Stochastic analysis of particle-fluid two-phase flows

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Abstract This paper is devoted to exploring approaches to understanding the stochastic characteristics of particle-fluid two-phase flow. By quantifying the forces dominating the particle motion and modelling the less important and/or unclear forces as random forces, a stochastic differential equation is proposed to describe the complex behavior of a particle motion. An exploratory simulation has shown satisfactory agreement with phase doppler particle analyzer (PDPA) measurements, which indicates that stochastic analysis is a potential approach for revealing the details of particle-fluid flow phenomena.

Keywords: **two-phase flow, random force, stochastic simulation and modelling.**

Particle-fluid two-phase flow exists extensively both in nature and in technological processes. Heterogeneity and randomness are the most prominent characteristics of the flow and they are also the baffling problems in accurate hydrodynamic modelling and fundamental flow analysis. This presents difficulties not only in studying the physics of multiphase flow but also in determining the performance of engineering devices[1]. The development of a variety of physical and chemical processes has to take a long way to come into industrialization; and in the scale-up of the multiphase flow processes still exists a very complicated task. The accurate modelling and the better understanding of the flow properties are urgently needed for improving existing processes and for designing optimal future processes involving applications of particle-fluid flows.

At present, four different models, such as the empirical and semi-empirical model, the two-phase structural model, the pseudo-fluid model and the discrete particle (trajectory) model, are commonly utilized to conduct the modelling of particle-fluid flows. These models have both advantages and disadvantages. However, none of them seems to be able to describe the complex phenomena of the particle-fluid flows perfectly. The reason for this is that the existing models and the experiments carried out are too simple to grasp the multi-parameter coupling and randomness of multiphase flows. The lack of effective mathematic-physical methodology and microscopic measuring instruments can also create difficulties in developing quantitative models. The discrete particle model, which resembles physical phenomena of immersing particles into fluid directly, should provide precise and exhaustive description of most flows in engineering theoretically. However, existing models have deficiencies in considering all the forces affecting the motion of a particle. Simplified treatments of this problem unavoidably damage the validity of the models. Even for a very simple flow, the computation involved is again too expensive to achieve engineering purposes.

In this note, a new modelling approach is proposed based on the stochastic analysis of the motion of particles. An exploratory simulation has shown a satisfactory agreement with PDPA measurements.

1 Analysis

The motion of a particle is a combined result of the various forces applied to it. The entire behavior of a moving particle should be grasped if every such force could be quantified. However, we only have a little knowledge of them. It has been quite hard to enumerate all forces influencing the motion of a particle up to now. Every force differs significantly in each flow. Therefore, it is impossible and also not necessary to compute all the force's effects quantitatively in a specific flow. The most feasible way is to treat each force according to its contributions to the particle motion, that is, to quantify the forces dominating the particle's motion and then to model the less important and/or presently unclear forces as random forces. In this way, a stochastic differential equation can be established to describe the complex motion of a particle.

For example, in most publications the motion equation for a small, rigid sphere particle in an unbounded, uniform and incompressible turbulent gas flow is generally simplified as:

$$\frac{dU_p}{dt}=\frac{U_g-U_p}{\tau_p}-g, \tag{1}$$

where U_g and U_p are the velocities of the gas and particle respectively; τ_p is the particle relaxed time; g is acceleration of the gravity; t is the time. The subscripts g and p denote the gas and particle respectively. Obviously, many forces are neglected in eq. (1). The motion equation only presents the balance of the fluid drag and the particle gravity. This negligence is usually justifiable and offers convenience to most incompressible gas-particle systems since the particle density is much greater than the fluid density. But, eq. (1) is not adequate for a precise description of a detailed particle flow structure and, in some case, it may even lead to some incorrect results.

Recent researches have shown that more additional terms should be added to the particle motion equation in order to reflect the influences of the neglected forces. This means the following stochastic differential equation should be used:

$$\frac{dU_p}{dt}=\frac{U_g-U_p}{\tau_p}-g+F(t). \tag{2}$$

In principle, eq. (2) can be thought as a general particle motion equation, which should be more precise than eq. (1) in characterizing the complex particle motion if the stochastic parts could be determined. Evidently, the random force identification varies with particle-fluid systems and with flow conditions as well.

2 Exemplification

Particles suspended in a flowing fluid are simultaneously subject to various forces. The force analysis is certainly very complex. Consequently, the governing equation for the motion of a particle varies with each flow situation and the general expression of the random forces cannot be worked out at present. In order to test the feasibility of the above proposed approach, a relatively simple particle-fluid flow in a dilute circulating fluidized bed (CFB) riser is analyzed in this section and simulated in next section as an exemplification. In a dilute riser flow, gas drag force, gravity of particles and the Saffman force near the riser wall are thought to dominate the motion of particles. These forces contribute to the deterministic terms in the particle motion equation. Other forces, such as collisions among particles and fluid turbulence, which were not considered in the deterministic part, are the component parts of the stochastic term and can be integrally modelled by a random force *F(t)*. Therefore, the following stochastic differential equations are established for a particle flowing in a CFB riser:

Axially:

$$\frac{\pi}{6} d_p{}^3 \rho_p \frac{dU_p}{dt} = C_D \frac{\pi d_p{}^2}{4} \frac{\rho_g}{2} \left| U_g - U_p \right| (U_g - U_p) - \frac{\pi}{6} d_p{}^3 \rho_p g + F_x(t). \tag{3a}$$

Horizontally:

$$\frac{\pi}{6} d_p{}^3 \rho_p \frac{dV_p}{dt} = \begin{cases} C_D \frac{\pi}{4} d_p{}^2 \frac{\rho_g}{2} \left| V_g - V_p \right| (V_g - V_p) + 1.615 \mu_g d_p{}^2 (U_g' - U_p) \sqrt{\frac{dU_g}{dy} \frac{1}{\upsilon_g}} + F_Y(t), & \\ & \text{for} \quad y^+ < 30, \\ C_D \frac{\pi}{4} d_p^2 \frac{\rho_g}{2} \left| V_g - V_p \right| (V_g - V_p) + F_Y(t) & \text{for} \quad y^+ \geqslant 30. \end{cases} \tag{3b}$$

Laterally:

$$\frac{\pi}{6} d_p^3 \rho_p \frac{dW_p}{dt} = C_D \frac{\pi d_p^2}{4} \frac{\rho_g}{2} \left| W_g - W_p \right| (W_g - W_p) + F_z(t), \tag{3c}$$

where, U, V and W are the axial velocity, horizontal velocity and lateral velocity respectively. C_D is the gas drag coefficient, $C_D = \frac{24}{\mathrm{Re}_p}$ for $\mathrm{Re}_p < 1$; $C_D = \frac{24}{\mathrm{Re}_p}(1 + 0.15 \mathrm{Re}_p{}^{0.687})$ for $1 < \mathrm{Re}_p < 1000$, where

$\mathrm{Re}_p = \frac{d_p \rho_g \left| U_g - U_p \right|}{\mu_g}$, $y^+ = \frac{y}{y^*} = y \frac{u^*}{\upsilon_g}$, y is the distance from the particle to the riser wall. u^* is the friction velocity between the fluid and the riser wall, d_p is the particle diameter, ρ is its density, μ_g and v_g are the gas viscosities. $F_X(t), F_Y(t)$ and $F_Z(t)$ are the three components of the random force F(t). Since the Reynolds' number Re_p is usually not very small in most CFB riser flows, eq. (3) tend to be strongly nonlinear. The solution of the corresponding nonlinear equations depend on the properties of nonlinear terms and random terms.

In eq. (3), many forces, such as, the pressure gradient force, virtual mass force, Basset history integral, Faxen's modification to Stokes' drag force, are not quantitatively considered. These forces, of course, have made contributions to the randomness of the particle-flow. One common character of these forces is that the effect of each force is directly proportional to the square or cube of the particle size. A better understanding of these forces is helpful for determining the random force *F*(*t*). Furthermore, the motion of a small particle is often modelled as a kind of Gaussian white noise process, such as Brownian motion in physics. Thus, as the first attempt, the random force $F(t)$ in eq. (3) could be assumed as a modified Gaussian random process.

$$F(t)=\frac{\pi}{6}d_p^3\rho_p\frac{u^{*3}}{\upsilon_g}B\left(\frac{u^{*2}}{\upsilon_g}t\right), \tag{4}$$

where $B\left(\frac{u^{*2}}{\upsilon_g}t\right)$ is the Gaussian white noise process.

3 Simulation

The governing eq. (3) can be solved by Monte Carlo method when the random force term $F(t)$ is expressed as a modified white noise process. In order to compare the simulation results with the measurements by the phase doppler particle analyzer (PDPA)[2], the same CFB riser model was selected for simulation and for PDPA measurement. This riser model has a rectangular cross-section of 100 mm ×15 mm and a height of 2.4 m. The particles are glass beads with diameters ranging from 2 to 90 μm and the particle density ρ_p is 2 470 kg/m^3. The collision between particles and the CFB riser wall are modelled as sliding collisions according to Sommerfeld's research[3].

Fig. 1 shows a comparison of axial particle velocity profiles gained by simulation and by the PDPA measurements. These are the typical flow characteristics in the fully developed zone of a CFB riser. W is the half width of the riser section. Fig. 2 depicts the local particle mean diameter profiles acquired by the PDPA measurements and by simulation. The results obtained, when we ignor the random forces, are also illustrated in these two figures. It is manifest that the simulated profiles are much better when the random force is considered. These preliminary simulations demonstrate the validation of the present approach and the rationality of the presumed random force (eq. (4)). Fig. 3 is the comparison of the temporal evolution of the local axial particle velocities obtained by simulation and the PDPA measurements. The simulation and experiment fit each other quite well. Further work is still under the way.

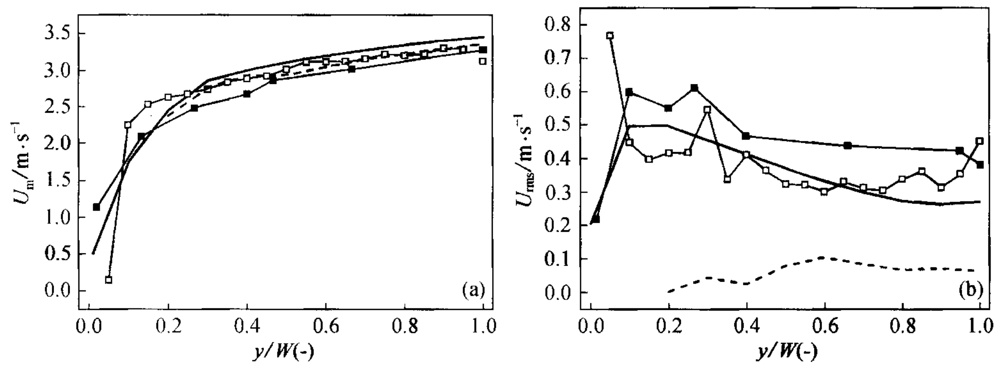

Fig. 1. Comparison of axial particle velocity profiles between simulation and PDPA measurements. (a) Local average axial particle velocity; (b) local average axial particle rms velocity. ——, Gas without particles; ■, PDPA measured; □, simulated with random force; -----, simulated without random force.

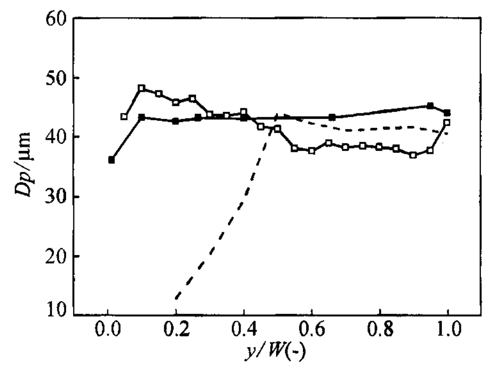

Fig. 2. Comparison of mean particle diameter profiles gained by simulation and by PDPA measurements. ■, PDPA measured; □, simulated with random force; ---, simulated without random force.

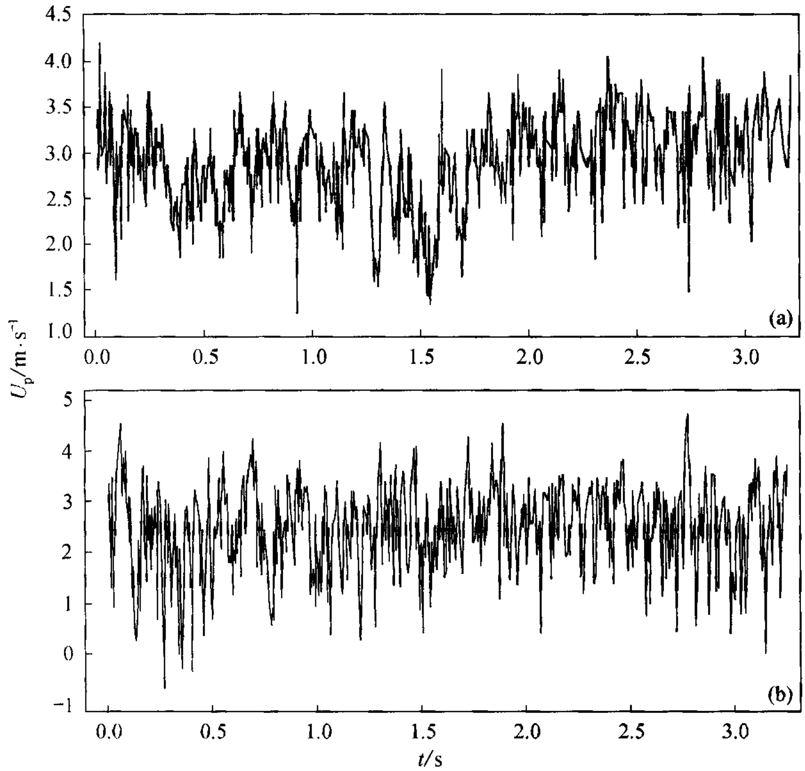

Fig. 3. Comparison of the temporal evolution of local axial particle velocities obtained by simulation (a) and by PDPA measurement (b).

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