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THE FRACTIONAL DIMENSION IDENTIFICATION METHOD OF CRITICAL BIFURCATED PARAMETERS OF BEARING-ROTOR SYSTEM *

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Abstract: *The stable problem of rotor system, seen in many fields, has been cared for more. Nowadays the reasons of most losing stability are caused by nonlinear behaviors. This presents higher requirements to the designing of motor system: considering nonlinear elements, avoiding the unstable parameter points or regions where nonlinear phenomena will be presented. If a family of time series of the unknown nonlinear dynamical system can only be got (may be polluted by noise), how to identify the change of motive properties at different parameters? In this paper, through the study of Jeffcott rotor system, the result that using the figures between the fractional dimension of time-serial and parameter can be gained, and the critical bifurcated parameters of bearing-rotor dynamical system can be identified.*

Key words: fluid film bearing-rotor system; bifurcation; fractional dimension
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Introduction

Bifurcation and fractal theory are two important branches of nonlinear science. The bifurcation is: for continuous-time dynamical system $\dot{x} = f(x, t, \alpha)$ or discrete dynamical system $x \mapsto g(x, \alpha)$ (where g is map $g: R^n \times R \to R^n$), when the parameter α changes continuously and passes α_0, the topological structure of above continuous-time or discrete dynamical system changes suddenly, then define that system appear bifurcation at parameter α_0. There are many types of bifurcation, for example: Hopf bifurcation, Period-doubling bifurcation (Sub-harmonic bifurcation) and Quasi-period bifurcation etc.. The study object of fractal theory is the irregular and non-differentiable geometrical body which is generated by nonlinear system, the evident characteristic of fractional structure is the similitude between part and whole, its parameter of

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characteristic target is fractional dimension. Which dimension isn't the simple supplement to the Euclidean space dimension, for it has much new information: fractional dimension not only measures the self-similitude property of fractional structure, but also is one of main basis used to distinguish chaotic movement from others; and its value may be either integral, or fraction. Normal forms of fractional dimension have: Capacity dimension (Hausdorff dimension), Information dimension, Correlation dimension and Generalized dimension etc..[1~3] In all fractal case, the calculation of fractional dimension is key problem in fractal theory. In this paper, during identifying critical parameters or regions of bifurcation and chaotic model, we would encounter the process of time series fractional dimension calculation which is determined by different r, but it can't affect the identifying result, for the identifying method only relates to the fractional dimension changing trend, it may be proved below.

We may use many tools or algorithms as Floquet exponents[4] (Variable Floquet exponents accords to different bifurcated type, for example, if Floquet exponents has a couple of imaginary roots, then Holp bifurcation takes place.), Poincaré maps[5] (Different bifurcation case may be found through Poincaré maps, for example, if the Poincaré map is a circle, then quasi-period bifurcation takes place) etc. to decide the bifurcation of nonlinear dynamical system. But for one unknown nonlinear system (period, Jacobi matrix can't be obtained), only a group of nth-order time serial at different parameters can be obtained, then the above methods are invalid. For above consideration, in this paper, we present a method that using the time serial fractional dimension trending figures to identify the critical bifurcated parameters of nonlinear dynamical system. Finally this method is used to identify the critical bifurcated parameters of a Jeffcott fluid film bearing-rotor system, and get a satisfactory result, it implies that this method can present some reference for the engineering designing.

1 Calculation of Time Series Capacity Dimension

If there is a time series: $\{x_1, x_2, \cdots, x_N\}$ which dimensions aren't limted, we may get the fractional dimension of time series[3] as follows.

For any r, then calculate how many numbers of $(x_i, x_j)\mid_{i\neq j}$ which distance $\| x_i - x_j \|$ is less than r, finally calculate the percentage that $(x_i, x_j)\mid_{i\neq j}$ which distance $\| x_i - x_j \|$ is less than r to N^2, and definite the percentage as function $c(r)$:

$$c(r) = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\theta(r - \| x_i - x_j \|), \tag{1}$$

where $\theta(x)$ is Heaviside function:

$$\theta(x) = \begin{cases} 1 & x \geqslant 0, \\ 0 & x < 0. \end{cases} \tag{2}$$

The selecting of r isn't random, if r is too larger or too less, then $c(r)$ can't describe the behavior of system. Appropriately reduce the r, probably there is the following relation between r and $c(r)$:

$$c(r) = r^D. \tag{3}$$

Then the D is capacity fractional dimension of time series that we need, from formula (3):

$$D = \lim_{r \to 0} \frac{\lg c(r)}{\lg r}. \tag{4}$$

If time series is one variable (one dimension), the distance:

$$\| x_i - x_j \| = \mathrm{abs}(x_i - x_j);$$

If time series is two variables (two dimensions), the distance:

$$\| x_i - x_j \| = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2};$$

If time series is nth-order variables (nth dimension), the distance:

$$\| x_i - x_j \| = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + \cdots + (x_{in} - x_{jn})^2}.$$

2 The Critical Bifurcated Parameters Identification of Jeffcott Rotor System

For the complexity of fluid film bearing-rotor system, it includes abundante nonlinear behaviors. The rotor dynamic nonstable problem which caused by fluid force in the bearing troubles many making and using manufactures for a long time[5], so the rotor system nonstable problem has been the focal problem that be cared by many experts. This requires that we should consider the stability not only in linear field but also in nonlinear field during the designing of rotor system and try to avoid the appearance of nonlinear behavior (for example chaos, bifurcation) which often cause the system to lose stability. How to decide the property of different parameters or regions? One identifying critical bifurcated parameters with the trending figures of time series fractional dimensions has been presented in [6], simultaneously this method is proved theoretically through the critical bifurcated parameters identifying of May model[7] (May model is 1 dimension autonomous system). If this method still is valid for 2 dimension or n dimension non-autonomous system? For this consideration, in this paper, this idea is used to identify the critical parameters of a n dimension non-autonomous system —one plate Jeffcott Fluid film bearing-rotor System, finally gain satisfactory result. The procedures of identification are introduced as follows.

2.1 Model establishing of Jeffcott rotor system

The mechanical model of one plate Jeffcott fluid film bearing-rotor system, is showed in Fig.1 and Fig.2, its dynamical equations is[5]

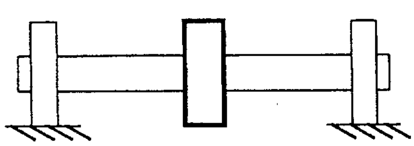

Fig.1 Rotor model

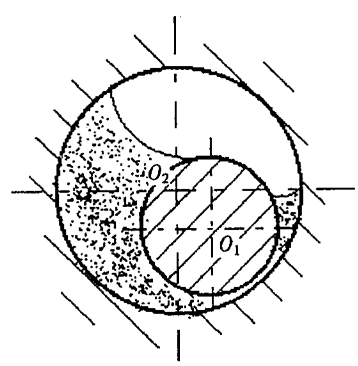

Fig.2 Fluid film

1) The equations of axle center O_1 is

$$\left.\begin{aligned} m_1\ddot{x}_1 + k_1(x_1 - x_3) + d_1(\dot{x}_1 - \dot{x}_3) &= -f_{1x}, \\ m_1\ddot{y}_1 + k_1(y_1 - y_3) + d_1(\dot{y}_1 - \dot{y}_3) &= -f_{1x} - m_1 g; \end{aligned}\right\} \tag{5}$$

2) The equations of bearing center O_2 is

$$\left.\begin{aligned} m_0\ddot{x}_2 + k_2x_2 &= -f_{1x}, \\ m_0\ddot{y}_2 + k_2y_2 &= -f_{1y} - m_0 g; \end{aligned}\right\} \tag{6}$$

3) The equations of concentrated mass center O_3 is

$$\left.\begin{aligned} m\ddot{x}_3 + k_1(x_3 - x_1) + d_1(\dot{x}_3 - \dot{x}_1) &= m\rho\omega^2\cos\omega t, \\ m\ddot{y}_3 + k_1(y_3 - y_1) + d_1(\dot{y}_3 - \dot{y}_1) &= m\rho\omega^2\sin\omega t - mg, \end{aligned}\right\} \tag{7}$$

where $f_{1x} = P_r\cos\psi_1 + P_t\sin\psi_1$, $f_{1y} = P_r\sin\psi_1 - P_t\cos\psi_1$ (P_r, P_t is the film force).

2.2 The critical bifurcated parameters identification of Jeffcott rotor system

For the one-plate Jeffcott flexible rotor model, through calculating, we know it may appear period-doubling bifurcation (this may be seen clearly from the figures of frequency spectrum), when get the structural parameters listed in Table 1.

Table 1 The Structural Parameters of rotor system

name	rotate speed	bearing mass	film viscosity	mass eccentricity	bearing interval	bear length	bearing radius	bearing seat stiffness
Value	4 100	24	1.02e-3	1.52e-5	3.25e-3	0.055	0.038	0.86e8

Through numerical integration to the dynamical model of Jeffcott given above, we may get the imbalance response at the structural parameters listed in Table 1. The response time serial of axle center is $\{[x_{1i}, y_{1i}]^{\mathrm{T}} \mid i = 1,2,\cdots,n\}$. Let $u_i = [x_{1i}, y_{1i}]^{\mathrm{T}}$, then the response time serial of axle center can be expressed as $\{u_1, u_2, \cdots, u_n\}$, time serial $\{u_1, u_2, \cdots, u_n\}$ is two dimension time serial. Getting a serial of value in the designing domain of rotor, through numerical integral (4th order Runge-Kutta), we may get a family of time series, then draw the Poincaré map: Fig.3. In Fig.3 (a), the interval of plate mass is 5kg, in Fig.3 (b), the interval of plate mass is 0.5kg. From Fig. 3, we may find that the critical bifurcated plate mass value is 238kg.

Getting K different plate mass m, we may gain time serial $\{u_{ik} \mid i = 0,1,\cdots,n\}_{k=1,2,\cdots,K}$, for one r:

$$\min[\max(\| u_{ik} - u_{jk} \|_{k=1,2,\cdots,K})] \geqslant r \geqslant \max[\min(\| u_{ik} - u_{jk} \|_{k=1,2,\cdots,K})]$$
$$(i \neq j \mid i,j = 0,1,\cdots,n).$$

Using the calculating method of one dimension time serial introduced above, we may compute K capacity dimensions of $\{u_1, u_2, \cdots, u_n\}$ responding to different m :

$$D_k = \lg c_k(r)/\lg(r) \qquad (k = 1,2,\cdots,K). \tag{8}$$

During computing the distance $\| u_i - u_j \|_{i\neq j}$, we should use the calculation formula of two dimension time serial

$$\| u_i - u_j \| = \sqrt{(x_{1i} - x_{1j})^2 + (y_{1i} - y_{1j})^2}.$$

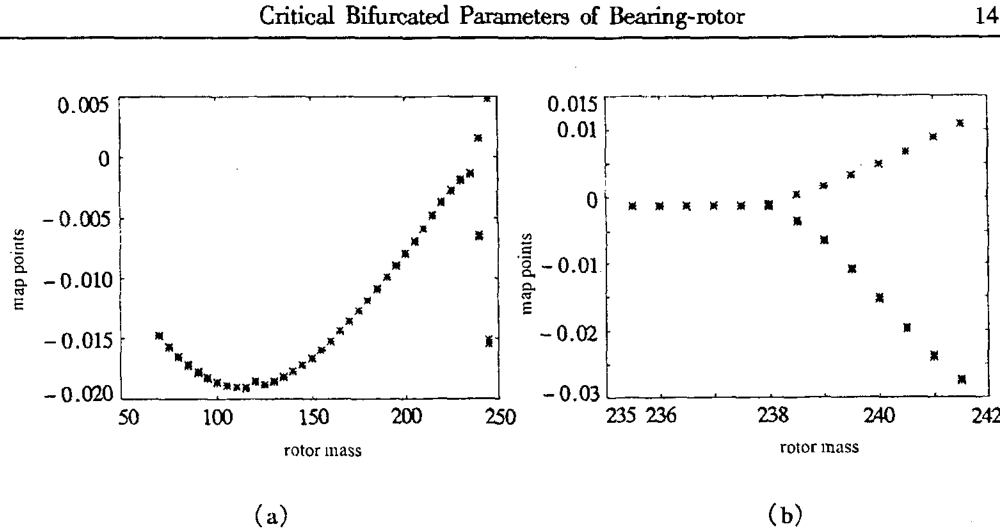

Fig.3 Plate mass-map point figure

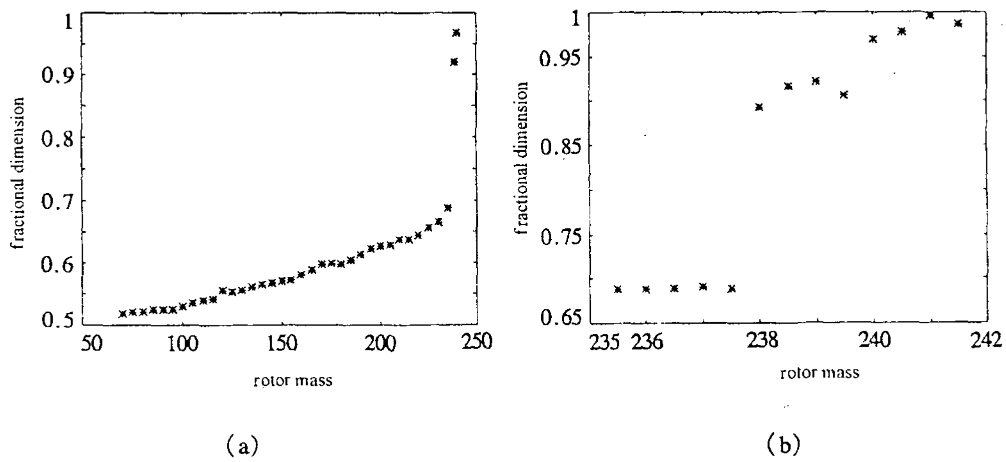

Fig.4 Plate mass-fractional figure

During the practical identifying, for considering identification precise and identification time, generally we need process identifying through two phases: firstly processing initial identifying, so as to decide the approximate region of bifurcation presenting; then processing exact identifying to the region got by first phase. In this example, firstly process initial identifying: the valid value of m is (75 kg, 245 kg), interval Δm is 5 kg, get Fig.4(a) between fractional dimensions and plate mass m; then process exact identifying: the valid value of m is (235 kg, 245 kg), interval Δ m is 0.5 kg, get Fig.4 (b) between fractional dimensions and plate mass m. From Fig.4, may find there exist changing suddenly where the plate mass gets 238 kg, that is to say, we may find the critical bifurcated plate mass is about 238 kg from Fig.4 too. Through using the identifying result to contrast with the Poincaré maps showed in Fig.3, we may say that this method is valid to identify the critical bifurcated parameter.

3 Conclusion

Through the above calculation and analysis results to fluid film bearing-rotor dynamical systems which can appear bifurcation, we may arrive to the conclusion: the characteristic parameter of fractal—fractional dimension may be a identifying tool whether the nonlinear dynamical system has bifurcation taking place at some parameter region, furthermore may present some theory supporting for the engineering designing of bearing-rotor dynamical system.

References:

[1] Zheng Huiyong, Liu Guaqiang, Dai Guanzhong. Fractal, chaos of nonlinear dynamics system and their application[J]. *Journal of Nonlinear Dynamics in Science and Technology*, 1996, 3(2): 182 ~ 190. (in Chinese)

[2] Barnsley M F. *Fractal Everywhere*[M]. New York: Academic Press Inc, 1988.

[3] Gong Yunfan, Xu Jianxue. Chaos signal and noise [J]. *Signal Processing*, 1997, **13**(2): 112 ~ 118. (in Chinese)

[4] Long Yunjia. *Study of Chaotic Vibration: Theory and Applying* [M]. Beijing: Tsinghua University Press, 1996. (in Chinese)

[5] Zhang Jiazhong. The nonlinear dynamic behaviors of the fluid film bearing-rotor system with squeeze film damper: Stability and bifurcation[D]. Doctor thesis. Xi'an: Xi'an Jiaotong University. (in Chinese)

[6] Zhao Yucheng, Xu Qingyu. Bifurcation parameter identification by fractionary character of time series[J]. *Journal of Xi'an Jiaotong University*, 1998, **32**(8): 106 ~ 107. (in Chinese)

[7] Liu Shida, Liu Shikuo. *Fractal and Fractional Dimension Introduction*[M]. Beijing: China Meteorological Press, 1992. (in Chinese)

[8] Guo Youzhong, Zhou Huanwen. Bifurcation, strange attractor, intermittence and chaos [J]. *Advances in Mechanics*, 1984, **14**(3): 255 ~ 274. (in Chinese)

[9] Grassberger P. Generalized dimensions of strange attractors[J]. *Physics Letters*, 1983, **97**A(7): 227 ~ 230.

[10] Grassberger P. On the fractal dimension of the Henon attractor[J]. *Physics Letters*, 1983, **97**A(7): 224 ~ 227.

[11] Parker T S, Chua L. *Practical Numerical Algorithms for Chaotic Systems*[M]. New York: Springer-Verlag, Inc, 1992.