## INTEGRATED PROCEDURE FOR IDENTIFICATION AND CONTROL OF MDOF STRUCTURES

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**ABSTRACT:** An integrated procedure based on a direct adaptive control algorithm is applied to structural systems for both vibration suppression and damage detection. The wider class of noncollocated actuator-sensor schemes is investigated through parameterized linear functions of the state variables that preserve the minimum phase property of the system. A larger number of mechanical parameters are shown to be identifiable in non-collocated configurations. Proper output selection allowing for model reference control and tracking error based parameters estimation under persistent excitation is described. Using full-state feedback, these capabilities are effectively exploited for oscillation reduction and health monitoring of uncertain multi-degree-of-freedom (MDOF) shear-type structures.

## INTRODUCTION

The technical science of mechatronics—developing systems composed by integrated mechanical elements, control logic, and electronic components—has been fastly evolving during the last decades. Vibration suppression and health monitoring of large flexible structures are recent structural engineering applications of this multidisciplinary field. In this respect, a structural system equipped with sensors and actuators may be designed to enable the identification of the relevant structural parameters and the robust reduction of the oscillations during a dynamical event. Nevertheless, in most of the investigations the two aspects have been treated separately according to the primary objective pursued.

On one hand, on-line identification procedures, developed in the context of system theory [e.g., Isermann et al. (1974)], have recently received attention for applications to linear and nonlinear multi-degree-of-freedom (MDOF) structural systems [e.g., Ghanem and Shinozuka (1995)]. Most of these procedures consist of prediction error-based estimators, where the errors between predicted and measured output are used to update estimates of the parameters. The stability proof, parameter boundedness, smallness condition for the estimation error and speed of adaptation delineate the differences in the available methodologies [e.g., Ioannou and Datta (1991)]. Recently, some specific studies in the structural context have been carried out for time-dependent degrading structures (Lin et al. 1990) and for nonlinear chain-like MDOF hysteretic systems (Smyth et al. 1999). In both cases, a least-squares based procedure has been implemented. Good accuracy in the stiffness degradation estimate is obtained in the former, whereas the use of forgetting factors and the effects of both persistent excitation and under- or overparameterization are some of the results obtained in the latter.

On the other hand, adaptive control procedures for structural systems have been investigated with the main goal of bringing some state variable combinations of an uncertain dynamical system to track a desired behavior relying on on-line adjustment of control parameters. The use of these methodologies has been proposed in different engineering applications (Poh and Baz 1996; Ghanem et al. 1997; Sun and Stelson 1997; Gattulli and Ghanem 1999; Gattulli and Romeo 1999). In particular, Ghanem et al. (1997) implemented an adaptive control procedure in a nonlinear single-degree-of-freedom system modeling a device that relies on an electrorheological fluid for its force-resisting mechanism. Gattulli and Ghanem (1999) proposed the use of a direct adaptive procedure to mitigate hydrodynamic vortex induced oscillations. Experimental validation of the direct adaptive controller has been pursued for a large aerospace flexible structure with six collocated actuator/sensor pairs (Ih et al. 1993a,b).

Recently Ray and Tian (1999) proposed a sensitivity enhancing procedure based on feedback control for damage detection pointing out the potential use of feedback for vibration suppression and health monitoring. According to this point of view, the present work proposes the use of an integrated procedure for robust control of oscillations and damage detection of MDOF linear structural systems. The methodology is founded on direct model reference adaptive control (MRAC), highlighting the key role played by the output selection in both collocated and noncollocated actuator/sensor configurations. Indeed, a proper output choice assures a linear dependence on the on-line identifiable mechanical parameters, and it allows detection of their variations due to damage under persistent excitation. Moreover, the number of the latter parameters is shown to be increased in noncollocated configurations. A qualitative analysis of the uncontrolled dynamics together with a Lyapunov-based controller design guarantee overall stability. It is assumed that the number of actuators is smaller than the number of sensors, which is the case in a general configuration for structural control in earthquake engineering applications (Soong 1990; Gattulli et al. 1994). The control strategy is currently based on full-state feedback (i.e., displacements and velocities). Applications for aseismic protection of an existing structure could be tackled by modifying the procedure to rely only on acceleration measurements (Dyke et al. 1996).

The present paper is organized as follows. The governing relations of a general structural system are reported in the second section, and control canonical forms are recalled in the third section. In particular, such transformations permit one to separate from the assignable dynamics the internal dynamics unaffected by the control input. The relevant steps of the control procedure based on sliding mode and model reference concepts are presented in the fourth section. An on-line identification procedure based on the asymptotic convergence to zero of the tracking errors is delineated in the fifth section. The last section is devoted to the applications of the procedure to sheartype structures.

### **GOVERNING RELATIONS**

It is assumed that structural oscillations induced by dynamic loads are described by a linear discrete model with  $n_a$  Lagran-

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gian degrees of freedom. A set of  $n_q$  linear ordinary differential equations of the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{E}\mathbf{u} + \mathbf{F}\mathbf{w}$$
(1)

represents the governing relations of the dynamical motion. The vector **q** describes the  $n_q$  displacements of a discrete set of points of the structural system from a reference configuration; the vector **u** contains the *m* control actions; and the vector **w** represents the  $n_q$  components of the dynamic loads. The  $(n_q \times n_q)$  mass, damping, and stiffness matrices are represented by **M**, **C**, and **K**, respectively. The allocation matrices for the control and the external actions are expressed by the matrices **E**  $(n_q \times m)$  and **F**  $(n_q \times n_q)$ , respectively. The equation of motion [(1)] can be rewritten in the state space form as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{H}\mathbf{w} \tag{2}$$

where  $\mathbf{x} = (\mathbf{q}, \dot{\mathbf{q}})$  is the  $n = 2n_q$  dimensional state vector. A vector  $\mathbf{y}$ , representing the  $p \le n$  outputs, can be obtained by a linear combination of the state variables through the following observation equation:

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{3}$$

where  $\mathbf{C} = (p \times n)$  observation matrix. In particular, the state space matrix **A** and allocation matrices **B** and **H** are given by

$$\mathbf{A} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{pmatrix}$$
(4*a*)

$$\mathbf{B} = \begin{pmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{E} \end{pmatrix}; \quad \mathbf{H} = \begin{pmatrix} \mathbf{0} \\ -\mathbf{M}^{-1}\mathbf{F} \end{pmatrix}$$
(4*b*,*c*)

#### **CONTROL CANONICAL FORMS**

The present section summarizes some basic control theory aspects [e.g., Isidori (1995) and Stengel (1994)] useful for applications of MRAC in structural contexts. MRAC procedures aim to lead the actual system to follow a desired reference system. Complete matching between actual and reference outputs can be pursued when their number is equal to the number of inputs (Ih et al. 1993a). Noncollocated input-output pairs can be properly selected using control canonical forms. Indeed, these representations highlight the relation between input and output and the existence of internal dynamics not affected by the control input. Therefore, we exploit the internal dynamics stability to select a class of plausible outputs.

Introducing the linear transformation,  $\eta = \mathbf{T}\mathbf{x}$ , a control canonical form of (2) and (3), can be written

$$\dot{\boldsymbol{\eta}} = A\boldsymbol{\eta} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{H}\boldsymbol{w} \tag{5}$$

$$\mathbf{y} = C\mathbf{\eta} \tag{6}$$

with  $A = TAT^{-1}$ ; B = TB; H = TH; and  $C = CT^{-1}$ . For single-input single-output (SISO) systems, in which *u* and *y* are scalar quantities (m = p = 1), the ( $n \times n$ ) linear transformation matrix **T** can be defined as

$$\mathbf{T} = \begin{pmatrix} \mathbf{c} \\ \mathbf{cA} \\ \cdots \\ \mathbf{cA}^{r-1} \\ \mathbf{\phi}_{1} \\ \cdots \\ \mathbf{\phi}_{n-r} \end{pmatrix}$$
(7)

where **c** is the  $(1 \times n)$  observation row vector;  $\phi_i$  is the  $(n - r \times 1)$  arbitrary row vectors chosen such that  $\phi_i \mathbf{b} = 0$  are satisfied; and  $\mathbf{b} = (n \times 1)$  control allocation vector. The transformed system is partitioned in *r* and n - r equations where

the vector  $\mathbf{\eta} = (\mathbf{\eta}_1; \mathbf{\eta}_2)^T$ , whereas the submatrices  $A_{11}$  and  $A_{12}$  and the subvector  $\mathbf{b}_1$  have the following special form:

$$\boldsymbol{A}_{11} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_1 & a_2 & a_3 & \cdots & a_r \end{pmatrix}$$
(8*a*)

$$\boldsymbol{A}_{12} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_{r+1} & a_{r+2} & a_{r+3} & \cdots & a_n \end{pmatrix}; \quad \boldsymbol{b}_1 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ b_r \end{pmatrix} \quad (8b,c)$$

while  $A_{21}$ ,  $A_{22}$ ,  $H_1$ , and  $H_2$  are generally nonsparse submatrices; and  $b_2$  = null vector. The observation row vector **c** becomes  $c = (1, 0, 0 \cdots 0)$ . In the transformed system, the control variable affects only the *r*th equation, whereas the transfer function between *u* and *y* is directly related to the coefficients of the transformed state space matrix. Indeed, the *n* coefficients  $a_i$  and the n - r coefficients  $d_j$  appearing in the transfer function G(s)

$$G(s) = \frac{y(s)}{u(s)} = g \frac{\det(A_{22} - s\mathbf{I})}{\det(A - s\mathbf{I})} = g \frac{d_0 + d_1s + \dots + d_{n-r}s^{n-r}}{a_0 + a_1s + \dots + a_ns^n}$$
(9)

are related with those of the *r*th row of the state matrix **A** through the constant  $g = cA^{r-1}b$ . In (9), *r* is known as the relative degree of the transfer function, and the transformation is complete when r = n.

Multi-input multi-output (MIMO) systems can also be represented in control canonical forms. In both SISO and MIMO cases, the transformed system puts into evidence the existence of an internal dynamics represented by the evolution of a subvector  $\eta_2$  of proper dimension that is not directly affected by the control. Indeed, choosing a control that brings the first subvector  $\eta_1$  to zero, the evolution of  $\eta_2$  depends on the spectral contents of the submatrix  $A_{22}$ . Eq. (9) shows that the spectral characteristics of the matrix  $A_{22}$  are identical to the zeros of the corresponding transfer function G(s) (Isidori 1995). Systems where the selected input-output pair causes stable internal dynamics are called minimum phase. The developed procedure is concerned with minimum phase systems characterized by generic actuator/sensor configurations.

# CONTROL STRATEGY VIA SLIDING MODE AND REFERENCE MODEL

The control is designed to reach an asymptotic tracking of some suitable variables selected from the ones representing the system dynamics belonging to a reduced subdimensional space. The tracking of a reference multidimensional trajectory  $\mathbf{y}_d \in \mathfrak{R}^p$  is achieved by asymptotic zero convergence of the multidimensional error between the actual and reference system outputs. This request can be satisfied only if the control variables are able to assign eigenvectors and eigenvalues of a subdimensional dynamical system  $\in \mathfrak{R}^p$ . It is currently known that eigenvectors assignment in a *p*-dimensional subspace needs at least a *p*-dimensional control vector (Stengel 1994). Thus, the developed procedure is restricted to systems with p = m.

#### **Reference Model**

Despite the imperfect knowledge about the chosen mathematical model for the structural system, the present procedure aims to devise a controller that will steer the system in some desired fashion, for example, so that the system response will approach or track a desired reference response. The latter can be obtained from a reference dynamical system analogous to the actual one. We will refer to such a model as the reference model described by the following equations:

$$\dot{\mathbf{x}}_d = \mathbf{A}_d \mathbf{x}_d + \mathbf{H}_d \mathbf{w}_d; \quad \mathbf{y}_d = \mathbf{C}_d \mathbf{x}_d \tag{10a,b}$$

where the vector  $\mathbf{y}_d \in \Re^p$  denotes the desired trajectories that the controlled system will be designed to track.

#### **Sliding Mode Control**

The control algorithm is based on the convergence of the output actual state y(t) to the desired target state  $y_d(t)$ . Therefore, defining  $e(t) = y(t) - y_d(t)$ , the tracking error vector **e** is introduced

$$\mathbf{e}(t) = \left(e, \frac{d}{dt}e, \dots, \frac{d^{r-1}}{dt^{r-1}}e\right)$$
(11)

The combined error vector s(t) is defined as

$$\mathbf{r}(\mathbf{e}, t) = \left(\frac{d}{dt} + \lambda_0, \dots, \frac{d}{dt} + \lambda_{r-1}\right) \mathbf{e}$$
 (12)

where the positive parameter  $\lambda_i$ , with i = 0, ..., r - 1, representing the relative weights, is used to fine-tune the controller. Obviously, for s(t) being identically zero, the tracking error vector  $\mathbf{e}(t)$  goes exponentially to zero. This observation justifies the design of a control algorithm that keeps s(t) at or near zero (Slotine and Li 1991). This goal is achieved sliding along the line

$$\dot{s}(t) + k_c s(t) = 0$$
 (13)

where the weighting parameter  $k_c$  defines the convergence rate. Therefore the control law is designed such that the *r*th equation of (5) matches (13). In the following, we will consider the significant case r = 2, generally occurring in SISO systems governed by second-order ordinary differential equations. Thus, by looking at the second equation of the transformed system represented by (5), an asymptotic tracking of the reference output can be achieved through the control law

$$\boldsymbol{\mu} = 1/b_2(\boldsymbol{y}_d - \boldsymbol{\lambda}\boldsymbol{\dot{e}} - \boldsymbol{k}_c \boldsymbol{s} - \mathbf{c}\mathbf{A}^2\mathbf{x} - \boldsymbol{h}_2\mathbf{w})$$
(14)

where the state space vector  $\mathbf{x}$  and the external force vector  $\mathbf{w}$  are the full-state feedback and feedforward terms, respectively.

### ON-LINE IDENTIFICATION PROCEDURE BASED ON TRACKING ERROR

In the above analysis, it has been assumed that the values of the parameter  $a_i$  entering the mass, viscous, and stiffness matrices were known and time invariant. Introducing uncertainties in these coefficients, in general, will not permit the synthesis of (13). Thus, assuming that only estimates of these coefficients are available, some conditions on updating these estimates will prove necessary for the stable operation of the controlled system.

## Parameters Estimator with Exponential Variable Gains

Due to the chosen reference model based control algorithm, prediction error and tracking error constitute possible sources of parameters information, and even a combination of the two errors can be used (Slotine and Li 1991). The on-line identification procedure considered here is based on tracking error.

Denoting parameter estimates by  $\hat{a}_i$  and the external force vector estimation by  $\hat{\mathbf{w}}$ , a control law with the actual quantities replaced by their estimates is used at this stage, yielding

$$u = 1/b_2(\ddot{y}_d - \lambda \dot{e} - k_c s - \mathbf{c} \hat{\mathbf{A}}^2 \mathbf{x} - \boldsymbol{h}_2 \hat{\mathbf{w}})$$
(15)

From (15) it follows that the estimates of the parameters enter nonlinearly in the feedback control law. Nevertheless, by selecting the output in terms of displacements, a linear function of stiffness and damping estimates can still be assured provided that the masses are known. In this case, the unknown parameters can be introduced through the  $(1 \times n)$  vector  $\tilde{\boldsymbol{\theta}}^{T} =$  $(\tilde{a}_{1}, \tilde{a}_{2}, \tilde{a}_{3}, \ldots, \tilde{a}_{n})$ , where  $\tilde{a}_{i} = \hat{a}_{i} - a_{i}$  are the parameters errors. Substituting (15) into (5) gives the following equation governing the dynamics of the combined output error:

$$\dot{s}(t) + k_c s = \hat{\boldsymbol{\theta}}^T \mathbf{x} + \boldsymbol{h}_2 \tilde{\mathbf{w}}$$
(16)

where  $\tilde{\mathbf{w}}$  = vector of colored noise modeling errors between the estimate of the external excitations and its actual value and possible errors in the measurements of the output variables or even in the dynamical model itself.

The convergence of s(t) to zero, in this case, is not unconditional and depends on the values of  $\tilde{a}_i$ . A Lyapunov function argument guarantees that a sufficient condition for the asymptotic decay of s(t) can be ensured by imposing the following adaptation law to the uncertain parameters

$$\tilde{\boldsymbol{\Theta}} = -s\boldsymbol{\gamma}^{T}\mathbf{I}\mathbf{x} \tag{17}$$

Due to the relevance of the initial combined tracking error *s*, a variable gain vector  $\gamma$  has been introduced in the procedure. In particular, an exponential term with saturation has been adopted for each gain  $\gamma_i$  such as

$$\gamma_i(t) = \begin{cases} \gamma_{0i} e^{\sigma_i t} & \text{if } \gamma_i \le \bar{\gamma}_i \\ \bar{\gamma}_i \end{cases}$$
(18)

where  $\gamma_{0i}$ ,  $\bar{\gamma}_i$ , and  $\sigma_i$  = positive constants. The introduced variable gain vector enables the initial value of *s*, with respect to its smaller value after a certain time *t*, to be weighted differently; such gain evolution is introduced to avoid large initial chattering of the estimated parameters.

#### Lyapunov Asymptotic Stability

Asymptotic convergence of the motion of MIMO systems can be guaranteed in the presence of the above control. In particular, it will be shown that the fixed point of the controlled dynamical system, represented by the origin of the hyperplane ( $\mathbf{e}$ ,  $\dot{\mathbf{e}}$ ), is asymptotically stable. Thus, we start by choosing a Lyapunov function of the form

$$V = \mathbf{s}^T \mathbf{P} \mathbf{s} + \hat{\mathbf{\Theta}}^T \hat{\mathbf{\Theta}}$$
(19)

where  $\mathbf{P}$  = positive definite matrix such that *V* is globally positive definite  $\forall \mathbf{s} \in \Re^m$ , and  $\forall \tilde{\mathbf{\theta}} \in \Re^n$ . The first-order equations [(16) and (17)] governing  $\mathbf{s}$  and  $\tilde{\mathbf{\theta}}$  can be rewritten

$$\dot{\mathbf{s}} = -\mathbf{K}_c \mathbf{s} + \mathbf{H}_c \tilde{\mathbf{\theta}}^T \mathbf{x} + \mathbf{H}_c \tilde{\mathbf{w}}$$
(20)

$$\tilde{\boldsymbol{\Theta}} = -\mathbf{s}\boldsymbol{\Gamma}\mathbf{x} \tag{21}$$

where  $\mathbf{K}_c$  = diagonal matrix composed by the  $k_c$  constant of (16); and  $\mathbf{H}_c$  = matrix that indicates the parameters combination. By choosing a symmetric  $\mathbf{K}_c$  that satisfies the following equation:

$$\mathbf{K}_{c}^{T}\mathbf{P} + \mathbf{P}\mathbf{K}_{c} = \mathbf{Q}$$
(22)

with  $\mathbf{Q}$  being positive definite, the derivative of (19) can be rewritten

$$\dot{V} = -\mathbf{s}^{T}\mathbf{Q}\mathbf{s} + 2\mathbf{s}(\mathbf{P}\mathbf{H}_{c} - \mathbf{\Gamma})\tilde{\mathbf{\theta}}^{T}\mathbf{x}$$
(23)

A monotonically decreasing Lyapunov function V guarantees the global stability of the controlled dynamical system. The scalar quantity  $\dot{V}$  can be forced to be negative definite by a proper choice of  $\Gamma$ . Indeed, once **Q** and **K**<sub>c</sub> are selected, a matrix  $\Gamma$  can be found by guaranteeing that the second term of (23) is smaller than zero. Thus, a relation between tracking and adaptation speed has been found.

In the absence of modeling errors, the previous adaptive laws assure parameters convergence of the complete vector  $\hat{\mathbf{\theta}}$ to the true value  $\mathbf{\theta}$  provided that the vector  $\mathbf{x}$  is persistently exciting (Ioannou and Datta 1991).

### **APPLICATIONS**

The numerical investigations presented in this section refer to linear shear-type structural models. At first, the output selection criterion relying on the linear transformations introduced in the third section is discussed; then, based on the selected output, the adaptive control scheme for a 3-DOF system is implemented for both vibration suppression and damage detection.

#### Stability of Uncontrolled Internal Dynamics in Shear-Type Structures

The set of output variables representing the reference trajectories to be tracked is selected among those sets that assure the stability of the system internal dynamics. As discussed in the third section, such dynamics is made evident in control canonical form representations. Given an SISO transformed system, it would be sufficient to analyze the spectrum of the

TABLE 1. 2-DOF Model Parameters

DOF (1)	<i>m</i> (kN s²/m) (2)	c (kN s/m) (3)	<i>k</i> (kN/m) (4)
1	1.0	2.42	800.0
2	2.0	3.52	800.0



$$y = (\alpha + 1)x_1 + \alpha x_2 \tag{24}$$

where  $\alpha$  = displacements ( $x_1$ ,  $x_2$ ) combinations parameter. The analytical stability regions are shown in  $\mu$ ,  $\alpha$ -plane in Fig. 1 for different positions of the actuator;  $\mu$  represents the ratio between the masses of the two floors. Thus, given a mass ratio value  $\mu$ , the values of  $\alpha$  corresponding to points in the dashed regions indicate output choices assuring overall stability. Combinations of the state variables involving both displacements ( $x_1$ ,  $x_2$ ) and velocities ( $x_3$ ,  $x_4$ ) can also be taken into account through the observation expression

$$y = (\alpha + 1)x_1 + \alpha x_2 + (\beta + 1)x_3 + \beta x_4$$
(25)

where  $\beta$  = velocities combinations parameter. For these output choices the relative degree of the system reduces to 1, and three eigenvalues belong to the spectrum of the submatrices governing the internal dynamics. In Fig. 2, the numerically evaluated region of the stability of the internal dynamics is reported in the  $\alpha$ ,  $\beta$ -plane for the actuator position A (Fig. 1). As expected, the choices of the output corresponding to collocated cases, represented by a black point in the figures, are always stable.

Analogous analyses are performed for the stability of the internal dynamics of the 3-DOF model sketched in Fig. 3(a) (Soong 1990). In this case, the actuator acts only on the first floor, and an observation expression involving displacements combinations of the three floors is considered

$$y = (\delta + 1)x_1 + \delta x_2 + \alpha x_3$$
 (26)

In Fig. 4, the stability regions for the four eigenvalues are shown in the  $\alpha$ ,  $\delta$ -plane, where  $\alpha$ ,  $\delta$  in this case define the displacements combinations according to (26). The applica-



FIG. 1. Stable Regions (Hatched Areas) for 2-DOF Models in  $\mu$ ,  $\alpha$ -Plane; Two Cases of Control Force Location: A—Internal Force; B—External Force



FIG. 2. Numerical Stability Regions for Actuator Position A in  $\alpha$ ,  $\beta$ -Plane; • = Collocated Case

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FIG. 3. (a) MDOF Actively Controlled Structural System; (b) Uncontrolled Displacements under White Noise Excitation [Actual Model (Dashed Line); Reference Model (Solid Line)]



FIG. 4. Numerical Stability Regions for 3-DOF Model Function in  $\alpha$ ,  $\delta$ -Plane; • = Collocated Case

DOF (1)	<i>m</i> (kN s²/m) (2)	с (kN s/m) (3)	ξ (4)	<i>k</i> (kN/m) (5)		
(a) Actual Model						
1 2 3	0.9823 0.9823 0.9823	0.4677 0.1837 0.3879	0.001 0.005 0.005	1,965.0 1,946.0 1,184.1		
(b) Reference Model						
1 2 3	0.9823 0.9823 0.9823	25.00 0.500 0.500	0.03 0.05 0.05	1,965.0 1,946.0 1,184.1		

TABLE 2. 3-DOF Model Parameters

tions of the adaptive control procedure that will be described in the following sections refer to the latter 3-DOF system whose mechanical parameters are reported in Table 2.

# Vibration Suppression and On-Line Identification through Model Reference Control

The results reported in this section aim at validating the procedure for vibration suppression and on-line identification purposes for a collocated scheme [ $\alpha = 0$  and  $\delta = 0$ , in (26)] and a noncollocated scheme [ $\alpha = 1.0$  and  $\delta = 2.0$ , in (26)]. Aiming at reducing structural oscillations, a reference model characterized by high modal dampings has been selected (Table 2). Thus, the control action permits output tracking by

increasing the actual structural damping values by one order of magnitude. The significant differences in the level of oscillations between actual and reference models under white noise excitation are shown for a selected time interval in Fig. 3(b).

The achievements of the integrated procedure in terms of vibration suppression are expressed through the error vector  $\boldsymbol{\varepsilon}$  $= \mathbf{q} - \mathbf{q}_d$  between actual and reference displacements [Figs. 5(a and b)]. In particular, the running average of the maximum absolute values of the displacements errors are reported for both collocated and noncollocated cases. In the collocated case [Fig. 5(d)], the reference output is represented by the first-floor displacement; the error between reference and actual output converges to zero, and the errors between the other displacements oscillate around a constant small mean value. The actual and reference floors' displacements in a time interval are shown in Figs. 5(c-e) where a good matching is evident. On one hand, the strategy is successful in terms of displacement reduction through accurate output tracking; on the other hand, the selected output cannot allow for a complete identification of the structural parameters because the third-floor state variables do not appear in the control law [(14)]. As a matter of fact, the procedure enables only first- and second-floor stiffnesses and dampings to be identified accurately, because they are directly involved in the action of the collocated actuatorsensor pair. The performance of the on-line identification is reported in Fig. 6 for a 25% initial error in all the parameters. Different simulations have shown that an enhancement in terms of initial chattering can be obtained using the variable



FIG. 5. Average Maximum Displacements Errors [First Floor (Solid Line); Second Floor (Dotted Line); Third Floor (Dashed Line)]: (a) Collocated; (b) Noncollocated. Floors' Displacement Time Histories [Actual Model (Dashed Line); Reference Model (Solid Line)]: (c-e) Collocated; (f-h) Noncollocated



FIG. 6. Collocated Scheme Showing On-Line Identification of: (a) Stiffness Parameter; (b) Damping Parameter

gain given by (18). In the noncollocated case, good tracking performance and associated displacement reduction are also obtained. In particular, the floors' displacement errors decrease [Fig. 5(b)] as the estimated parameters converge to the actual ones (Fig. 7). The shown time interval indicates a perfect matching for the first two floors [Figs. 5(g and h)], whereas the third floor actual displacement differs from the reference one [Fig. 5(f)]. The difference mainly consists of a phase shift due to the poor estimate of the third floor damping parameter

 $c_3$  [Fig. 7(b)]; notwithstanding this, the desired amplitude reduction is obtained. The chosen displacement combination [ $\alpha = 1$ ;  $\delta = 2$  in (26)] involve all of the displacement variables, therefore the whole set of mechanical parameters is identifiable. The identification performance for the noncollocated case is shown in Fig. 7 for a 25% initial error in all of the parameters. Accurate estimate of all of the stiffnesses [Fig. 7(a)] and of the first and second dampings [Fig. 7(b)] are achieved. In this case a careful selection of the  $\lambda$ ,  $k_c$ , and  $\gamma_i(t)$  gains min-



FIG. 7. Noncollocated Scheme Showing On-Line Identification of: (a) Stiffness Parameter; (b) Damping Parameter

imizes the initial and final chattering in the evolution of the parameters.

#### Damage Detection and Simultaneous Corrective Control Action

In this section the ability of the procedure to detect damage occurring in the structural system is exploited. It is generally recognized that damage in structures appears as degradation of system characteristics, such as stiffness and/or damping (Lin et al. 1990). In particular, structures are apt to suffer damage caused by different events such as strong environmental loads, sudden impacts, and degradation due to longtime exposure. By modeling damage as an abrupt reduction in the stiffness and damping coefficients, the procedure can be effective for identifying these structural changes. Based on a good knowledge of the actual system mechanical parameters, a reference model close to the actual one can be designed. Thus, the initial small control action compensates only the small differences between actual and reference model. When damage occurs, the procedure detects the amplitude of the sudden reduction of stiffness and damping and simultaneously compensates such degradation through the control. Indeed the actual system is forced to behave as the reference model that represents its initial undamaged conditions.

A numerical experiment has been carried out to demonstrate the performance of the procedure. The reference model has been selected with a 2% lower estimate of the mechanical parameters. At the time instant t = 100 s in the first and second floors, instantaneous 25 and 15% reductions occur in damping and stiffness parameters, respectively. Fig. 8 shows the identification performance of the procedure. In Fig. 8(a) the exact detection of a change in the stiffness parameters is obtained after a small transient; the initial overshoot can be regulated through the gains  $\lambda$ ,  $k_c$ , and  $\gamma_i$ . Despite a larger overshoot, the damping parameters are also accurately identified [Fig. 8(b)]. In Fig. 9 the control efforts needed to detect and compensate damage are reported together with the displacement of the first floor [Fig. 9(a)]. Fig. 9(b) shows that the control action is scarce before damage occurs, and then a sudden large control effort for a small transient is required. Afterward, the control compensates the loss of stiffness and damping leading the structure to behave as undamaged. The control-displacement cycles during the described three phases are depicted in Figs. 9(c-e). Fig. 9(c) shows that during the first time interval the control acts to reduce the stiffness (negative slope in the cycle)



FIG. 8. On-Line Identification in Presence of Abrupt Variation of: (a) Stiffness; (b) Damping



FIG. 9. Control Performance in Presence of Abrupt Changes in Mechanical Parameters: (a) First-Floor Displacement Time-History; (b) Control Force Time-History; (c–e) Control-Displacement Cycles during Time-Intervals  $\Delta t_1$ ,  $\Delta t_2$ , and  $\Delta t_3$ , Respectively

to drive the actual system to behave as the reference one. Large control efforts are required during the abrupt parameters variations, and a cycle mainly characterized by a viscous behavior takes place [Fig. 9(d)]. A stiffness increment is eventually furnished in the third phase to compensate for the reduction in the actual system, and the intensity of control efforts depends on the level of damage [Fig. 9(e)].

#### CONCLUSIONS

An integrated procedure is used in this paper enabling reference model tracking and parameters identification. A general framework for the application of the procedure to collocated and noncollocated control schemes has been provided. The algorithm relies on a direct MRAC procedure where the fullstate feedback is employed to address a dual goal—vibration suppression and damage detection. This is accomplished in the former by tracking a reference output of an arbitrary model with desired damping characteristics and in the latter by detecting on-line mechanical parameters variations. During the control process, after a designed transient interval, the relevant mechanical parameters of the structure are monitored through the on-line estimation algorithm. Applications to shear-type models have been considered showing that an opportune selection of a reduced measure of the complete state variables guarantees exact output reference tracking. Moreover, it has been shown that the complete actual state vector turns out to be rather close to the reference one.

Numerous issues still need to be investigated. Acceleration feedback, nonlinear parameters dependence in the control law, time delay, and noise contamination are aspects that need to be tackled to make the procedure implementable in full-scale systems. In this respect, small-scale experiments would represent an initial step providing meaningful insights into these problems.

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#### APPENDIX. REFERENCES

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