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Thermohydrodynamics of the Ocean

Analysis of the intensification of tsunami waves near the South Coast of Crimea*

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Abstract — We perform the numerical analysis of the intensification of tsunami waves in the course of their propagation from the open part of the Black Sea to the shelf zone. For this purpose, we use a one-dimensional model of nonlinear long waves taking into account the effect of bottom friction. We study four profiles of the bottom corresponding to the south coast of the Crimean Peninsula and establish the predominant role of the bottom pattern and insignificant contribution of nonlinearity to the transformation of waves in the process of their propagation in the direction of the coast. Down to depths of 50 m, all changes in the height of waves are described by the Green law. For the evaluation of vertical run-up of waves, it is important to take into account nonlinear effects. The highest vertical run-ups of waves are observed in the parts of the shelf zone located near Yalta and Alushta.

INTRODUCTION

Tsunami waves become especially dangerous in the coastal zone of the World Ocean because their height significantly increases in the shallow-water regions. This transformation of waves propagating to the coast is explained, first of all, by a decrease in the depth of the basin. For this reason, the quantitative characteristics of the intensification of waves depend on the topography of the bottom in the zone of the continental slope and in the shelf zone and reflect local distinctive features of the basin.

One-dimensional numerical models taking into account the changes in the depth of the basin corresponding to sections of the bottom topography in the direction normal to isobaths are extensively used in theoretical investigations of transformations of long surface waves propagating to the coast. Unlike two- and three-dimensional models, they are capable of giving high spatial resolutions. The application of one-dimensional long-wave models made it possible to study general physical regularities of the evolution of waves in the coastal zone taking into account nonlinear effects, dissipation, and dispersion and perform computer experiments aimed at the evaluation of the level of danger caused by tsunami for the west coast of the Pacific Ocean [1–5].

In the present work, a nonlinear one-dimensional model of long waves with bottom friction is used for the numerical analysis of the intensification of long tsunami-type waves propagating from the abyssal region to the shelf zone of the Black Sea. We analyze four profiles of the bottom in the direction normal to the coast corresponding to four points of the south coast of the Crimean Peninsula. This enabled us to estimate the degrees of intensification of waves for different parts of the coast loc-

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ated to the north of the Crimean zone of seismic activity in the Black-Sea Depression. Note that several tsunami waves were observed and instrumentally recorded just in this region [6]. The results of numerical experiments aimed at the analysis of propagation and intensification of tsunami waves in the shelf zone near Yalta and taking into account the effects of nonlinearity and bottom friction are presented in refs 7 and 8.

HYDRODYNAMIC MODEL

We consider a basin of variable depth $H = H(x)$ semiinfinite along the horizontal x-axis. In the vertical plane Oxz, this basin occupies the region $-\infty < x < L$, $-H(x) < z < 0$, where z is the vertical coordinate. For $x = L$, the basin is bounded by a vertical wall and $H(L) > 0$. For $x < 0$, the depth of the basin is constant and we have $H_0 = H(0)$. An isolated surface wave $\zeta = \zeta_0(x + \lambda/2 - C_0 t)$ of length λ, where $\zeta(x, t)$ are vertical displacements of the free surface of the liquid from its equilibrium position $z = 0$, $t \geq 0$ is time, and $\zeta_0(\xi) = 0$ for $|\xi| \geq 0.5\lambda$, propagates with a positive velocity C_0 from the abyssal part of the basin $x < 0$ to the region $0 \leq x \leq L$.

Within the framework of the nonlinear theory of long waves without dispersion, we analyze the intensification of an isolated surface wave $\zeta = \zeta_0$ with horizontal length λ propagating from the abyssal part of the basin to the shelf zone and its run-up on the vertical wall for $x = L$. We consider the distributions of depths typical of the south part of the Crimean shelf zone and assume that the bottom friction is described by a quadratic function of the velocity.

Under the assumptions made above, the long-wave motions of water in the region $0 \leq x \leq L$ are described by the following set of two equations [5, 9]:

$$u_t + u\,u_{xx} = -g\zeta_x - rD^{-1}u|u| \quad \text{and} \quad \zeta_t + (Du)_x = 0, \tag{1}$$

where $u(x, t)$ is the horizontal velocity averaged over the depth, $D = H + \zeta$, $r = 0.0026$ is the coefficient of bottom friction, and g is the acceleration of gravity.

On the open boundary $x = 0$, we must also set special conditions. For the interval $0 \leq t \leq T$ $(T = \lambda/C_0)$ when the incident wave passes through the point $x = 0$ into the analyzed region, these conditions are taken in the form

$$\zeta = \zeta_0\left(\frac{\lambda}{2} - C_0 t\right) \quad \text{and} \quad u = \frac{C_0}{H_0}\zeta_0\left(\frac{\lambda}{2} - C_0 t\right) \quad (x = 0). \tag{2}$$

At the same time, for $t > T$, they are replaced by

$$u_t - C_0 u_x = 0 \quad \text{and} \quad \zeta = -\frac{H_0}{C_0}u \quad (x = 0). \tag{3}$$

On the right boundary of the region $x = L$, we use the condition of impermeability

$$u(L, t) = 0, \quad t > 0, \tag{4}$$

guaranteeing the total reflection of waves.

Problem (1)–(4) must be supplemented with the initial conditions

$$u(x, 0) = \zeta(x, 0) = 0, \quad 0 \le x \le L, \tag{5}$$

according to which the liquid in the region $0 \le x \le L$ is not perturbed for $t \le 0$.

Conditions (2) mean that, in the abyssal part of the basin $x \le 0$, where the depth is constant and equal to H_0, the effects of nonlinearity and dissipation are weak. Hence, for free long waves propagating to the right, we have $u = (C_0/H_0)\zeta$ and $C_0 = \sqrt{gH_0}$ [5, 9]. The first condition in (3) ensures that the linear waves can freely leave the region $0 \le x \le L$ for $t > T$ and the second condition establishes the relationship between ζ and u on the open boundary $x = 0$ of the investigated region. In the cases where the waves cannot be regarded as linear in the vicinity of the point $x = 0$, conditions (2) and (3) are somewhat modified [3, 5].

NUMERICAL ALGORITHM AND INITIAL DATA

We solve the initial-boundary-value problem (1)–(5) by the method of finite differences. For this purpose, in the segment $0 \le x \le L$, we consider a uniform mesh $x_j = j\delta$ $(j = 0, 1, \ldots, N;\ \delta N = L)$, where δ is the step of the mesh. The point $x_0 = 0$ corresponds to the sea (open) boundary of the investigated region and the last point $x_N = L$ corresponds to the vertical side wall. The values of u and H are specified at $N + 1$ nodes of the mesh x_j and the values of ζ are given at N middle points of cells of the mesh $x_{j+1/2} = x_j + 0.5\delta$. We use the explicit-implicit finite difference scheme described and applied to the analysis of storm surges and tsunami waves in [3, 7–9]. According to this procedure, we approximate equations (1) to within terms of the same order of smallness as $O(\delta^2, \tau)$, where τ is the time step. For waves in the basin whose geometry is described above, the difference scheme is stable provided that $C_0\tau/\delta < 1$ [3, 9].

We analyze the transformation of waves propagating from the abyssal part of the basin to the coast for four tabulated laws of changes in the depth of the basin along rectilinear sections I–IV made in the Black-Sea Depression near the South Coast of Crimea and depicted in Fig. 1. The corresponding profiles of the bottom are displayed in Fig. 2. The vertical side wall at $x = L$ corresponds to an isobath of 10 m. The indicated sections were made near Cape Feolent (I), Cape Sarych (II), Yalta (III), and Alushta (IV). The bottom topography is characterized by the continental slope with a width of 15–40 km, where the depth of the sea decreases to 100 m, and the shelf zone 10–20 km in width, where the depth changes from 100 to 10 m.

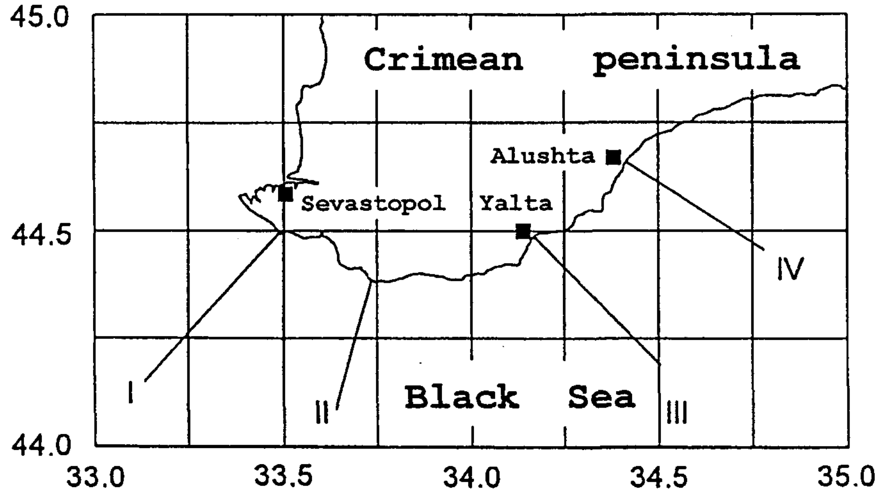

Figure 1. Investigated geographic region. In numerical experiments, we use the distributions of depths corresponding to sections I–IV in the Black-Sea Depression.

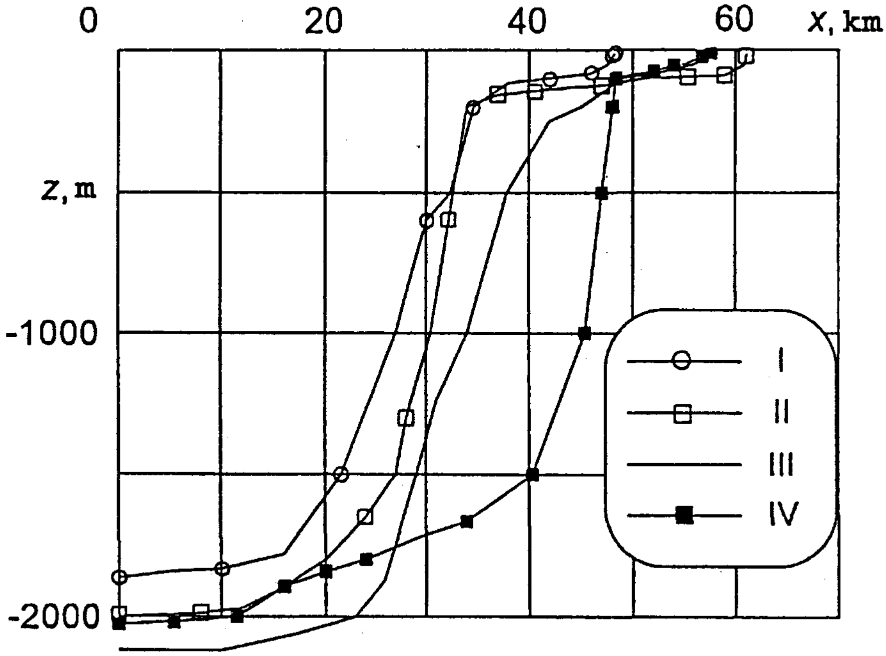

Figure 2. Profiles of depth of the sea along sections I–IV corresponding to the following points on the coast of the Black Sea: Cape Feolent (I), Cape Sarych (II), Yalta (III), and Alushta (IV).

We simulate the shape of an isolated symmetric wave in the abyssal part of the basin by the following smooth function:

$$\zeta_0(\xi) = a_0 \begin{cases} \cos^2 \dfrac{\pi\xi}{\lambda}, & |\xi| \le \dfrac{\lambda}{2}, \\ 0, & |\xi| > \dfrac{\lambda}{2}, \end{cases}.$$

In numerical experiments, the length λ and height $a_0 > 0$ of the incident wave in the open part of the sea varied within the ranges $15\,\text{km} < \lambda < 50\,\text{km}$ and $0 < a_0 < 1\,\text{m}$.

The steps of integration δ and τ were chosen according to the results of numerical experiments to guarantee the stability of the difference procedure and take into account significant "compression" of waves in the shelf zone. The major part of numerical experiments was carried out for $N \approx 800$ ($\delta \le 75\,\text{m}$) and $\tau = 0.25\,\text{s}$. In this case, it is possible to guarantee the preservation of mass and total energy (for $r = 0$) of water in the region $0 \le x \le L$ when the wave completely enters this region by replacing equalities (3) for $t > T$ with the condition of total reflection of waves $u(0, t) = 0$.

NUMERICAL ANALYSIS OF THE HEIGHT OF TSUNAMI WAVES

We study the behavior of the height of tsunami waves in the course of their propagation to the coast for the indicated distributions of depth of the basin (Figs 1 and 2).

For all profiles of the bottom, the character of transformation of the shape of waves in the course of their propagation to the coast is qualitatively similar to that described in [7, 8]. Thus, over the continental slope and shelf, we observe the compression of waves, their height increases and velocity of propagation decreases. Moreover, in the shelf zone, the wave becomes asymmetric: its front side becomes steeper than the back side.

The behavior of the height $A = \max_x \zeta(x, t)$ of a wave propagating to the coast as a function of the local depth of the basin H in the vicinity of its peak is described by the curves plotted in Fig. 3. Despite the difference between the profiles of the bottom I–IV, the difference between the dependences of A on H is insignificant if we exclude the regions of the shelf located in the immediate vicinity of the side boundary of the basin. On passing over the continental slope, the height of the wave is almost doubled. In the course of propagation of the wave to the coast in the shelf zone, its height also increases and, at a depth of $30\,\text{m}$, becomes as high as $2.5\,a_0$.

Within the framework of the hydrodynamic model (1)–(5), the transformation of waves is determined by the following three factors: the bottom topography of the basin, the nonlinearity of the wave process, and energy losses caused by bottom friction. According to the estimates made in [7, 8] for the bottom profile III, the influence of nonlinearity and dissipation on the shape of waves propagating to the coast is weak.

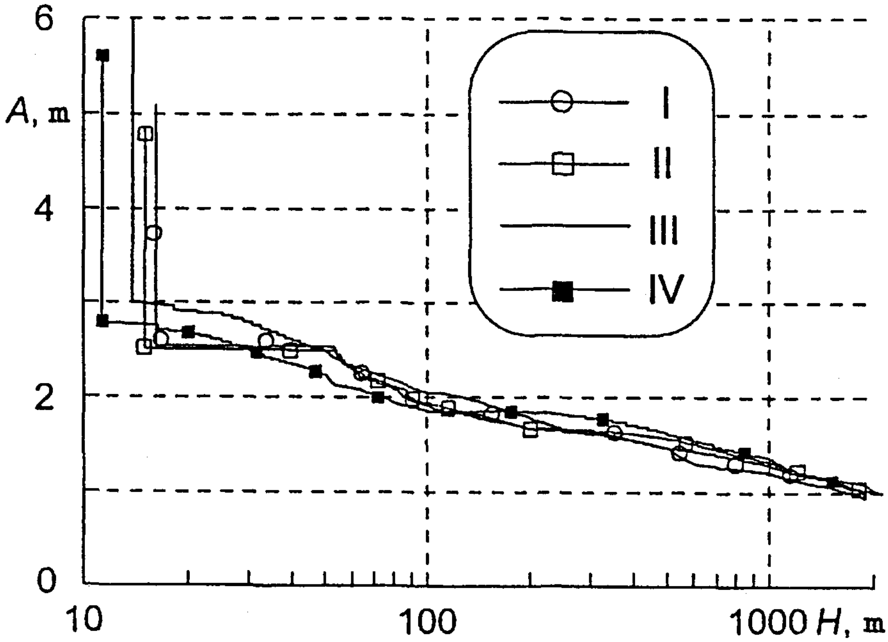

Figure 3. Variation of the height of a long wave ($\lambda = 30$ km, $a_0 = 1$ m) in the course of its propagation to the coast along sections I–IV in the Black-Sea Depression.

Table 1.
Maximal elevations of the sea level caused by the wave ($a_0 = 1$ m, $\lambda = 15$ km) passing through points with depths of 500, 100, 50, and 10 m in sections I–IV

Region	Maximum elevation of the sea level (m)											
	$H = 500$ m			$H = 100$ m			$H = 50$ m			$H = 10$ m		
	ND	*L*0	*G*	*ND*	*L*0	*G*	*ND*	*L*0	*G*	*ND*	*L*0	*G*
I	1.45	1.47	1.39	2.01	2.03	2.08	2.57	2.71	2.47	5.93	5.99	3.36
II	1.52	1.52	1.41	2.03	2.02	2.11	3.21	2.98	2.51	6.87	5.68	3.47
III	1.46	1.46	1.44	2.09	2.11	2.15	2.50	2.57	2.55	9.47	6.65	3.59
IV	1.57	1.57	1.42	1.95	1.97	2.12	2.34	2.37	2.53	9.11	6.51	3.69

***Note*:** Numerical calculations were carried out for the following three models: nonlinear-dissipative model (1)–(5) (*ND*), linear model without bottom friction (*L*0), and Green law (6) (*G*).

At the same time, the contribution of nonlinearity to the vertical run-up of tsunami waves is more significant.

The results presented in Table 1 demonstrate that these conclusions remain valid in the case of propagation of waves along sections I, II, and IV. As could be expected, the contribution of nonlinearity and bottom friction to the height of long waves increases as the waves approach the coast and is maximum in the shallow-water region. For an isobath of 100 m, the indicated contribution is about 1%. At a depth of 50 m, it does not exceed 10%. At the same time, the joint contribution of nonlinearity and dissipation to the vertical run-up of waves (corresponding to the depth $H = 10\,\mathrm{m}$ in Table 1) can be as high as 42% (section III).

The Green law gives a quite simple formula for the height of waves propagating to the coast [1, 2, 5]. Indeed, according to this law, the height of waves at a point x satisfies the relation

$$A(x) = a_0\left[\frac{H_0}{H(x)}\right]^{1/4}. \tag{6}$$

This formula is asymptotic. It is approximately true for long linear waves in a basin whose depth slowly varies on scales of the same order as the wavelength. As follows from Table 1, relation (6) holds with sufficiently high accuracy for all considered profiles of the bottom at least down to depths of about 50 m.

From the viewpoint of estimation of danger caused by tsunami for the south coast of the Crimean Peninsula, it is of interest to determine the vertical run-up of waves on the coast. Within the framework of the applied hydrodynamic model, this means that we determine the maximal elevation of the sea level $h = \max_x \zeta(L, t)$ near the vertical side wall corresponding to an isobath of 10 m. The vertical run-up depends on the height a_0 and length λ of the incident wave.

The results of calculations of h are presented in Table 1 and Fig. 4. For the initial wavelengths $15\,\mathrm{km} \le \lambda \le 50\,\mathrm{km}$, the vertical run-up of waves in the considered directions of propagation of tsunami is 4.2–9.5 times greater than the height of isolated waves coming to the continental slope. Clearly, it decreases as the wavelength λ increases and as the height of the wave a_0 in the abyssal part of the basin decreases. However, it should be emphasized that, for actual underwater earthquakes, the quantities λ and a_0 are not independent. They are specified by the magnitude of the earthquake M [2]. The problem of dependence of heights and vertical run-ups of tsunami waves on M requires special investigations.

For long linear waves in a basin with constant depth, the run-up h of waves on the vertical wall is equal to $2a_0$. Therefore, the fact that h decreases as λ increases established in numerical experiments (Fig. 4) is explained by the combined influence of nonlinearity and the bottom topography on the wave process.

The above-mentioned results of numerical experiments enable us to compare the intensities of tsunami in different parts of the South Coast of Crimea. As follows from Figs 3 and 4, for the same parameters of waves in the abyssal part of the Black Sea, the maximum intensification of waves is observed near Yalta and Alushta. This is especially well pronounced if we consider the vertical run-ups of waves.

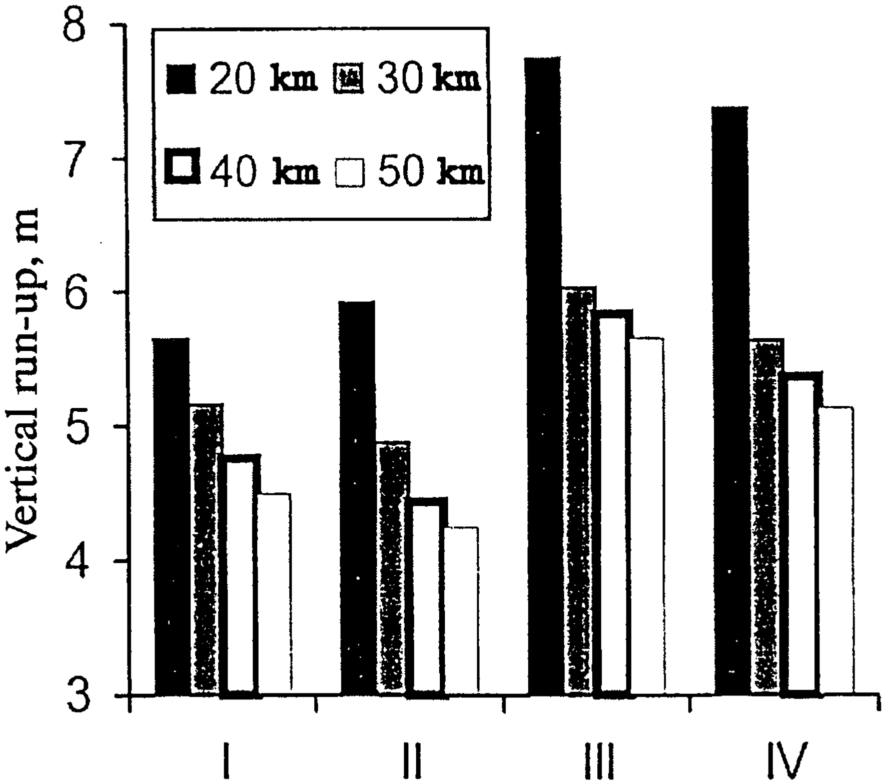

Figure 4. Run-ups of tsunami waves with initial wavelengths λ of 20, 30, 40, and 50km and height $a_0 = 1$ m on the vertical wall for the bottom profiles I–IV.

CONCLUSIONS

We analyzed the process of intensification of isolated tsunami waves propagating from the open part of the Black Sea into the shelf zone. Our investigation was based on the numerical solution of one-dimensional equations for long waves taking into account the effects of nonlinearity and bottom friction (regarded as a quadratic function of the velocity) by the finite-difference method. The numerical analyses were performed for profiles of the bottom (in the direction normal to isobaths) corresponding to four points of the south coast of the Crimean Peninsula. We used the condition of total reflection of waves from an isobath of 10 m (vertical side wall).

It is shown that, in this region of the sea, the determining role in the transformation of isolated waves propagating to the coast is played by the bottom topography. In the course of propagation, the nonlinear effects are weak and the growth of wave amplitude in the region of continental slope and sea side of the shelf zone is fairly well described by the Green law (6).

The contribution of nonlinearity to the run-up of waves on the vertical side wall located at an isobath of 10 m is more pronounced. For the wavelengths of tsunami 15 km $\le \lambda \le$ 50 km in the open part of the sea, the vertical run-up is 4.2–9.5 times greater than the initial height of the wave and decreases as wavelength λ increases and as the height of the incident wave decreases.

We compared the intensities of tsunami for various regions of the South Coast of Crimea and showed that the intensification of waves propagating to the coast was especially pronounced near Yalta and Alushta.

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