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THE BIAXIAL TENSION OF AN ELASTOPLASTIC SPACE WITH A PRISMATIC INCLUSION

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The problem on the elastoplastic state of a thick isotropic plate (the case of plane deformation) is solved. A larger prismatic inclusion is made a close fit in a polygonal hole in the plate. The plate is stretched at infinity by constant mutually perpendicular forces. The problem is solved by the small-parameter method and by the theory of ideal plasticity. The axisymmetric state of the plane with a circular hole stiffened by a round ring with a constant force applied to its inner contour is considered as a zero approximation. Some specific shapes of the hole and reinforcing elastic rigid ring are considered.

The problem on the elastoplastic state of an infinite plane with a circular hole stretched at infinity by mutually perpendicular forces, and the problem on the biaxial tension of a thick plate with an elliptic hole were solved in [3]. The elastoplastic state of an eccentric set on an elastic shaft was determined in [2]. The stress–strain state of a thick plate with an elliptic rod–inclusion making an interference fit in an elliptic hole was determined in [4].

The objective of the present work is to investigate the elastoplastic state of a thick plate with a polygonal hole into which a larger prismatic inclusion is tightly fitted. The plate is stretched at infinity by mutually perpendicular forces of intensities P_1 and P_2.

The problem is solved by the small-parameter method and by the theory of ideal plasticity in a cylindrical coordinate system r, θ, z. The axis z is directed along the cylinder axis, and the origin is at the center of the cylinder. We have the case of plane deformation shown in Fig. 1, where *1* is the elastic zone of the plate, *2* is the plastic zone of the plate, and *3* is the elastic ring. In the plane perpendicular to the axis Oz, the equation of the polygon bounding the hole in the plate prior to deformation has the form

$$\rho_1 = \alpha (1 + \delta d_1 \cos m \theta - \dots) . \tag{1}$$

The equation of the polygon bounding the inclusion prior to deformation is

$$\rho_2 = \alpha_1 (1 + \delta d_1 \cos m \theta - \dots) . \tag{2}$$

The equation of the polygon bounding the inner hole of the ring is

$$\rho_3 = \beta (1 + \delta d_2 \cos m \theta - \dots) , \tag{3}$$

where $\rho = r/r_{s0}$, $\alpha = \alpha/r_{s0}$, $\alpha_1 = \alpha_1/r_{s0}$, and r_{s0} is the radius of the plastic zone in the axisymmetric case, m is the number of angles of the polygon bounding the hole and inclusion prior to deformation, n is the number of angles of the polygon bounding the inner hole of the inclusion, d_1 and d_2 are dimensionless constants, and δ is a parameter, small in comparison with unity and characterizing the degree to which the polygon differs from a circle and the perturbation of static boundary conditions, $\alpha_1 > \alpha$.

We will search for the solution in the form

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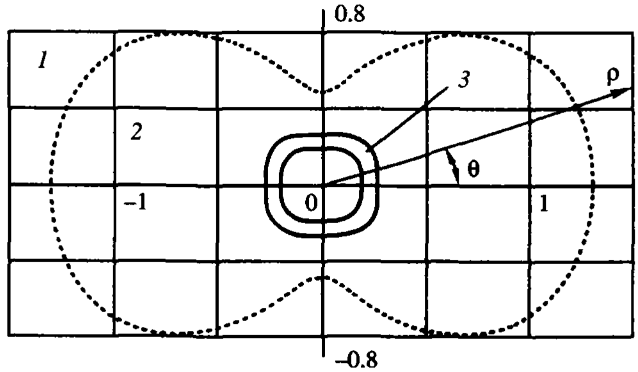

Fig. 1

$$\sigma_\rho=\sigma_\rho^0+\delta\sigma_\rho^1,\quad \sigma_\theta=\sigma_\theta^0+\delta\sigma_\theta^1,\quad \tau_{\rho\theta}=\tau_{\rho\theta}^0+\delta\tau_{\rho\theta}^1,\quad \sigma_z=\frac{1}{2}(\sigma_\rho+\sigma_\theta),$$

$$u=u^0+\delta u^1,\quad \vartheta=\vartheta^0+\delta\vartheta^1,\quad \rho_k=R_0+\delta R_1,\quad \rho_s=1+\delta\zeta, \tag{4}$$

where σ_{ij} are the components of the stress tensor, u and ϑ are displacements along the axes r and θ, respectively, ρ_s is the boundary of the plastic zone, and ρ_k is the line of contact between the inclusion and plate. Quantities having dimension of stresses are referred to the kth shear yield strength of the material of the plate. The displacements are referred to r_{s0}.

We assume that the pressure appearing on the contacting surfaces of the plate and inclusion is reduced to the normal pressure Q acting on the contours of the hole and inclusion. Let us consider the case of an elastic inclusion.

As a zero approximation, we consider the axisymmetric state of the plane with a circular hole of radius α tightly filled with a ring of outer radius α_1 and inner β and loaded by a force of intensity P_0/k.

The resultant structure is uniformly stretched at infinity by the force $p=\dfrac{P_1+P_2}{2k}$. Following [3] and allowing for the incompressibility of the material, for the plane we have

$$\left.\begin{array}{l}\sigma_\rho^{0e}\\ \sigma_\theta^{0e}\end{array}\right\}=p\mp\frac{1}{\rho^2},\quad \tau_{\rho\theta}^{0e}=0,\quad u^{0e}=\frac{k}{2G\rho},\quad \vartheta^{0e}=0, \tag{5}$$

$$\sigma_\rho^{0e}=-q+2\ln\frac{\rho}{\alpha},\quad \sigma_\theta^{0p}=\sigma_\rho^{0p}+2,\quad \tau_{\rho\theta}^{0p}=0,\quad u^{0e}=u^{0e},\quad \vartheta^{0p}=0, \tag{6}$$

where $q=Q/k$, $P_1\neq P_2$, and G is the shear modulus of the material of the plate. The subscript "e" stands for the elastic zone, and "p" for the plastic zone. For the elastic ring, we have

$$\sigma_{\rho B}^0=\frac{1}{\beta^2-\alpha_1^2}\left(q\alpha_1^2-P_0\beta^2-(q-P_0)\frac{\alpha_1^2\beta^2}{\rho^2}\right),$$

$$\sigma_{\theta B}^0=\frac{1}{\beta^2-\alpha_1^2}\left(q\alpha_1^2-P_0\beta^2+(q-P_0)\frac{\alpha_1^2\beta^2}{\rho^2}\right),$$

$$u_B^0=\frac{k}{2G_1}\frac{(q-P_0)\alpha_1^2\beta^2}{(\beta^2-\alpha_1^2)\rho},\quad \tau_{\rho\theta B}^0=0,\quad \vartheta_B^0=0, \tag{7}$$

where G_1 is the shear modulus of the inclusion and P_0 is the force applied to the inner contour of the ring.

Since the quantity $\varepsilon = \alpha_1 - \alpha$ is small, we can assume that the contact line coincides with the boundary of the inclusion, i.e.,

$$R_0 = \alpha_1, \quad R_1 = \alpha_1 d_1 \cos m\theta. \tag{8}$$

From the conditions of compatibility of the deformations of the plane and ring along the contact line and from the condition of conjugation on the boundary of the plastic zone, we have

$$q = 1 - p + 2\ln\frac{r_{s0}}{\alpha},$$

$$\frac{r_{s0}^2(\beta^2 - \alpha_1^2)}{2\alpha\alpha_1\beta^2} = \frac{G(\alpha_1 - \alpha)(\beta^2 - \alpha_1^2)}{\alpha_1\beta^2 k} + \frac{G}{2G_1}(1 - p - 2\ln\alpha) - \frac{P_0 G}{2G_1}\ln r_{s0}. \tag{9}$$

From (9), we determine q and r_{s0}.

We proceed to the determination of the first approximation for unknown quantities. The boundary conditions at infinity have the form

$$\sigma_\rho^{1e\infty} = -d_3\cos 2\theta, \quad \tau_{\rho\theta}^{1e\infty} = d_3\sin 2\theta, \tag{10}$$

where $\delta d_3 = \dfrac{P_1 - P_2}{2k}$ and d_3 is a dimensionless constant.

On the boundary of the plastic zone, according to [3], we have

$$\sigma_\rho^{1e} = \sigma_\rho^{1p}, \quad \tau_{\rho\theta}^{1e} = \tau_{\rho\theta}^{1p}, \quad u^{1e} = u^{1p}, \quad \vartheta^{1e} = \vartheta^{1p},$$

$$\zeta = \frac{1}{4}(\sigma_\theta^{1e} - \sigma_\theta^{1p}) \quad \text{for} \quad \rho = 1. \tag{11}$$

If the inclusion makes an interference fit in the hole, and the friction along the contact line is absent, then along the contact line we have

$$\sigma_\rho^{1p} + \frac{d\sigma_\rho^{0p}}{d\rho}R_1 = \sigma_{\rho\beta}^1 + \frac{d\sigma_{\rho B}^0}{d\rho}R_1, \quad \tau_{\rho\theta B}^1 - (\sigma_{\theta B}^0 - \sigma_{\rho B}^0)s_1 = 0,$$

$$\tau_{\rho\theta B}^{1p} - (\sigma_\theta^{0p} - \sigma_\rho^{0p})s_1 = 0 \quad \text{for} \quad \rho = R_0, \tag{12}$$

$$\left.\left(u^{1p} + \frac{du^{0p}}{d\rho}\alpha d_1\cos m\theta\right)\right|_{\rho=\alpha} = \left.\left(u_B^1 + \frac{du_B^0}{d\rho}\alpha_1 d_1\cos m\theta\right)\right|_{\rho=\alpha_1}, \tag{13}$$

where $s_1 = R_1/R_0$.

If the inclusion is tightly soldered in the hole, we have

$$\sigma_\rho^{1p} + \frac{d\sigma_\rho^{0p}}{d\rho}R_1 = \sigma_{\rho\beta}^1 + \frac{d\sigma_{\rho B}^0}{d\rho}R_1, \quad \tau_{\rho\theta B}^{1p} - (\sigma_{\theta B}^{0p} - \sigma_{\rho B}^{0p})s_1 = \tau_{\rho\theta B}^1 - (\sigma_{\theta B}^0 - \sigma_{\rho B}^0)s_1,$$

$$\vartheta^{1p} + u^{0p}s_1 = \vartheta_B^1 + u_B^0 s_1 \quad \text{for} \quad \rho = R_0, \tag{12*}$$

$$\left(u^{1p} + \frac{d u^{0p}}{d\rho} \alpha\, d_1 \cos m\theta \right)\Bigg|_{\rho=\alpha} = \left(u_B^1 + \frac{d u_B^0}{d\rho} \alpha_1\, d_1 \cos m\theta \right)\Bigg|_{\rho=\alpha_1} . \tag{13*}$$

Thus, we assume that the jump of the displacement vector is identical in value for all points of the contour of the inclusion and directed along the normal to the contour at any of its point.

Following [1] and satisfying conditions (10), in the elastic zone of the plate we have

$$\begin{aligned}
\sigma_\rho^{1e} = &-(d_3 + 6 a_{21} \rho^{-4} + 4 a_{22} \rho^{-2}) \cos 2\theta - (m (m+1) a_{m1} \rho^{-m-2}) \\
&+ (m-1)(m+2) a_{m2} \rho^{-m} \cos m\theta \\
&- (n (n+1) a_{n1} \rho^{-n-2} + (n-1)(n+2) a_{n2} \rho^{-n}) \cos n\theta,
\end{aligned} \tag{14}$$

$$\begin{aligned}
\sigma_\theta^{1e} = &(d_3 + 6 a_{21} \rho^{-4}) \cos 2\theta + (m (m+1) a_{m1} \rho^{-m-2}) \\
&+ (m-2)(m-1) a_{m2} \rho^{-m} \cos m\theta \\
&+ (n (n+1) a_{n1} \rho^{-n-2} + (n-2)(n-1) a_{n2} p^{-n}) \cos n\theta,
\end{aligned}$$

$$\begin{aligned}
\tau_{\rho\theta}^{1e} = &(d_3 - 6 a_{21} \rho^{-4} - 2 a_{22} \rho^{-2}) \sin 2\theta - m (m+1) a_{m1} \rho^{-m-2} \\
&+ (m-1) a_{m2} \rho^{-m} \sin m\theta - n ((n+1) a_{n1} \rho^{-n-2} + (n-1) a_{n2} \rho^{-n}) \sin n\theta,
\end{aligned}$$

$$\begin{aligned}
u^{1e} = &\frac{k}{3G} \left(-\frac{3}{2} \rho\, d_3 + 3a_{21}\rho^{-3} + 3a_{22}\rho^{-1} \right) \cos 2\theta + \frac{mk}{2G} (a_{m1}\rho^{-m-1} \\
&+ a_{m2}\rho^{-m+1}) \cos m\theta + \frac{nk}{2G} (a_{n1}\rho^{-n-1} + a_{n2}\, \rho^{n+1}) \cos n\theta,
\end{aligned}$$

$$\begin{aligned}
\vartheta^{1e} = &\frac{k}{3G} \left(\frac{3}{2} \rho\, d_3 + 3a_{21}\rho^{-3} \right) + 3a_{21}\rho^{-1} \sin 2\theta + \frac{k}{2G} (m a_{m1}\rho^{-m-1} \\
&+ (m-2) a_{m2}\rho^{-m+1}) \sin m\theta + \frac{k}{2G} (n a_{n1}\, \rho^{-n-1} + (n-2) a_{2n}\, \rho^{-n+1}) \sin n\theta,
\end{aligned} \tag{15}$$

where a_{21}, a_{22}, a_{m1}, a_{m2}, a_{n1}, and a_{n2} are unknown constants.

Following [3] and satisfying condition (11), in the plastic zone of the plate we have

$$\begin{aligned}
\sigma_\rho^{1p} = \sigma_\theta^{1p} = &-\frac{2}{\rho} \left[d_3 \cos \left(\gamma_1 - \frac{\pi}{3} \right) - 2\sqrt{3}\, a_{21} \sin \left(\gamma_1 - \frac{\pi}{3} \right) + 2a_{22} \cos \gamma_1 \right] \cos 2\theta \\
&+ \frac{1}{\rho} [m (\sqrt{m^2-1} \sin \gamma_2 - (m+1) \cos \gamma_2) a_{m1} \\
&+ (\sqrt{m^2-1} (m-2) \sin \gamma_2 - (m-1)(m+2) \cos \gamma_2) a_{m2}] \cos m\theta \\
&+ \frac{1}{\rho} [n (\sqrt{n^2-1} \sin \gamma_3 - (n+1) \cos \gamma_3) a_{n1} + (\sqrt{n^2-1} (n-2) \sin \gamma_3 \\
&- (n-1)(n+2) \cos \gamma_3) a_{n2}] \cos n\theta,
\end{aligned}$$

$$\tau^{1p}_{\rho\theta}=-\frac{2}{\rho}\left[-d_3\cos\left(\gamma_1+\frac{\pi}{3}\right)+2\sqrt{3}\,a_{21}\sin\left(\gamma_1+\frac{\pi}{3}\right)+2a_{22}\cos\left(\gamma_1-\frac{\pi}{3}\right)\right]\cos 2\theta$$

$$-\frac{1}{\rho}m\,[\,a_{m1}\,(\,m+1\,)\cos\gamma_2+\sqrt{m^2-1}\,\sin\gamma_2$$

$$+(\,(\,m-1\,)\cos\gamma_2+\sqrt{m^2-1}\,\sin\gamma_2\,)\,a_{m2}\,]\sin m\,\theta$$

$$-\frac{1}{\rho}n\,[\,a_{n1}\,(\,n+1\,)\cos\gamma_2+\sqrt{n^2-1}\,\sin\gamma_3$$

$$+(\,(\,n-1\,)\cos\gamma_3+\sqrt{n^2-1}\,\sin\gamma_3\,)\,a_{n2}\,]\sin n\,\theta, \tag{16}$$

where

$$\gamma_1=\sqrt{3}\ln\rho,\quad \gamma_2=\sqrt{m^2-1}\ln\rho,\quad \gamma_3=\sqrt{n^2-1}\ln\rho\,.$$

Considering the case of plane deformation and assuming that the material is incompressible, we have the following system of linear equations for determination of displacements in the plastic zone:

$$\frac{\partial\vartheta^{1p}}{\partial\rho}-\frac{\vartheta^{1p}}{\rho}+\frac{1}{\rho}\frac{\partial u^{1p}}{\partial\theta}=-2\frac{\partial u^{0p}}{\partial\rho}\tau^{1p}_{\rho\theta},\quad \frac{\partial u^{1p}}{\partial\rho}-\frac{u^{1p}}{\rho}+\frac{1}{\rho}\frac{\partial u^{1p}}{\partial\theta}=0\,. \tag{17}$$

Solving this system and satisfying condition (11), we have

$$u^{1p}=-\frac{k}{G}[\,d_3M_1(\rho)+a_{21}N_1(\rho)-a_{22}N_3(\rho)\,]\cos 2\theta$$

$$-\frac{k}{4G}[\,a_{m1}N_{1m}(\rho)-a_{m2}N_{3m}(\rho)\,]\cos m\theta-\frac{k}{4G}[\,a_{n1}N_{1n}(\rho)-a_{n2}N_{3n}(\rho)\,]\cos n\theta, \tag{18}$$

$$\vartheta^{1p}=-\frac{k}{G}[\,d_3M_2(\rho)+a_{21}N_2(\rho)-a_{22}N_4(\rho)\,]\sin 2\theta$$

$$-\frac{k}{4G}[\,a_{m1}N_{2m}(\rho)-a_{m2}N_{4m}(\rho)\,]\sin m\theta-\frac{k}{4G}[\,a_{n1}N_{2n}(\rho)-a_{n2}N_{4n}(\rho)\,]\sin n\theta,$$

where

$$N_{iz}(\rho)=z\,(\,z-1\,)\cos\chi+(-1)^i z\sqrt{(z^2-1)}\,\sin\chi$$

$$-(-1)^i\frac{z}{\rho^2}(\sqrt{z^2-1}\,\sin\chi+(-1)^i(\,z+1\,)\cos\chi),\quad i=1,2,$$

$$N_{3z}(\rho)=\frac{(z^3-z^2+z-2)}{\sqrt{z^2-1}}\sin\chi-(\,z+1\,)(\,z-2\,)\cos\chi$$

$$-\frac{1}{\rho^2}((\,z-2\,)\sqrt{z^2-1}\,\sin\chi-(\,z-1\,)(\,z+2\,)\cos\chi),$$

$$N_{4z}(\rho)=(\,2z-z^2-2\,)\cos\chi-\frac{(z^3-4z+2)}{\sqrt{z^2-1}}\sin\chi$$

$$+\frac{z}{\rho^2}(\sqrt{z^2-1}\,\sin\chi+(\,z-1\,)\cos\chi), \tag{19}$$

$$M_i(\rho)=\frac{\sqrt{3}}{2}\sin\left(\gamma_1-(-1)^i\frac{\pi}{3}\right)+(-1)^i\frac{1}{2\rho^2}\cos\left(\gamma_1+(-1)^i\frac{\pi}{3}\right),\quad i=1,2,$$

where $z = 2, m, n$ and $\chi = \gamma_1, \gamma_2, \gamma_3$.

Following [1], for the elastic ring, we have

$$\begin{aligned}
\sigma^1_{\rho B} = & -2[c_1(1+3\beta^4\rho^{-4}-4\beta^2\rho^{-2})+6c_2(\beta^4\rho^{-2}-\beta^6\rho^{-4})]\cos 2\theta \\
& -[m(m-1)c_{1m}(\rho^{m-2}+(m+1)\beta^{2m}\rho^{-m-2}-(m+2)\beta^{2m-2}\rho^{-m}) \\
& +(m+1)c_{2m}(m^2\beta^{2m+2}\rho^{-m-2}+(m-2)\rho^m-(m-1)(m+2)\beta^{2m}\rho^{-m})]\cos m\theta \\
& +[n(n-1)c_{1n}((n+2)\beta^{2n-2}\rho^{-n}+(n+1)\beta^{2n}\rho^{-n-2}-\rho^{n-2}) \\
& +(n+1)c_{2n}(n^2\beta^{2n+2}\rho^{-n-2}-(n-2)\rho^n+(n-1)(n+2)\beta^{2n}\rho^{-n}) \\
& +A((1+n\beta)(n+2)\beta^n\rho^{-n}+n(n\beta+2\beta+1)\beta^{n+2}\rho^{-n-2})]\cos n\theta, \qquad (20)\\
\sigma^1_{\theta B} = & [2c_1(1+3\beta^4\rho^{-4})+3c_2(4\beta^6\rho^{-4}+4\rho^2)]\cos 2\theta \\
& -[m(m-1)c_{1m}(\rho^{m-2}+(m+1)\beta^{2m}\rho^{-m-2}-(m-2)\beta^{2m-2}\rho^{-m}) \\
& +(m+1)c_{2m}(m^2\beta^{2m+2}\rho^{-m-2}+(m+2)\rho^m-(m-1)(m-2)\beta^{2m}\rho^{-m})]\cos m\theta \\
& +[n(n-1)c_{1n}(\rho^{n-2}-(n+1)\beta^{2n}\rho^{-n-2}-(n-2)\beta^{2n-2}\rho^{-n}) \\
& +(n+1)c_{2n}((n+2)\rho^n-n^2\beta^{2n+2}\rho^{-n-2}-(n-1)(n-2)\beta^{2n}\rho^{-n})+ \\
& -A(n(n\beta+2\beta+1)\beta^{n+2}\rho^{-n-2}+(n-2)(1+n\beta)\beta^n\rho^{-n})]\cos n\theta, \\
\tau^1_{\rho\theta B} = & 2[c_1(1-3\beta^4\rho^{-4})+2\beta^2\rho^{-2}+3c_2(\rho^2-2\beta^6\rho^{-4}+\beta^4\rho^{-2})]\sin 2\theta \\
& +[m(m-1)c_{1m}(\rho^{m-2}+m\beta^{2m-2}\rho^{-m}-(m+1)\beta^{2m}\rho^{-m-2}) \\
& +m(m+1)c_{2m}(\rho^m(m-1)\beta^{2m}\rho^{-m})-m\beta^{2m+2}\rho^{-m-2}]\sin m\theta \\
& +[n(n-1)c_{1n}(\rho^{n-2}+(n+1)\beta^{2n}\rho^{-n-2}+n\beta^{2n-2}\rho^{-n}) \\
& +n(n+1)c_{2n}(\rho^n+(n-1)\beta^{2n}\rho^{-n}+n\beta^{2n+2}\rho^{-n-2}) \\
& +A(n(n\beta+2\beta+1)\beta^{n+2}\rho^{-n-2}+n(1+n\beta)\beta^n\rho^{-n})]\sin n\theta, \\
u^1_B = & \frac{k}{G_1}[c_1(\beta^4\rho^{-3}-2\beta^2\rho^{-1}-\rho)+c_2(2\beta^6\rho^{-3}-\rho^{-3}-3\beta^4\rho^{-1})]\cos 2\theta \\
& +\frac{km}{2G_1}[c_{1m}((m-1)\beta^{2m}\rho^{-m-1}-m\beta^{2m-2}\rho^{-m+1}-\rho^{m-1}) \\
& +c_{2m}(m\beta^{2m+2}\rho^{-m-1}-\rho^{m+1}-(m+1)\beta^{2m}\rho^{-m+1})]\cos m\theta
\end{aligned}$$

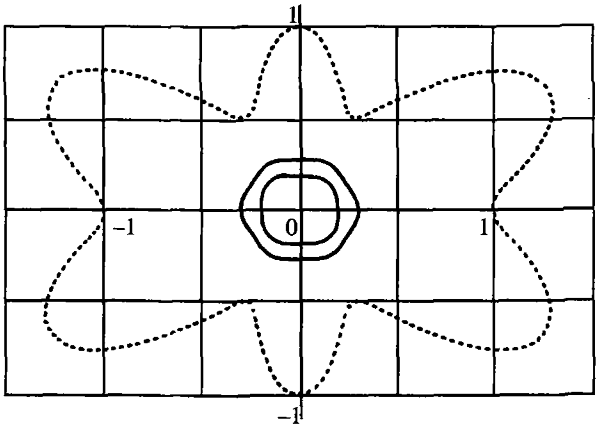

Fig. 2

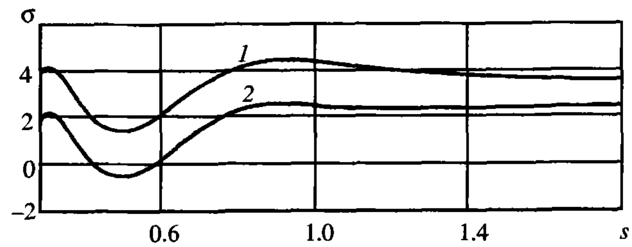

Fig. 3

$$
\begin{aligned}
&-\frac{k}{2G_1}\Big[n\,c_{1n}(\rho^{n-1}+(n-1)\beta^{2n}\rho^{-n-1}+n\beta^{2n-2}\rho^{-n+1})\\
&+n\,c_{2n}(\rho^{n+1}+(n+1)\beta^{2n}\rho^{-n+1}+n\beta^{2n+2}\rho^{-n-1})\\
&+A\left(n\frac{(1+n\beta)}{(n-1)}\beta^{n}\rho^{-n+1}+n\frac{(1+n\beta+2\beta)}{(n+1)}\beta^{n+2}\rho^{-n-1}\right)\Big]\cos n\theta,\\
&\vartheta_B^1=\frac{k}{G_1}[c_1(\rho+\beta^4\rho^{-3})+2c_2(\beta^6\rho^{-3}+\rho^3)]\sin 2\theta\\
&+\frac{k}{2G_1}[m\,c_{1m}(\rho^{m-1}+(m-1)\beta^{2m}\rho^{-m-1}-(m-2)\beta^{2m-2}\rho^{-m+1})\\
&+c_{2m}(m^2\beta^{2m+2}+(m+2)\rho^{m+1}-(m-2)(m+1)\beta^{2m}\rho^{-m+1})]\sin m\theta\\
&+\frac{k}{2G_1}\Big[n\,c_{1n}(\rho^{n-1}+(n-1)\beta^{2n}\rho^{-n-1}-(n-2)\beta^{2n-2}\rho^{-n+1})\\
&+c_{2n}((n+2)\rho^{n+1}-n(n-2)\beta^{2n-2}\rho^{-n+1}-n^2\beta^{2n+2}\rho^{-n-1})\\
&-A\left((n-2)\frac{1+n\beta}{(n-1)}\beta^{n}\rho^{-n-2}+\left((1+n\beta)-\frac{1-n\beta}{(n+1)}\right)\beta^{n+2}\rho^{-n-1}\right)\Big]\sin n\theta,
\end{aligned}
\tag{21}
$$

where

$$A=(p_0-q)\frac{\alpha_1^2 d_2}{\beta^2-\alpha_1^2},$$

c_1, c_2, c_{1m}, c_{2m}, c_{1n}, and c_{2n} are unknown constants.

From (11), (14), and (16), we have

$$\zeta=\left(\frac{d_3}{2}+3a_{21}+a_{22}\right)\cos 2\theta+\frac{m}{2}((m+1)a_{m1}+(m-1)a_{m2})\cos m\theta$$

$$+\frac{n}{2}((n+1)a_{n1}+(n-1)a_{n2})\cos n\theta\,. \tag{22}$$

Using conditions (12) and (13) for determination of c_1, c_2, c_{1m}, c_{2m}, c_{1n}, c_{2n}, a_{21}, a_{22}, a_{m1}, a_{n1}, and a_{n2}, we obtain a system of equations, which, being solved, yields the desired characteristics.

For $d_3 = 0$, we have the case of uniform stretching of the structure at infinity, for $d_1 = 0$, we have a circular hole in the plate and the circular outer boundary of the inclusion, for $d_2 = 0$, we have the circular inner boundary of the inclusion, and for $G_1 = \infty$, we have the case of a rigid inclusion. Let us illustrate the method by an example.

Example. Let

$$\alpha = 0.02\text{ m},\quad \alpha_1 = 0.021\text{ m},\quad \sigma = \sigma_1 = 820\text{ MN/m}^2,\quad d_1 = 1,\quad d_2 = -2,\quad d_3 = 8.$$

$$P_1 = 3\,k,\quad P_2 = 2.8\,k,\quad k = 12/\sqrt{3}\text{ MN/m}^2,\quad P_0 = 2,\quad m = 6,\quad n = 4,\quad \delta = 0.04.$$

Figure 2 shows the boundary of the plastic zone ($r_{s0} = 0.07402$). Figure 3 presents the distribution of stresses in the plate for $\theta = 0$, the expressions for the stresses and displacements being omitted for brevity. In Fig. 3, curve *1* is for σ_θ and curve *2* for σ_ρ.

Perturbation of the inner and outer contours of the prismatic inclusion affects significantly the shape of the elastoplastic boundary.

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