilibrium condition states that the stand should be preserved as long as the flow of amenity and the expected "capital gains" $(1/dt)E_t[dV(S_t)]$ provide a return equal to the social discount rate r. Applying a generalization of Itô's lemma (Merton), we find

$$(12) \quad rV(S_t) = A + \gamma S_t \frac{dV(S_t)}{dS_t} + \frac{\sigma^2}{2}S_t^2\frac{d^2V(S_t)}{dS_t^2} + \lambda\varepsilon_{Y_t}\{V(S_tY_t) - V(S_t)\}.$$

$V(S_t)$ can be written as the sum of two terms: $V(S_t) = V_P(S_t) + \varphi(S_t)$. The term $V_P(S_t)$ is a particular solution of (12). It represents the present value of the constant flow of amenity A, and thus

$$(13) \quad V_P(S_t) = \frac{A}{r}.$$

The term $\varphi(S_t)$ represents the value of the option to cut the stand. We know from option theory that in the continuation region it verifies the homogeneous equation associated with (12):

$$(14) \quad r\varphi(S_t) = \gamma S_t \frac{d\varphi(S_t)}{dS} + \frac{\sigma^2}{2}S_t^2\frac{d^2\varphi(S_t)}{dS_t^2} + \lambda\varepsilon_{Y_t}\{\varphi(S_tY_t) - \varphi(S_t)\}.$$

We also assume that if S_t ever becomes 0, it is zero forever so:

$$(15) \quad \varphi(0) = 0.$$

To find S^*, we need the continuity and the smooth-pasting conditions, which require V and V' to be smooth across the stopping frontier (Dixit and Pindyck):

$$(16) \quad V_P(S^*) + \varphi(S^*) = S^* + \frac{L}{r}$$
$$\frac{dV_P(S^*)}{dS} + \frac{d\varphi(S^*)}{dS} = 1.$$

The continuity and the smooth-pasting conditions can be rewritten in terms of φ to get

$$(17)\quad \varphi(S^*) = S^* - \frac{A-L}{r}$$

$$(18)\quad \frac{d\varphi}{dS}(S^*) = 1.$$

Equation (17) is in fact valid in the entire stopping region because when $S_t \geq S^*$ cutting should take place immediately.

Equation (14) with boundary conditions (15), (17), and (18) cannot be solved analytically, so we extend Galerkin's method (Delves and Mohamed) to solve it numerically. To our knowledge, this has not been done before in resource economics. See the appendix for details.

If, however, S_t follows a GBM, equation (14) (where λ has been set to 0) with boundary conditions (15), (17), and (18) is easily solved. Trying a power function in S, we find

$$(19)\quad \varphi(S) = C_0 S^{\eta}$$

where C_0 is a constant to be determined jointly with S^* and η is given by

$$(20)\quad \eta = \frac{-(\gamma - 0.5\sigma^2) + \sqrt{(\gamma - 0.5\sigma^2)^2 + 2r\sigma^2}}{\sigma^2}.$$

The expression of V_P in equation (13) is still valid so, plugging (13) and (19) into (16) and solving for S^*, we find

$$(21)\quad S^* = \frac{\eta}{\eta - 1}\frac{A-L}{r}.$$

To assess the error one could make by fitting a GBM to data generated by a jump-GBM process, we estimate numerically $S^*_{\text{Jump-GBM}}$, the stopping value for a jump-GBM, and compare it with S^*_{GBM}, the stopping value for a GBM calculated from equation (21). We use our sample data for Douglas Fir and True Firs and various discount rates. By choosing appropriate units, we assume that the difference $A-L$ equals 1.[11]

A necessary condition to have a finite stopping value S^* is that r be larger than the expected growth rate of stumpage price. For the GBM, the expected growth rate of stumpage prices is γ, the infinitesimal drift in equation (1). For the jump-GBM model, we also need to account for the contribution of the jumps, so the expected growth rate is (Merton)

$$(22)\quad \varepsilon\left(\frac{dS_t}{S_t}\right) = \gamma + \lambda\left(e^{\theta+\delta^2/2} - 1\right)$$

where again $\gamma = \mu + 0.5\sigma^2$. Using parameter values reported in table 2, we find that the annual expected stumpage price growth rates for Douglas Fir and True Firs are 25.5% and 37.5% respectively for the jump-GBM. These high values force us to adopt high discount rates to make cutting worthwhile. They should be interpreted with caution given that our parameters are estimated from a small sample that covers a pretty eventful period in the Pacific Northwest. We want to take these events into account, however, if only to correctly estimate the parameters of the underlying GBM (μ and σ) or if we believe that similar events could occur in the future.

Like Haight and Holmes, we find that the empirical stumpage price DGP has important implications for harvesting timber. From table 4a, we see that neglecting jumps when they are indeed present can either lead to cutting too early (for Douglas Fir) or too late (for True Firs). To explain this difference, recall that we have multiplicative, lognormally distributed jumps. Their expected value is thus $\exp(\theta + 0.5\delta^2)$, which equals $1.17 > 1$ for Douglas Fir and $0.90 < 1$ for True Firs. Jumps thus tend to increase S_t for the former and to decrease S_t for the later. Also note that old-growth forest would not be cut with a GBM when the rate of increase of stumpage value is greater than the discount rate (the "$+\infty$" for True Firs when the discount rate is 2% above r_{critical}). A look at the % change between the stopping values calculated with the GBM and with the jump-GBM shows that the difference between the two can be quite substantial (over 500% for True Firs when the discount rate is 4% above r_{critical}, for example).

To check the robustness of these results, we conduct a simulation study. For both Douglas Fir and True Firs, we generate 100,000 samples of 97 stumpage prices using the jump-GBM parameters reported in table 2. For each sample, we estimate the GBM parameters μ and σ and calculate the corresponding value of S^*_{GBM} and the relative error $(S^*_{\text{Jump-GBM}} - S^*_{\text{GBM}})/S^*_{\text{Jump-GBM}}$. Table 4b presents three quartiles of the distribution of relative errors for different interest rates. "$-\infty$" means that the stand of old-growth

[11] If A-L were negative, we see from (21) that it would be optimal to cut immediately.

Table 4. Comparison of Stopping Values Based on (a) Sample Data and (b) A Simulation Study

(a)	Discount Rate in Excess of r_{critical}	S^*_{GBM}	$S^*_{\text{Jump-GBM}}$	% Change
Douglas Fir	2%	265	428	38%
	4%	172	216	20%
	6%	127	144	12%
True Firs	2%	$+\infty$	485	–
	4%	1601	246	−551%
	6%	427	163	−160%

(b)	Discount Rate in Excess of r_{critical}	Quartiles of the Distribution of Relative Errors: $100\%(S^*_{\text{Jump-GBM}} - S^*_{\text{GBM}})/S^*_{\text{Jump-GBM}}$		
		25th Percentile	50th Percentile	75th Percentile
Douglas Fir	2%	$-\infty$	48%	85%
	4%	$-\infty$	27%	74%
	6%	$-\infty$	18%	65%
True Firs	2%	$-\infty$	$-\infty$	83%
	4%	$-\infty$	−161%	71%
	6%	$-\infty$	−82%	62%

Note: In table 4, $r_{\text{critical}} = \gamma + \lambda(e^{\theta+0.5\delta^2} - 1)$ is the minimum discount rate that makes cutting the stand of old-growth forest worthwhile with the jump-GBM model. In table 4a, S^*_{GBM} and $S^*_{\text{Jump-GBM}}$ are the values of the stand of old-growth forest at which it is optimal to harvest under the GBM and jump-GBM models respectively. They are calculated using the parameter values shown in table 2, assuming that amenity value is unity. In table 4b, simulated samples are generated with the jump/GBM parameters from table 2. Results are for 100,000 samples of 97 points each (97 is the size of our stumpage price samples). "$-\infty$" means that the stand of old-growth forest would not be cut if stumpage prices follow a GBM, because it appreciates too quickly.

forest would not be cut if stumpage prices follow a GBM, because it appreciates too quickly; this happens more than 25% of the time for the parameters considered. Looking at the 50th percentile, we also see that with the GBM we would cut too early for Douglas Fir and too late for True Firs, although the GBM may also lead us to cut too early on occasion for True Firs (75th percentile). Finally, we note that the difference between the stopping values of both models tends to decrease as the discount rate increases. These results confirm that omitting jumps when they are present can lead to large errors.

Conclusions

In this article, we reconsider the representation of natural resource prices by continuous processes by allowing for the presence of jumps, which can be due to the arrival of discrete events that cause large price changes. We also allow for ARCH effects, which like jumps have been found to generate increments in log prices with more extreme values than the normal distribution.

First, we propose a LR-based methodology, based on combining bounds with Monte Carlo tests, that gives finite-sample, level-exact p-values when testing for jumps or ARCH effects. This is particularly useful in natural resources because sample sizes are often small. We then analyze four quarterly time series of stumpage prices from Pacific Northwest National Forests. Controlling for seasonal variations and a definition change in stumpage prices, we find evidence of jumps and ARCH effects.

Second, we revisit the tree-cutting problem for old-growth forest when stumpage prices follow a GBM with jumps. We present an algebraic method to solve this autonomous, infinite-horizon stopping problem, which turns a complex Bellman equation into a system of linear equations. We then show that ignoring jumps, when they are indeed present, may lead to significantly suboptimal decisions to harvest old-growth timber. In empirical work, this illustrates the importance of investigating the presence of jumps in natural resource prices.

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Appendix

Galerkin's Method

With Galerkin's method (Delves and Mohamed), we solve numerically for the unknown function $f(x)$ in the integro-differential equation with boundary conditions:

$$\text{(A.1)}\quad P(x)f''(x)+Q(x)f'(x)+R(x)f(x) + \int_{-1}^{1} k(x,u)f(u)\,du = g(x) \qquad -1\le x\le 1$$

$$\text{(A.2)}\quad \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} f(-1) \\ f(1) \end{pmatrix} + \begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}\begin{pmatrix} f'(-1) \\ f'(1) \end{pmatrix} = \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}.$$

$P(x)$, $Q(x)$, $R(x)$, $k(x,u)$, and $g(x)$ are "well behaved" functions.

We first replace $P(x)$, $Q(x)$, $R(x)$, $g(x)$, $f(x)$, $f'(x)$, and $f''(x)$ by their truncated Chebychev decomposition:

$$P(x)=\sum_0^N{}' p_j T_j(x) \qquad Q(x)=\sum_0^N{}' q_j T_j(x)$$

$$R(x)=\sum_0^N{}' r_j T_j(x) \qquad g(x)=\sum_0^N{}' g_j T_j(x)$$

$$f(x)=\sum_0^N{}' a_j T_j(x) \qquad f'(x)=\sum_0^N{}' a'_j T_j(x)$$

$$f''(x)=\sum_0^N{}' a''_j T_j(x).$$

$\sum'$ indicates that the first term of the summation is halved, and $\sum''$ indicates that both the first and the last terms are halved. $T_j(x)\equiv\cos(j\arccos(x))$ is the jth Chebychev polynomial. For example, to find the g_i's $(1\le i\le N)$, we calculate

$$g_i=\frac{2}{N}\sum_{k=0}^{N}{}'' g\left(\cos\frac{kp}{N}\right)\cos\frac{ikp}{N}.$$

Delves and Mohamed show that $a_j=\frac{1}{2}(a'_{j-1}-a'_{j+1})$, and $a'_j=\frac{1}{2}(a''_{j-1}-a''_{j+1}), j\ge 1$. In vector form (where $\mathbf{a}^{(1)}$ is an $N\times 1$ vector, $\mathbf{A}$ is an $N\times(N+1)$ matrix, and $\mathbf{a}'$ is an $(N+1)\times 1$ vector)

$$\mathbf{a}^{(1)}=\mathbf{A}\mathbf{a}' \quad \mathbf{a}^{(1)}\equiv\begin{pmatrix} a_1 \\ a_2 \\ \vdots \end{pmatrix} \quad \mathbf{a}'=\begin{pmatrix} a'_0 \\ a'_1 \\ \vdots \end{pmatrix}$$

$$A_{i,i}=\frac{1}{2i} \quad A_{i,i+2}=\frac{-1}{2i}$$
$$A_{i,j}=0 \quad \text{otherwise.}$$

Equation (A.2) links a_0 to the a'_j, and a'_0 to the a''_j. For example, if $c_{11}+c_{12}\neq 0$, we have

$$a_0=\frac{2}{c_{11}+c_{12}}\{e_1-[(d_{11}\quad d_{12})\mathbf{T} + (c_{11}\quad c_{12})\mathbf{T}^{(1)}\mathbf{A}]\mathbf{a}'\}$$

$$a'_0=2(f_{21}e_1+f_{22}e_2) + \left[\mathbf{h}^t-2(f_{21}\quad f_{22})\left\{(\mathbf{C}\mathbf{T}^{(2)}\mathbf{A}_{11}+\mathbf{D}\mathbf{T}^{(1)})\mathbf{A} + \mathbf{D}\begin{pmatrix}0.5\\0.5\end{pmatrix}\mathbf{h}^t\right\}\right]\mathbf{a}''.$$

- $\mathbf{T}\equiv\begin{pmatrix} 0.5T_0(-1) & T_1(-1) & T_2(-1)\cdots \\ 0.5T_0(1) & T_1(1) & T_2(1)\cdots \end{pmatrix}$ is a $2\times(N+1)$ matrix, $\mathbf{T}^{(k)}$ is $\mathbf{T}$ without its first k columns;
- $\mathbf{A}_{11}$ is $\mathbf{A}$ without its first line and column;
- $\mathbf{h}^t$ is the $1\times(N+1)$ vector: $(0\ \frac{1}{4}\ 0\ -\frac{1}{4}\ 0\ \cdots\ 0)$;
- $\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}$ is the inverse of $\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} 0.5 & -1 \\ 0.5 & 1 \end{pmatrix}+\begin{pmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$ which is required to be nonsingular to have a well posed problem; and

- $\mathbf{a}, \mathbf{a}'$, and $\mathbf{a}''$ are the vectors of Chebychev coefficients for $f(x), f'(x)$, and $f''(x)$ respectively.

We find:

$$\text{(A.3)} \quad \mathbf{a} = \mathbf{A}'\mathbf{a}' + \boldsymbol{\mu}, \qquad \mathbf{a}' = \mathbf{A}''\mathbf{a}'' + \boldsymbol{\eta}$$

with

$$\boldsymbol{\mu}^t = \left(\frac{2e_1}{c_{11}+c_{12}} \quad 0\cdots 0\right) \qquad \text{and}$$
$$\boldsymbol{\eta}^t = \left(2(f_{21}e_1 + f_{22}e_2) \ 0\cdots 0\right).$$

For $0 \le i \le N$, we then multiply the resulting equation by $T_i(x)/(\sqrt{1-x^2})$ and integrate between -1 and 1. We obtain

$$\text{(A.4)} \quad \mathbf{P}\mathbf{a}'' + \mathbf{Q}\mathbf{a}' + (\mathbf{R} + \lambda\mathbf{B})\mathbf{a} = \mathbf{g}.$$

For $0 \le i \le N$, $0 < j \le N$, the coefficients of $\mathbf{P}, \mathbf{Q}, \mathbf{R}$, and $\mathbf{B}$ are

$$P_{i0} = \frac{p_i}{2} \qquad Q_{i0} = \frac{q_i}{2} \qquad R_{i0} = \frac{r_i}{2}$$
$$B_{i0} = \frac{\pi}{N^2}\sum_{s=1}^{N-1}\sin\left(\frac{s\pi}{N}\right) \times \sum_{r=0}^{N}{}'' k\left(\cos\left(\frac{r\pi}{N}\right), \cos\left(\frac{s\pi}{N}\right)\right) \times \cos\left(\frac{ri\pi}{N}\right)$$
$$P_{ij} = \frac{p_{i+j} + p_{|i-j|}}{2} \qquad R_{ij} = \frac{r_{i+j} + r_{|i-j|}}{2}$$
$$Q_{ij} = \frac{q_{i+j} + q_{|i-j|}}{2}$$
$$B_{ij} = \frac{2\pi}{N^2}\sum_{s=1}^{N-1}\cos\left(\frac{sj\pi}{N}\right)\sin\left(\frac{s\pi}{N}\right) \times \sum_{r=0}^{N}{}'' k\left(\cos\left(\frac{r\pi}{N}\right), \cos\left(\frac{s\pi}{N}\right)\right) \times \cos\left(\frac{ri\pi}{N}\right).$$

Substituting (A.3) into (A.4) gives the linear system in $\mathbf{a}''$:

$$\text{(A.5)} \quad [\mathbf{P} + (\mathbf{Q} + (\mathbf{R} + \lambda\mathbf{B})\mathbf{A}')\mathbf{A}'']\mathbf{a}'' = \mathbf{g} - (\mathbf{Q} + (\mathbf{R} + \lambda\mathbf{B})\mathbf{A}')\boldsymbol{\eta} - (\mathbf{R} + \lambda\mathbf{B})\boldsymbol{\mu}.$$

Once we know $\mathbf{a}''$, we calculate $\mathbf{a}$ from $\mathbf{a} = \mathbf{A}'(\mathbf{A}''\mathbf{a}'' + \boldsymbol{\eta}) + \boldsymbol{\mu}$.

Here, we want to solve for $\varphi(\bullet)$ and for S^* in (14) to (18). We first change variables: $s \equiv \mathrm{Ln}(S)$ (and so $s^* = \mathrm{Ln}(S^*)$), $\psi(\mathrm{Ln}(S)) \equiv \varphi(S)$, $\mathrm{Ln}(Y) \equiv Z$. We replace $-\infty$ by $s_{\inf}$. A second change of variables

$$w \equiv \frac{2}{s^* - s_{\inf}}s - \frac{s^* + s_{\inf}}{s^* - s_{\inf}}$$
$$f(w) \equiv \psi(s) \qquad g(w) \equiv G(s)$$

leads to

$$\text{(A.6)} \quad P(w)f''(w) + Q(w)f'(w) + R(w)f(w) + \int_{-1}^{1} k(w,u)f(u)\,du = g(w)$$
$$-1 \le w \le 1$$
$$f(-1) = 0$$
$$f'(1) = \frac{s^* - s_{\inf}}{2}e^{s^*}$$
$$f(1) = e^{s^*} - \frac{A-L}{r}$$

with

$$P(w) = \frac{\sigma^2}{2}\left(\frac{2}{s^* - s_{\inf}}\right)^2$$
$$Q(w) = \left(\gamma - \frac{\sigma^2}{2}\right)\frac{2}{s^* - s_{\inf}} \qquad R(w) = -(\lambda + r)$$
$$k(w,u) = \lambda\frac{s^* - s_{\inf}}{2\delta}\phi\left(\frac{s^* - s_{\inf}}{2\delta}(u - w) - \frac{\theta}{\delta}\right)$$
$$g(w) = \lambda\exp\left(\left\{\frac{s^* - s_{\inf}}{2}w + \frac{s^* + s_{\inf}}{2} + \frac{\delta^2}{2} + \theta\right\}\right) \times \left[\Phi\left(\frac{s^* - s_{\inf}}{2\delta}(1-w) - \frac{\theta}{\delta} - \delta\right) - 1\right] + \lambda\frac{A-L}{r} \times \left[1 - \Phi\left(\frac{s^* - s_{\inf}}{2\delta}(1-w) - \frac{\theta}{\delta}\right)\right].$$

$\phi(z)$ and $\Phi(z)$ are respectively the density and the cumulative distribution of the standard normal. Compared to (A.1), we have one extra boundary conditions because s^* is unknown. To solve, we extend Galerkin's method: we pick a value of s^*, find f from the first three equations of (A.6), check if the fourth equation of (A.6) is satisfied, and iterate with other values of s^* until it is. A GAUSS program, available upon request, was written to implement this procedure. With $s_{inf} = -4.0$ and $N = 60$ (number of terms in the Chebychev expansions), we obtain satisfactory numerical convergence.