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Full Color Computer-Generated Rainbow Hologram with Enlarged Viewing Angle

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The viewing angle of a computer-generated hologram (CGH) can be expanded by lens-less Fourier configuration. However, a laser is required to illuminate the CGH for reconstruction. We proposed earlier making a second hologram for white-light reconstruction; here, we propose a method to calculate CGH for a full-color rainbow hologram with enlarged viewing angle. First, we calculate the master hologram with three virtual slits whose positions correspond to red, green and blue wavelength. The transfer hologram can be made from the master CGH with a single exposure. In the experimental result, we obtain a full-color computer-generated rainbow hologram with 17° of viewing angle and 3.0 mm × 2.5 mm image size.

Key words: holography, computer-generated hologram, enlarged viewing angle, full color, rainbow hologram

1. Introduction

Photographic reduction is often used as a simple output facility for computer-generated holograms (CGH). However, its resolution limits the diffraction angle so that the viewing angle is inadequate for binocular observation. We investigate the use of a lens-less Fourier hologram[1)] for photographic reduction of the CGH because the viewing angle is independent from the resolution and proportional to the number of hologram pixels. Since the hologram is recorded with the reference point source beside the object, the illumination point source must be precisely located near the hologram for reconstruction. It is not practical to use such special illumination, and therefore, we designed a secondary hologram with collimated reference beam and the object beam from the reconstructed image of a lens-less Fourier type CGH.[2)] We have also extended this approach to computer-generated rainbow holograms.[2,3)]

We propose here a method to record a full color rainbow hologram with single exposure from a computer-generated hologram.[4)] For the optical rainbow hologram,[5)] a method of making a full color hologram has been proposed.[6,7)] However, the configuration of our computer-generated rainbow hologram is different from the optical rainbow hologram, so we propose a new approach, which is similar to the pseudo-color technique.[7)] The first hologram is calculated with three virtual slits at a single wavelength. The angle and distance of each slit is obtained to correspond to one of the three primary colors. Then, the second hologram can be made from the first hologram with a single exposure. From the preliminary experimental result, we obtained a full-color computer-generated rainbow hologram with 17° of viewing angle and 3.0 mm × 2.5 mm image size. The size and viewing angle can be easily increased by increasing the number of hologram pixels.

2. Theory

2.1 *Viewing angle of the lens-less Fourier computer-generated holograms*

If the CGH is calculated with a collimated reference beam, its viewing angle is the same as its diffraction angle. If the hologram is recorded with the collimated reference beam slanted vertically, then the hologram becomes on-axis horizontally. Therefore, the horizontal viewing angle 2Θ is obtained as

$$2\Theta = 2\sin^{-1}\frac{\lambda}{2d}, \tag{1}$$

where λ is the wavelength of the light, and d is the pixel pitch of the CGH. This means that the viewing angle depends on the pixel pitch of the CGH. In contrast, the lens-less Fourier hologram,[1)] recorded with a reference point source, makes the viewing angle independent of the pixel pitch. Geometry to determine the viewing angle of the lens-less Fourier hologram is shown in Fig. 1. Note that the hologram is vertically off-axis and the reference point source is located above or below the object area. The viewing angle 2θ is proportional to the number of hologram pixels P as expressed below[2)] (approximation is valid when the angle is small):

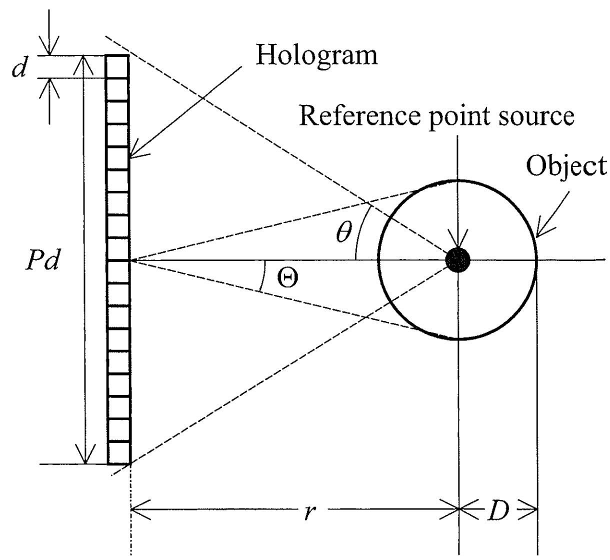

Fig. 1. Top view of the optical setup for the recording hologram with reference point source.

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$$2\theta = 2\tan^{-1}\frac{Pd}{2r} \approx \frac{P\lambda}{4D}, \tag{2}$$

where D is radius of the object area, P is the number of horizontal pixels, d is the pixel pitch and r is the distance between the hologram and center of the object. Equation (2) also shows that reducing image size can increase the viewing angle.

2.2 *Calculating master CGH*

In this paper, the objects are assumed to be a collection of self-illuminated points[9] and the object beam can be calculated as a summation of complex amplitudes from these points. In the actual calculation, fringes formed by each object point and the reference beam are summed up.[9] The final fringe $I(X, Y)$ on the hologram is obtained as,

$$I(X, Y) = 2\sum_{p=1}^{N}\frac{a_p}{r_p}\cos\left(kr_p + \phi_p + kr_{\mathrm{R}}\right), \tag{3}$$

where N is the number of object points, r_p is the distance between the p-th object point and the point (X, Y) on the hologram, r_{R} is the distance between the reference point source and the point (X, Y). Each object point has a real-valued amplitude a_p and a relative phase ϕ_p. The wave number k is defined as $k = 2\pi/\lambda$. Note that the factor $\exp(j\omega t)$ is not explicitly included. The values $I(X, Y)$ are shifted and scaled to fit an 8-bit gray scale representation. This hologram is called the master hologram or H1.

2.3 *Making the second hologram by optical means*

In the image reconstruction, the lens-less Fourier hologram requires a special illumination, a coherent point light source that must be located near the hologram. Since it is not practical to use such special illumination, we made a secondary hologram that can be reconstructed with collimated white-light. Wentzel *et al.* ealier reported the idea to make a secondary hologram from the CGH to increase the angle of illumination and output beam for comfortable viewing.[8] The optical setup for making the secondary hologram is illustrated in Fig. 2. The holographic plate is located at the center of the reconstructed image from the master CGH to make the image-plane hologram. A collimated reference beam is provided from the same side of the object beam for a transmission hologram or the opposite side for a reflection hologram. The secondary hologram is called a transfer hologram or H2.

We use a photographic film for the master hologram, although it is possible to make an automatic hologram printer if the film can be replaced by a spatial light modulator.

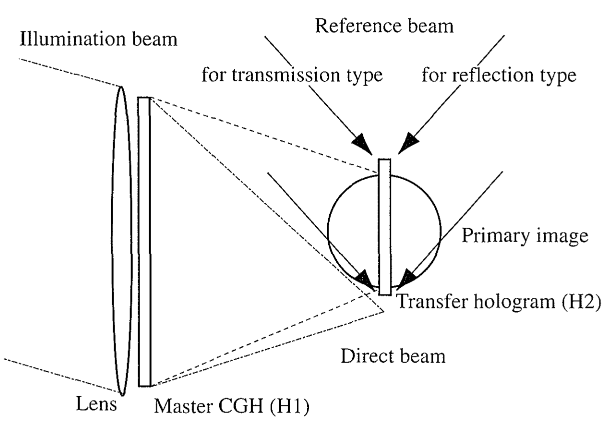


Fig. 2. Side view of the optical setup for recording an optical hologram from the reconstructed image of the CGH.

2.4 *Calculating master CGH for rainbow hologram*

A rainbow hologram[5] can also be produced when the master CGH is calculated in the proper manner;[2] the calculation geometry is shown in Fig. 3. A virtual slit, located at the viewing position, limits the calculation area on the hologram as shown. For each object point source, calculation is required only for the area where the light beam can pass through both the virtual slit and the object point. This limitation also reduces the computational time.[2] In the optical rainbow hologram, the slit is usually located on the master hologram. Since the width of the viewing area is the same as the horizontal width of the slit, the width of the master hologram should be the same as the viewing area. In contrast, the computer generated rainbow hologram uses the virtual slit and its width is independent of the master CGH. This means we can obtain a wide viewing angle from a small CGH.

The calculated master CGH is then transferred to the second hologram, which can be reconstructed with white-light, using the process described in Sect. 2.3.

2.5 *Calculating master CGH for full color rainbow hologram*

There are two major ways to make the optical full color rainbow hologram. The first is by three-color recording,[6] using red, green and blue laser light. The other method is

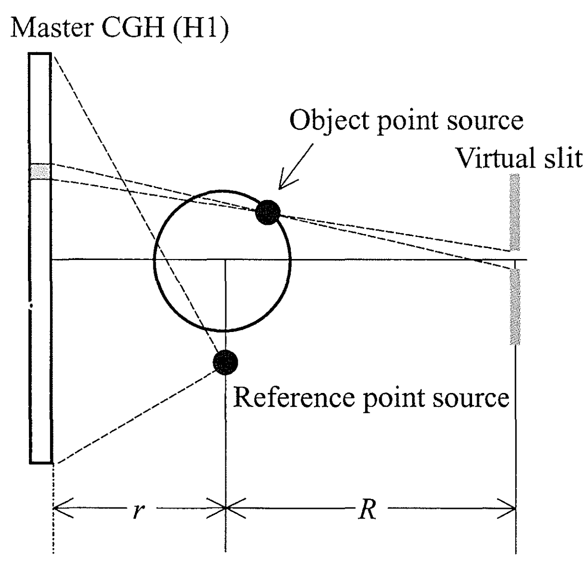


Fig. 3. Side view of the optical setup for calculating the master hologram of computer-generated rainbow hologram with enlarged viewing angle.

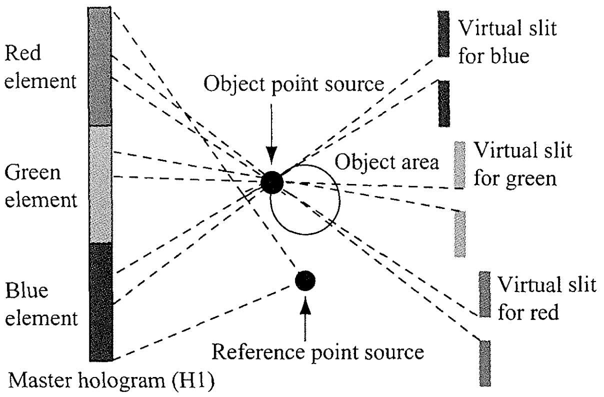


Fig. 4. Side view of the optical setup for calculating a master hologram for full color computer-generated rainbow hologram with enlarged viewing angle.

pseudocolor recording.[7)] Since color of the reconstructed image from the rainbow hologram changes when the viewer moves up and down, a multicolor image can be made by multiple exposure with different reference beam angles or different slit positions.

Here, we made a full color computer-generated rainbow hologram with different virtual slit positions. In the optical pseudocolor hologram, the slits are located on the master hologram, while the virtual slits are located separately from the master holograms as illustrated in Fig. 4. Positions of the virtual slits are determined in the following manner. The relation of the angles and the wavelengths in hologram recording and reconstruction is expressed as,[10)]

$$\sin\theta_{\text{out}} = \frac{\lambda_2}{\lambda_1}\left(\sin\theta_{\text{obj}} - \sin\theta_{\text{ref}}\right) + \sin\theta_{\text{ill}}, \tag{4}$$

where θ_{out}, θ_{obj}, θ_{ref} and θ_{ill} represent the beam angle of output, object, reference and illumination, respectively. These angles are shown in Fig. 5, the recording geometry

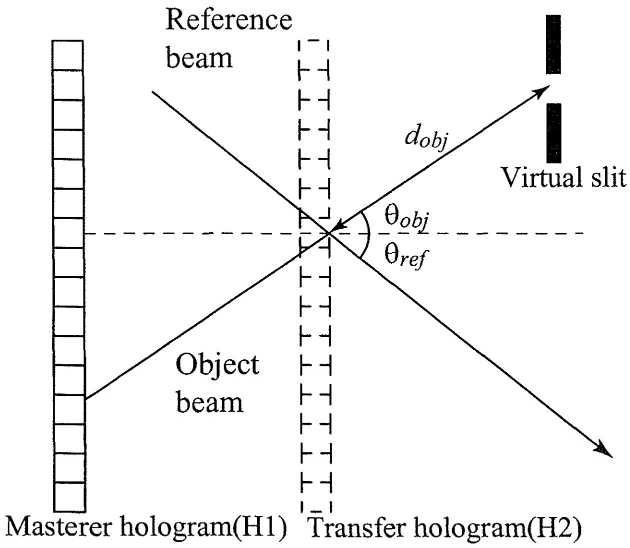


Fig. 5. Side view of the optical setup for recording the transfer hologram for full color computer-generated rainbow hologram with enlarged viewing angle.

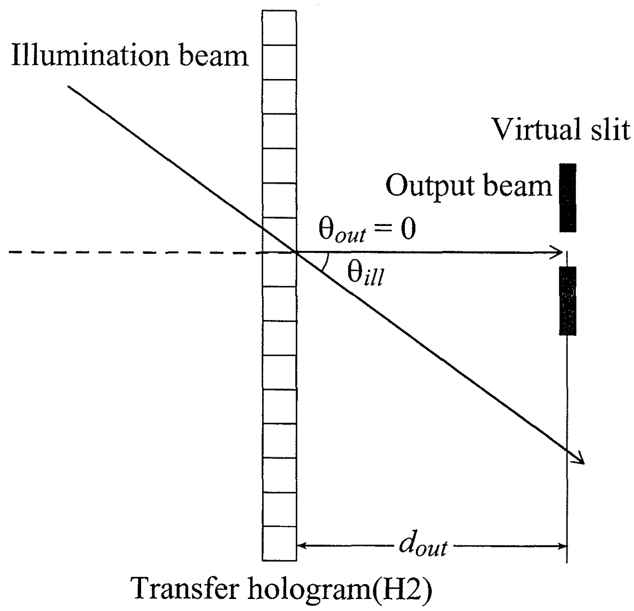


Fig. 6. Side view of the optical setup for reconstructing the transfer hologram for full color computer-generated rainbow hologram with enlarged viewing angle.

of the transfer hologram (H2), and in Fig. 6, the reconstruction geometry. For clarity, only a single virtual slit is shown in these figures. The wavelength in recording is λ_1 and that in reconstruction is denoted as λ_2. In the reconstruction, the output beam angle θ_{out} is usually set to $0°$ and the illumination beam angle θ_{ill} of $-45°$ is preferred. In the recording, the reference beam angle θ_{ref} is often set to the same angle as θ_{ill}, but it can be selected independently. From these angles and the recording wavelength λ_1, one can determine the object beam angle θ_{obj} for the virtual slit of the desired output wavelength λ_2.

The relation of the distances between the hologram and the location of beam sources in hologram recording and reconstruction is expressed as,[10)]

$$\frac{\cos^2\theta_{\text{out}}}{d_{\text{out}}} = \frac{\lambda_2}{\lambda_1}\left(\frac{\cos^2\theta_{\text{obj}}}{d_{\text{obj}}} - \frac{\cos^2\theta_{\text{ref}}}{d_{\text{ref}}}\right) + \frac{\cos^2\theta_{\text{ill}}}{d_{\text{ill}}}, \tag{5}$$

where d_{out}, d_{obj}, d_{ref} and d_{ill} represent the distance of output, object, reference and illumination, respectively. In the present paper, reference and illumination beams are assumed as collimated. Therefore, d_{ref} and d_{ill} are infinite, and Eq. (5) can be simplified as,

$$\frac{\cos^2\theta_{\text{out}}}{d_{\text{out}}} = \frac{\lambda_2}{\lambda_1}\frac{\cos^2\theta_{\text{obj}}}{d_{\text{obj}}}. \tag{6}$$

These distances are shown in Figs. 5 and 6. The distance d_{out} must be equal to the viewing distance in the hologram reconstruction. Then, the distance d_{obj} can be obtained from Eq. (6).

For the full color hologram, the angle of θ_{obj} and the distance of d_{obj} should be found for three primary wavelengths. The master hologram is then calculated with those parameters. The transfer hologram, or white-light reconstructable full color rainbow hologram, can be made with the process described in Sect. 2.3 at a single exposure.

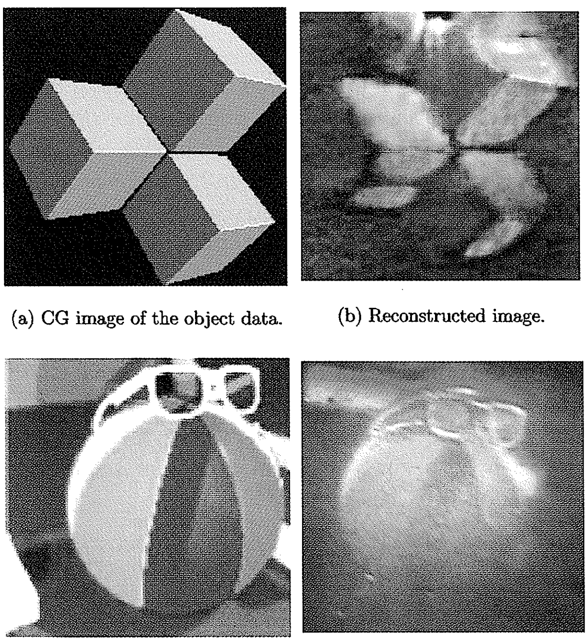
One problem of the pseudocolor method is color misregistration.[7] However, this can be compensated for by putting reverse distortion in the original object data.

3. Results

We use photographic reduction to make the master CGH. The computed holograms are first printed by a laser printer. Then, the hologram is taken by a single lens reflection camera with Fuji Film's "Minicopy film", a kind of a microfilm. After developing, the secondary hologram is exposed on AGFA's 8E75HD holographic plate by optical means as shown in Fig. 2.

Figure 7 shows the original object data and the white-light reconstructed images from the full color computer-generated rainbow hologram made by the proposed method. The original object data shown in Figs. 7(a) and 7(c) are represented as a collection of polygons. These data are converted to collections of self-illuminated points for CGH calculation. Total number of converted points is 5,378 for Fig. 7(b) and 6,816 for Fig. 7(d). Pixel number of the master CGHs is 4,800 horizontally $\times 7,200$ vertically and the size of the exposed CGHs on the film is about 24 mm $\times$ 36 mm. Therefore, the pixel pitch d of the master CGH is about 5.0 μm. Computational time is measured on a personal computer (CPU: Pentium II 800 MHz, OS: Windows 98, Compiler: Visual C++ 6.0), and takes about 15 min.

The sizes of the reconstructed image shown in Figs. 7(b) and 7(d) are 2.5 mm $\times$ 3.0 mm; these images are taken by CCD camera. Since the holograms are full color rainbow holograms, actual images are full color and display continuous perspective changes when the viewer moves left and right; color change occurs when the viewer moves up and down. The horizontal viewing angle is as wide as 17°, about 5 times wider than that of the CGH calculated with the collimated reference beam. This viewing angle corresponds to the viewing area of 149 mm at the viewing distance $d_{\rm out} = 500$ mm, and the viewer can appreciate adequate motion parallax.



(a) CG image of the object data. (b) Reconstructed image.

(c) CG image of the object data. (d) Reconstructed image.

Fig. 7. CG images and reconstructed images of the proposed full color rainbow hologram: (a) CG image of the object data. (b) Reconstructed image. (c) CG image of the object data. (d) Reconstructed image.

4. Conclusion

We have proposed a method to make a full color computer-generated rainbow hologram with enlarged viewing angle. The pseudocolor method for an optical hologram is adapted to a computer-generated hologram, and the full color hologram can be made with a single exposure from the master CGH whose resolution is relatively low. We have obtained a viewing angle as wide as 17°, 5 times wider than by the conventional method. Using the proposed technique, increasing pixel number of the holograms makes the viewing angle much wider. The photographic film for the master CGH can be replaced in the spatial light modulator for an automatic hologram printer. Our future plans are to improve image quality and make image size even larger.

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