(8) T. Glonek and J. R. Van Wazer, *J. Magn. Reson.*, **13**, 390 (1974).
(9) T. Glonek, J. R. Van Wazer, M. Mudgett, and T. C. Myers, *Inorg. Chem.*, **11**, 567 (1972).
(10) J. R. Van Wazer and T. Glonek in "Analytical Chemistry of Phosphorus Compounds", M. Halmann, Ed., Wiley, New York, N.Y., 1972, p 176 also p 167.
(11) G. M. Kosolapoff, "Organophosphorus Compounds", Wiley, New York, N.Y., 1950, Chapter 9, pp 220–277.
(12) T. Glonek, R. A. Kleps, E. J. Griffith, and T. C. Myers, *Phosphorus*, **5**, 157, 165 (1975).
(13) A. Szent-Gyorgi, "Bioenergetics", Academic Press, New York, N.Y., 1957.
(14) G. T. Rossetti and K. E. Van Holde, *Biochem. Biophys. Res. Commun.*, **26**, 717 (1967).
(15) W. E. Morgan, T. Glonek, and J. R. Van Wazer, *Inorg. Chem.*, **13**, 1382 (1974).

Stereodynamics of Acyclic Alcohols, Ethers, and *N,N*-Dimethylurethanes. Potential Barriers to Rotation about Carbon–Carbon and Carbon–Nitrogen Bonds

Steven Hoogasian,[1] C. Hackett Bushweller,*[2] Warren G. Anderson, and George Kingsley

Department of Chemistry, Worcester Polytechnic Institute, Worcester, Massachusetts 01609

(Received November 14, 1975)

Publication costs assisted by the National Science Foundation

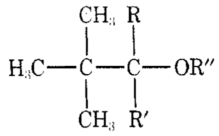
Examination of the ^{1}H DNMR spectra of a series of acyclic alcohols and methyl ethers showed changes consistent with slowing rotation about carbon–carbon single bonds and allowed an assessment of the relative hindering potential to *tert*-butyl rotation of hydroxyl, methoxyl, and various alkyl groups. Both hindered *tert*-butyl and *N,N*-dimethyl rotation were observed in an acyclic *N,N*-dimethylurethane and *N,N*-dimethylthiourethane.

Although the potential barrier to threefold rotation in ethane (2.9 kcal/mol) is well established experimentally,[3] a quantitative theoretical assessment of various energy components contributing to the barrier is elusive.[4] It is apparent that van der Waals repulsions can account for only a fraction of the barrier height in ethane and an orbital-control mechanism has been proposed to account for the hindering potential.[5] In more complicated systems,[6] van der Waals repulsions become more important and in many instances, the barriers to rotation about carbon–carbon single bonds are large enough to be accessible to measurement by the dynamic nuclear magnetic resonance (DNMR)[7] technique.

Although a large number of polyhalogenated ethanes and butanes have been examined by the DNMR method,[6] there has been little effort to assess the effect of hydroxyl or alkoxyl on the barrier to carbon–carbon single bond rotation.[8] This report concerns the DNMR measurement of the rate of *tert*-butyl rotation in a series of selectively deuterated acyclic alcohols, methyl ethers, and urethanes. The *tert*-butyl rotor was selected in order to provide a symmetrical threefold potential surface for rotation and eliminate the need to determine more than one potential barrier for a given compound. The *tert*-butyl group has also been useful in establishing stereodynamical trends in trialkylamines,[9] trialkylphosphines,[10] and trialkylphosphine complexes.[11,12]

Results

For the purpose of DNMR spectral simplification, a number of specifically deuteriated alcohols (1–5) and methyl ethers (6–9) were prepared by standard methods (see Experimental Section).



$$H_3C-\overset{CH_3}{\underset{CH_3}{C}}-\overset{R}{\underset{R'}{C}}-OR''$$

1, $R = R' = CD_3$; $R'' = D$
2, $R = CD_3$; $R' = CD_2CD_3$; $R'' = D$
3, $R = R' = CH_2CD_3$; $R'' = D$
4, $R = CD_3$; $R' = CH_2C_6H_5$; $R'' = D$
5, $R = CD_3$; $R' = t\text{-}C_4H_9$; $R'' = D$
6, $R = R' = CD_3$; $R'' = CH_3$
7, $R = CD_3$; $R' = CD_2CD_3$; $R'' = CH_3$
8, $R = R' = CH_2CD_3$; $R'' = CH_3$
9, $R = CD_3$; $R' = t\text{-}C_4H_9$; $R'' = CH_3$
10, $R = R' = CH_3$; $R'' = C(O)N(CH_3)_2$
11, $R = R' = CH_3$; $R'' = C(S)N(CH_3)_2$

Examination of the ^{1}H DNMR spectrum of 5 (4% v/v in CH_2CHCl) at −37.0 °C (Figure 1) revealed a singlet resonance for the *tert*-butyl protons (δ 1.045) consistent with rapid *tert*-butyl rotation on the DNMR time scale. At temperatures below −37.0 °C, the *tert*-butyl resonance broadens and is separated at −111.9 °C (Figure 1) into three singlet resonances at δ 0.980 (3 H), 1.046 (3 H), and 1.129 (3 H) consistent with slow *tert*-butyl rotation on the DNMR time scale and the symmetry experienced by a static *tert*-butyl (eq 1). Total DNMR line shape analyses at various

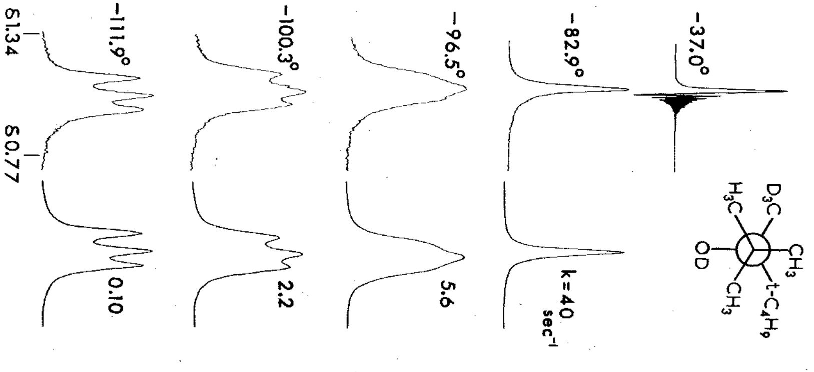
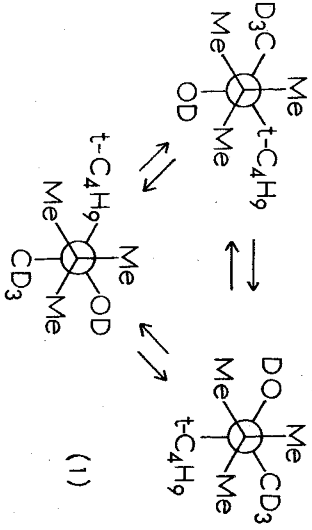


Figure 1. Experimental ^{1}H DNMR spectra (60 MHz) of the *tert*-butyl group of **5** (4% v/v in CH_2CHCl) and theoretical spectra calculated as a function of the rate of *tert*-butyl rotation (k = first-order rate constant for conversion of *one tert*-butyl rotamer to *one* other rotamer).



(1)

temperatures (Figure 1) using a substantially modified local version of computer program DNMR3[13] gave the rate of *tert*-butyl rotation at various temperatures. The activation parameters derived from a weighted least-squares fit of an Eyring plot and slow exchange *tert*-butyl chemical shifts are compiled in Table I.

Changes in the ^{1}H DNMR spectrum of **1** were observed at slightly lower temperatures than for **5** with the slow exchange spectrum of **1** at −124 °C (Figure 2) consistent with the symmetry of **1** and slow *tert*-butyl rotation. Analogous changes in the ^{1}H DNMR spectra were observed for **2** and **4**. Alcohol **3** did not show a separation of the ^{1}H *tert*-butyl resonance in both CH_2CHCl or $CBrF_3$ to about −150 °C due most likely to a small or zero chemical shift difference between the stereochemically nonequivalent methyls of the *tert*-butyl group. Pertinent data are compiled in Table I. Also compiled in Table I are activation parameters for *tert*-butyl rotation in **5** in a variety of solvent systems.

Examination of the ^{1}H DNMR spectra of the methyl ethers **7** (Figure 3; 5% v/v in CH_2CHCl) and **9** (Figure 4; 5%

TABLE I: *tert*-Butyl ^{1}H NMR Chemical Shifts at Slow Exchange and Activation Parameters for *tert*-Butyl Rotation in t-$C_4H_9(R)(R')COR''$

Compd	R	R′	R″	Solvent (v/v% of alcohol or derivative)	^{1}H chemical shifts, ppm from TMS	$\Delta H^\ddagger$, kcal/mol	$\Delta S^\ddagger$, gibbs	$\Delta G^\ddagger$, kcal/mol (−100 °C)
1	CD_3	CD_3	D	CH_2CHCl (4%)	0.873 (3 H); 0.969 (6 H)	8.5 ± 0.4	−1.4 ± 2.7	8.73 ± 0.10
2	CD_3	CD_2CD_3	D	CH_2CHCl (4%)	0.843 (3 H); 0.940 (3 H); 0.987 (3 H)	8.7 ± 0.6	−1.4 ± 3.4	8.91 ± 0.10
3	CH_2CD_3	CH_2CD_3	D	CH_2CHCl (4%)	No split			
4	CD_3	$CH_2C_6H_5$	D	CH_2CHCl (4%)	0.961 (6 H); 1.045 (3 H)	9.1 ± 0.4	0.8 ± 2.6	8.93 ± 0.10
5	CD_3	t-C_4H_9	D	CH_2CHCl (4%)	0.980 (3 H); 1.046 (3 H); 1.129 (3 H)	9.8 ± 0.8	1.5 ± 4.7	9.58 ± 0.10
				90:10 CH_2CHCl–CH_3OH (4%)	0.955 (3 H); 1.043 (3 H); 1.120 (3 H)	9.6 ± 0.2	−1.3 ± 1.2	9.77 ± 0.10
				75:25 CH_2CHCl–CH_3OH (4%)	0.946 (3 H); 1.036 (3 H); 1.109 (3 H)	9.9 ± 0.4	−0.6 ± 2.1	9.99 ± 0.10
				45:55 CH_2CHCl–CH_3OH (4%)	0.968 (3 H); 1.054 (3 H); 1.120 (3 H)	10.1 ± 0.7	0.6 ± 3.9	9.95 ± 0.10
				60:40 $(CH_3)_2O$–$(CH_3)_2NCHO$ (4%)	0.946 (3 H); 1.021 (3 H); 1.112 (3 H)	9.6 ± 0.4	−0.1 ± 2.3	9.57 ± 0.10
6	CD_3	CD_3	CH_3	CH_2CHCl (5%)	0.859 (3 H); 0.889 (6 H)			9.34 ± 0.40
7	CD_3	CD_2CD_3	CH_3	CH_2CHCl (5%)	0.821 (3 H); 0.922 (6 H)	8.5 ± 0.4	0.0 ± 2.8	8.49 ± 0.10
8	CH_2CD_3	CH_2CD_3	CH_3	CH_2CHCl (5%)	0.884 (3 H); 0.976 (6 H)	8.0 ± 0.2	0.2 ± 1.3	7.93 ± 0.10
9	CD_3	t-C_4H_9	CH_3	CH_2CHCl (5%)	0.913 (3 H); 0.983 (3 H); 1.089 (3 H)	9.6 ± 0.6	0.8 ± 3.4	9.43 ± 0.10
10	CH_3	CH_3	$C(O)N(CH_3)_2$	CH_2CHCl (5%)	0.863 (3 H); 0.992 (6 H)	9.2 ± 0.4	−0.8 ± 2.0	9.38 ± 0.10
11	CH_3	CH_3	$C(S)N(CH_3)_2$	CH_2CHCl (5%)	0.905 (3 H); 1.04 (6 H)	9.4 ± 0.4	−2.1 ± 2.2	9.80 ± 0.10

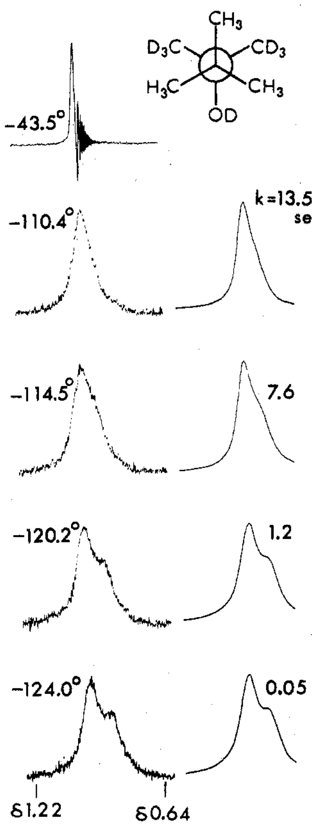


Figure 2. Experimental ^{1}H DNMR spectra (60 MHz) of the *tert*-butyl group of **1** (4% v/v in CH_2CHCl) and theoretical spectra calculated as a function of the rate of *tert*-butyl rotation (k = first-order rate constant for conversion of *one tert*-butyl rotamer to *one* other rotamer).

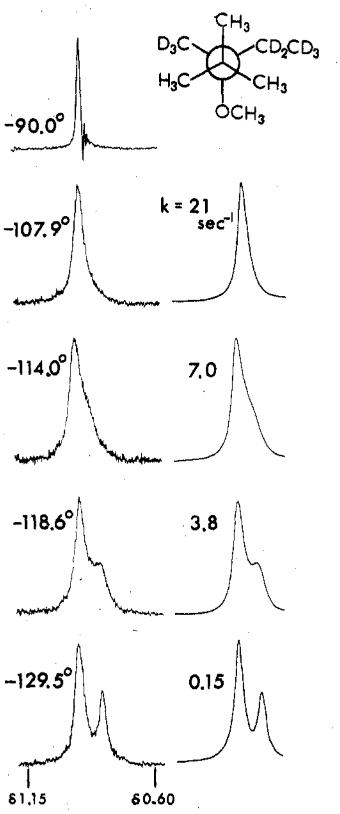


Figure 3. Experimental ^{1}H DNMR spectra (60 MHz) of the *tert*-butyl group of **7** (5% v/v in CH_2CHCl) and theoretical spectra calculated as a function of the rate of *tert*-butyl rotation (k = first-order rate constant for conversion of *one tert*-butyl rotamer to *one* other rotamer).

v/v in CH_2CHCl) also revealed changes at low temperature consistent with slowing *tert*-butyl rotation on the DNMR time scale. While it might be reasonable to expect three *tert*-butyl methyl resonances with slow *tert*-butyl rotation in **7**, only two peaks are resolved (Figure 3). It is apparent that the effective diamagnetic anisotropies of CD_3 and CD_2CD_3 toward *tert*-butyl are the same in **7**. A comparison of Figures 1 and 2 with Figures 3 and 4 also shows generally longer T_2 values (sharper lines) for the ethers as compared to the alcohols. In the case of **6**, the small chemical shift difference between the different *tert*-butyl methyl resonances precluded extraction of accurate $\Delta H^\ddagger$ and $\Delta S^\ddagger$ values from a complete line shape analysis and only a $\Delta G^\ddagger$ value is tabulated (Table I).

The ^{1}H DNMR spectra of urethane **10** (Figure 5; 5.5 wt % in CH_2Cl_2 above −20 °C and 5 wt % in CH_2CHCl below −20 °C) showed changes consistent with slowing rotation about the carbonyl carbon–nitrogen bond *and* with slowing *tert*-butyl rotation. The activation parameters for $N(CH_3)_2$ rotation in **10** ($\Delta H^\ddagger$ = 15.4 ± 1.4 kcal/mol; $\Delta S^\ddagger$ = 0.3 ± 5 gibbs; $\Delta G^\ddagger$ = 15.3 ± 0.1 kcal/mol at 6.3 °C) were determined from a complete ^{1}H DNMR line shape analysis. NMR and activation parameters for *tert*-butyl rotation in **10** are compiled in Table I. Similar changes in the ^{1}H DNMR spectra of thiourethane **11** were also observed (Figure 6; 5.5 wt % in CCl_2CCl_2 above 25 °C and 5 wt % in CH_2CHCl below −20 °C). Complete DNMR line shape analyses yielded the activation parameters for $N(CH_3)_2$ rotation ($\Delta H^\ddagger$ = 19.6 ± 0.8 kcal/mol; $\Delta S^\ddagger$ = 8 ± 2 gibbs; $\Delta G^\ddagger$ = 17.0 ± 0.1 kcal/mol at 53.0 °C) and *tert*-butyl rotation (Table I). The barriers to $N(CH_3)_2$ rotation in **10** and **11** seem typical[14] and are higher than those for *tert*-butyl rotation.

It should also be noted that for those compounds in Table I which show changes in the *tert*-butyl ^{1}H DNMR spectra at low temperatures, the slow exchange spectrum consists of two or three *singlets*. This is of course consistent with rapid rotation on the DNMR time scale of the *individual* methyls of each *tert*-butyl group.[15]

Discussion

A perusal of Table I reveals *some* trends which can be correlated with conformational parameters from other systems. For example, the conformational requirements of various alkyl groups as measured by preference for the equatorial conformer in the monosubstituted cyclohexane ($-\Delta G°$ or "A value")[16] reveals methyl (A = 1.7 kcal/mol) to be slightly smaller than ethyl (A = 1.8 kcal/mol)[16] which is in turn comparable in size to benzyl (A = 1.8 kcal/mol).[17] The sequence of barriers to *tert*-butyl rotation in proceeding from compound **1** to **2** to **4** parallels reasonably well the corresponding A values for methyl, ethyl, and benzyl.

However, the A value of *tert*-butyl (~5 kcal/mol) is very large compared to methyl or ethyl and the relatively small increase in the barrier to *tert*-butyl rotation in **5** as compared to **1**, **2**, or **4** (Table I) reveals the potential inadequacy of employing A values to predict *quantitative* trends in potential barriers to rotation. Unexpectedly low barriers to *tert*-butyl rotation have also been observed in 2,2,3,4,4-pentamethyl-3-chloropentane[6d] which is an analogue of **5**. Two approaches may be taken to rationalize the small barrier increase in **5**, one involving ground state geometry and

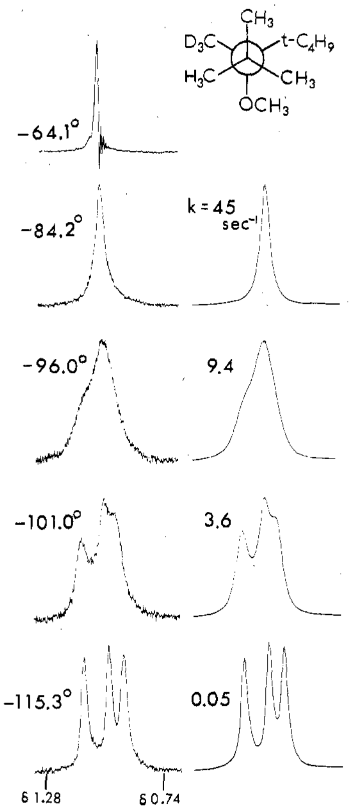


Figure 4. Experimental ^1H DNMR spectra (60 MHz) of the *tert*-butyl group of **9** (5% v/v in CH_2CHCl) and theoretical spectra calculated as a function of the rate of *tert*-butyl rotation (k = first-order rate constant for conversion of *one tert*-butyl rotamer to *one* other rotamer).

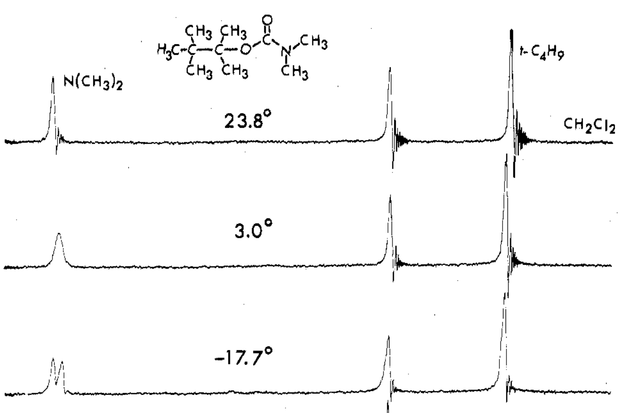


Figure 5. ^1H DNMR spectra (60 MHz) of **10** (5.5 wt % in CH_2Cl_2 from 23.8 to −17.7 °C and 5 wt % in CH_2CHCl from −85.9 to −116.1 °C).

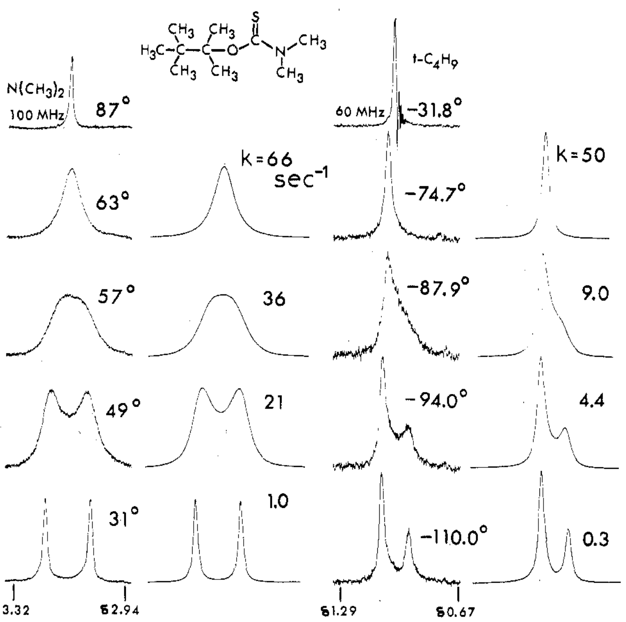
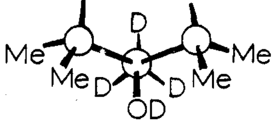


Figure 6. Experimental ^1H DNMR spectra (60 MHz) of the $N(CH_3)_2$ group of **11** (5.5 wt % in CCl_2CCl_2) and the *tert*-butyl group (5 wt % in CH_2CHCl) with theoretical spectra calculated as a function of the respective rates of rotation.

the other transition state geometry. Examination of a model of **5** keeping *all* bond angles at 109.5° and all vicinal substituents perfectly staggered (**12**; projection down C–CD_3 bond) shows severe 1,3-dimethyl repulsions analogous to the 1,3-dimethyl repulsions (~4 kcal/mol) in the diaxial chair conformation of *cis*-1,3-dimethylcyclohexane. In light of recent electron diffraction and molecular mechanics studies of tri-*tert*-butylmethane[18] and di-*tert*-butylmethane,[19] it is highly probable that the geometry of minimum potential energy for **5** is not **12** and that significant deviations from perfect staggering, from local T_d symmetry at each carbon, and normal carbon–carbon bond lengths (1.53 Å) occur. In tri-*tert*-butylmethane,[18] the central carbon–carbon bond length is stretched to 1.611 Å, the central (t-C_4H_9)C(t-C_4H_9) bond angle opens up to 116.2°, and each *tert*-butyl group is rotated about 10° away from perfect staggering. All of these adjustments in molecular geometry as compared with less hindered systems are consistent with relief of nonbonded repulsions involving the large *tert*-butyl groups and are analogous to trends observed in hindered *N*-*tert*-butyl-*N*,*N*-dialkylamines.[9] In di-*tert*-butylmethane,[19] the central carbon–carbon bond length is nearly normal at 1.545 Å while the central (t-C_4H_9)C(t-C_4H_9) bond angle opens up to 128°!



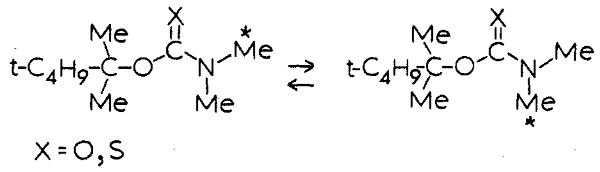
12

In light of these observations, it is virtually certain that the central carbon atom in the most stable geometry of **5** does not possess local T_d symmetry and that the central (t-C_4H_9)C(t-C_4H_9) bond angle will be greater than 109.5° and the *tert*-butyl group will rotate away from perfect staggering, i.e., rotate toward the transition state geometry for *tert*-butyl rotation. Any deviation from T_d symmetry at the central carbon of **5** also precludes achieving any geometry via *tert*-butyl rotation in which all three methyl groups of *tert*-butyl are perfectly eclipsed with the three other vicinal atoms on the central carbon of **5** bonded to hydroxyl. Thus, partial rotation of *tert*-butyl in the ground state and less-than-perfect eclipsing in the transition state for *tert*-butyl rotation would tend presumably to compress the energy difference between ground and transition states and lower the barrier. In addition to these effects of geometry on the barrier to *tert*-butyl rotation, it is very likely that the potential energy increase associated with one *tert*-butyl group in **5** rotating against the other may be optimized (i.e., minimized) via a *concomitant* rotation of both *tert*-butyl moieties. Other workers[6d] have suggested such a cog-wheel mechanism in analogues of **5**. All of these effects would tend to lower the barrier to *tert*-butyl rotation below what one would predict by analogy with the *A* value of *tert*-butyl.

It is noteworthy that the barrier to *tert*-butyl rotation in **5** in a variety of solvent systems (Table I) having different polarities and capacities to hydrogen bond varies to only a small degree indicating an almost negligible contribution of hydrogen bonding to restricting *tert*-butyl rotation in these relatively hindered systems.

In considering the methyl ethers (**6–9**; Table I) of interest in this study, a comparison of alcohols **1–5** with ethers **6–9** reveals hydroxyl to be roughly comparable to methoxyl in hindering *tert*-butyl rotation. However, in proceeding from ether **6** to **7** to **8**, there is a clearly defined *decrease* in the barrier to *tert*-butyl rotation as steric bulk around the central carbon *increases*. In ether **9**, the barrier then increases reflecting the presence of a much larger *tert*-butyl group. This inverse dependence of the barrier to *tert*-butyl rotation in ethers **6–8** on steric bulk is analogous to a similar but more pronounced dependence of the barrier to inversion–rotation about nitrogen in a series of *N*-*tert*-butyl-*N*,*N*-dialkylamines.[9] For the ethers **6–8**, such a trend is very likely due to yet-to-be-determined adjustments in ground state geometry resulting from the crowding of several bulky groups around the central atom. An important point to be made is that no simple approach can be taken to predict barrier trends in these encumbered systems without knowledge of the intimate details of ground state geometry. It is quite possible that methoxyl exerts a more pronounced buttressing effect than hydroxyl leading to more internal crowding in the ethers than in the alcohols and differences in preferred ground state geometry.

A number of studies have now appeared in which the DNMR method was used to measure the rate of rotation about carbon–carbon single bonds.[6] In addition, the rate of rotation about the carbonyl carbon–nitrogen bond of several urethanes has been measured using the same technique.[14] In light of these data and the barriers compiled in Table I, it was then possible to predict qualitatively the temperature dependence of the ^{1}H DNMR spectra of urethanes **10** and **11** (Table I; Figures 5 and 6). Separation of the $N(CH_3)_2$ resonance of **10** or **11** into two singlets of equal intensity at low temperatures is consistent with slowing the $N(CH_3)_2$ rotation (eq 2) and a significant degree of π bonding across the carbon–nitrogen bond.[14] The barriers to $N(CH_3)_2$ rotation in **10** ($\Delta H^{\ddagger} = 15.4 \pm 1.3$ kcal/mol; $\Delta S^{\ddagger} = 0.3 \pm 5$ gibbs; $\Delta G^{\ddagger} = 15.3 \pm 0.1$ kcal/mol at 6.3 °C) and **11** ($\Delta H^{\ddagger} = 19.6 \pm 0.8$ kcal/mol; $\Delta S^{\ddagger} = 8 \pm 2$ gibbs; $\Delta G^{\ddagger} = 17.0 \pm 0.1$ kcal/mol at 53 °C) are typical of such urethanes.[14] The barriers to *tert*-butyl rotation in **10** and **11** (Table I) reveal the carbamate moiety to be very similar to methoxyl and hydroxyl in its ability to restrict *tert*-butyl rotation.



(2)

TABLE II: Free Energies of Activation ($\Delta G^{\ddagger}$) for *tert*-Butyl Rotation in t-$C_4H_9C(CH_3)_2X$

X	$\Delta G^{\ddagger}$, kcal/mol	Ref
H	6.9	6e,f
F	8.0	20
Cl	10.4	6d,e
Br	10.7	6d,e
I	11.1	20
OH	8.7	This work
OCH_3	9.3	This work

Finally, it is interesting to compare the abilities of various functionalities incorporated into the same basic carbon skeleton to hinder *tert*-butyl rotation (Table II). It is not surprising to note that hydrogen is the least effective substituent in restricting rotation. The ability of halogens to hinder *tert*-butyl rotation (Table II) seems to be more a function of van der Waals radius and does *not* parallel *A* value trends.[16] Indeed the *A* values of methoxyl (0.55 kcal/mol)[16] and non-hydrogen-bonded hydroxyl (~0.6 kcal/mol)[16] are almost identical with chlorine (0.53 kcal/mol)[16] and yet hydroxyl and methoxyl are significantly less hindering to *tert*-butyl rotation than chlorine. These comparisons point up the possible pitfalls of using conformational parameters from *cyclic* compounds to predict trends in both conformational preferences and barriers to rotation in *acyclic* systems.

Experimental Section

The *60-MHz ^{1}H DNMR spectra* were obtained using a Varian HR-60A spectrometer equipped with a custom-built variable temperature probe.[21] The *100-MHz ^{1}H DNMR spectra* were obtained using a Varian HA-100-15 spectrometer equipped with a Varian variable temperature probe and temperature controller.

The theoretical ^{1}H DNMR spectra[13] were calculated using DEC PDP-10 and RCA Spectra 70/46 computers and plotted using a Calcomp plotter.

3,3-Dimethyl-2-butanone-1,1,1-d_3 (**13**) was prepared by repeated exchanges with refluxing deuterium oxide and a trace of sodium carbonate. *Alcohols* **1**, **2**, **4**, and **5** were prepared by reaction of CD_3MgI, CD_3CD_2MgI, $C_6H_5CH_2MgBr$, and t-C_4H_9MgBr respectively with **13** and worked up in deuterium oxide. *Alcohol* **3** was prepared by reaction of 2 M equivalents of CD_3CH_2MgI with the ethyl ester of pivalic acid followed by work-up in deuterium oxide.

Methyl ethers **6–9** were prepared from the corresponding alcohols above by initial conversion to the potassium alkoxide using the potassium hydride procedure of Brown[22] followed by treatment with methyl iodide.

Urethanes **10** and **11** were prepared by treating potassium alkoxide[22] of **1** (R = R′ = CH_3) with *N*,*N*-dimethylcarbamoyl chloride and *N*,*N*-dimethylthiocarbamoyl chloride, respectively.

Acknowledgment. We are grateful to the National Science Foundation (Grant Nos. GP18197 and MPS74-17544) for support and to the Worcester Area College Computation Center for donated computer time.

References and Notes

(1) National Science Foundation Undergraduate Research Participant, 1972–1973.

(2) Alfred P. Sloan Research Fellow, 1971–1974; Camille and Henry Dreyfus Teacher-Scholar, 1972–present; Visiting Associate Professor, University of Michigan, Ann Arbor, 1974–1975.

(3) J. D. Kemp and K. S. Pitzer, *J. Chem. Phys.*, **4**, 749 (1936); S. Weiss and G. E. Leroi, *ibid.*, **48**, 962 (1968), and references therein.

(4) I. R. Epstein and W. N. Lipscomb, *J. Am. Chem. Soc.*, **92**, 6094 (1970), and references therein.

(5) J. P. Lowe, *Science*, **179**, 527 (1973); L. Radom, W. J. Hehre, and J. A. Pople, *J. Am. Chem. Soc.*, **94**, 2371 (1972).

(6) (a) P. E. Stevenson, G. Bhat, C. H. Bushweller, and W. G. Anderson, *J. Am. Chem. Soc.*, **96**, 1067 (1974); (b) F. J. Weigart, M. B. Winstead, J. I. Garrels, and J. D. Roberts, *ibid.*, **92**, 7359 (1970); (c) R. A. Newmark and C. H. Sederholm, *J. Chem. Phys.*, **43**, 602 (1965); (d) B. L. Hawkins, W. Bremser, S. Borcic, and J. D. Roberts, *J. Am. Chem. Soc.*, **93**, 4472 (1971); (e) J. E. Anderson and H. Pearson, *ibid.*, **97**, 764 (1975); (f) C. H. Bushweller and W. G. Anderson, *Tetrahedron Lett.*, 1811 (1972); (g) C. H. Bushweller, S. Hoogasian, W. G. Anderson, and L. J. Letendre, *J. Chem. Soc., Chem. Commun.*, 152 (1975).

(7) G. Binsch, *Top. Stereochem.*, **3**, 97 (1968).

(8) C. H. Bushweller, S. Hoogasian, and W. G. Anderson, *Tetrahedron Lett.*, 547 (1974).

(9) C. H. Bushweller, W. G. Anderson, P. E. Stevenson, D. L. Burkey, and J. W. O'Neil, *J. Am. Chem. Soc.*, **96**, 3892 (1974).

(10) C. H. Bushweller and J. A. Brunelle, *J. Am. Chem. Soc.*, **95**, 5949 (1973).

(11) C. H. Bushweller and M. Z. Lourandos, *Inorg. Chem.*, **13**, 2514 (1974).

(12) C. H. Bushweller and J. A. Brunelle, *Tetrahedron Lett.*, 893 (1974).

(13) DNMR3 was written by D. A. Kleier and G. Binsch, *J. Magn. Reson.*, **3**, 146 (1970); Program 165, Quantum Chemistry Program Exchange, Indiana University, Bloomington, Ind. Our local modifications allow the use of larger rate constants, more configurations, more spins, and are described in detail in C. H. Bushweller, G. Bhat, L. J. Letendre, J. A. Brunelle, H. S. Bilofsky, H. Ruben, D. H. Templeton, and A. Zalkin, *J. Am. Chem. Soc.*, **97**, 65 (1975).

(14) C. H. Bushweller, P. E. Stevenson, J. Golini, and J. W. O'Neil, *J. Phys. Chem.*, **74**, 1155 (1970), and references therein.

(15) M. Nakamura, M. Ōki, and H. Nakanishi, *J. Am. Chem. Soc.*, **95**, 7169 (1973); J. E. Anderson and D. I. Rawson, *J. Chem. Soc., Chem. Commun.*, 830 (1973).

(16) F. R. Jensen and C. H. Bushweller, *Adv. Alicyclic Chem.*, **3**, 139 (1971), and references therein; F. R. Jensen, C. H. Bushweller, and B. H. Beck, *J. Am. Chem. Soc.*, **91**, 344 (1969).

(17) J. E. Anderson, *J. Chem. Soc., Perkin Trans. 2*, 10 (1974).

(18) H. B. Burgi and L. S. Bartell, *J. Am. Chem. Soc.*, **94**, 5236, 5239 (1972).

(19) W. Bradford, Ph.D. Thesis, University of Michigan, Ann Arbor, Mich., 1975.

(20) J. E. Anderson and H. Pearson, *Tetradron Lett.*, 2779 (1972).

(21) F. R. Jensen, L. A. Smith, C. H. Bushweller, and B. H. Beck, *Rev. Sci. Instrum.*, **43**, 894 (1972).

(22) C. A. Brown, *J. Am. Chem. Soc.*, **95**, 982 (1973).

COMMUNICATIONS TO THE EDITOR

Molecular Covolumes of Sphere and Ellipsoid of Revolution Combinations

Sir: In the analysis of equilibrium sedimentation, light scattering, or osmotic pressure results on polydisperse or interacting macromolecular systems it is required to express nonideality effects in terms of activity coefficients that are functions of solution composition. This implies that even for relatively simple interacting systems several self- and cross-term second virial coefficients must either be evaluated from the experimental results or assigned numerical values on the basis of molecular covolume and charge considerations. The former endeavor is formidable and has not been achieved without simplifying assumptions,[1,2] whereas the latter requires expressions for covolumes. Of particular interest in the protein field are covolume expressions for interactions between various combinations of spheres and ellipsoids of revolution (prolate or oblate). Recently, Ogston and Winzor[3] formulated expressions for sphere and prolate ellipsoid combinations, but left unanswered two basic questions. The first pertains to the corresponding expressions for oblate ellipsoids and the second to the correlation of their formulation with the Isihara[4] treatment of self-covolumes of ellipsoids of either type.

The basic set of equations considered by Ogston and Winzor[3] in relation to their Figure 1 was $\{(x^2/a^2) + (y^2/b^2) = 1, X = x + r \sin\theta, Y = y + r \cos\theta, dy/dx = \tan\theta\}$ where a and b refer to the semimajor and semiminor axes, respectively, of the ellipse, which is tangential at the point (x,y) to a circle of radius r and center (X,Y). This same set of equations applies to the sphere–oblate ellipsoid problem, the difference being that in this instance rotation of the ellipse is considered about the y axis. Formulation of the covolume expressions requires consideration of the volume enclosed by the locus of the point (X,Y) which may be formulated in terms of circular transverse sections. In these terms the molecular covolume U is given by

$$U = 2\int_{x=0}^{x=a} \pi Y^2\,\mathrm{d}X \qquad \text{(prolate)} \qquad (1a)$$

$$U = 2\int_{y=0}^{y=b} \pi X^2\,\mathrm{d}Y \qquad \text{(oblate)} \qquad (1b)$$

The indicated integrations may be performed directly once the quantities $Y^2\,\mathrm{d}X$ and $X^2\,\mathrm{d}Y$ are written (with the use of the basic set of equations) as functions of the respective single variables x and y. Indeed, the required integrals (12 in each case) are in standard form when the substitutions are made that $\epsilon^2 = 1 - (b^2/a^2)$ for the prolate case and