ON THE DESIGN OF ROBUST ORTHOGONAL ADAPTIVE DECISION FEEDBACK EQUALIZERS FOR UNCERTAIN DISPERSIVE CHANNELS

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Abstract— **This paper presents the theoretical aspects of a methodology for the design of robust orthogonal adaptive decision feedback equalizers. The dispersive transmission channel is assumed to have a transfer function type of description with small uncertainties in the parameters. A decision feedback equalizer is designed minimizing a mean square error objective function that takes into account the uncertain description of the channel. The equalizer is conceived with an orthogonal basis structure, so that the basis parameters inherit the robustness properties of the design to parameter perturbations. Adaptation of the coefficients that linearly combine the basis elements is also considered and the development of the adaptation algorithm is included. The resulting equalizer has a very flexible, modular and easy to implement structure. An example with comparisons of performance with FIR designs is included.**

Keywords— **Communications, Uncertainty modeling, Equalization, Robust Filtering**

I. INTRODUCTION

Channel equalization is nowadays an unavoidable signal processing procedure to apply if high-speed communication over severely band limited channels is to be established. This is particularly true when we are dealing with the transmission of digital information over copper wire lines such as in Digital Subscriber Line systems (xDSL). The lack of shielding, minimum conditioning and narrow bandwidths are the reasons for the heavy intersymbol interference (ISI) present in these channels (Starr *et al.*, 1999; Bingham, 2000). Linear Equalization (LE) and the more efficient Decision Feedback Equalization (DFE) are effective methods of reducing the ISI and constitute a good tradeoff between computational cost and performance (Belfiore and Park, 1979; Cioffi *et al.*, 1995).

The design of the equalizer relies in the a priori knowledge of the channel characteristics that can be materialized in a mathematical model. This model will normally represent a compromise between good practical representation of measured effects and mathematical tractability.

This paper presents a design procedure for an orthogonal adaptive DFE. The design requires good knowledge of the transmission channel, with a transfer function type of model, of known order but allowing the existence of small uncertainty in the parameters. The uncertainty is described from a statistical point of view and the design minimizes a mean square error objective function. This approach follows the lines of work of Sternad and Ahlén (1990, 1993) and Chen and Lin (1996). An orthogonal structure is then considered for the implementation of the DFE and a coefficient updating algorithm is developed based on this representation.

The paper is organized as follows. Section II introduces some notation, gives a description of the equalization problem and presents the channel model. Section III deals with the design procedure. Section IV, presents the implementation structure using orthogonal functions. Section V describes the development of an updating algorithm and an example is presented in Section VI. Finally, some conclusions are drawn in Section VII.

II. EQUALIZATION PROBLEM AND CHANNEL MODELING

Figure 1 shows the usual structure of a DFE. The random uncorrelated symbol sequence $a(k)$ of variance s_a^2 is dispersed in time by the channel $H(q^{-1},\boldsymbol{a})$ and is corrupted by noise. $D(q^{-1},\boldsymbol{b})$ is the linear filter that shapes the white noise sequence $n(k)$, of variance s_n^2. The sequences $a(k)$ and $n(k)$ are independent with $E\{a(k)n(k)\}=0$. The received signal enters the precursor portion of the equalizer $F(q^{-1})$ that deals with the non-causal effects of the ISI and noise. The detected symbols $\hat{a}(k)$ are filtered by $R(q^{-1})$ and fed back to the detector input. $R(q^{-1})$ is the causal part or post cursor portion of the DFE and it has the function of canceling the ISI introduced by the past symbols.

Both $H(\boldsymbol{a})$ and $D(\boldsymbol{b})$ are considered linear time invariant filters, of orders N and S respectively, with uncertainties in the parameter vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. They have causal impulse responses and belong to the space $\mathrm{H}_2(T)$ of square (Lebesgue) integrable functions on the unit circle $T=\{z:|z|=1\}$, which are analytic outside T, $\{z:|z|>1\}$. $\boldsymbol{a}$ and $\boldsymbol{b}$ are parameter vectors of the form

$a = a_0 + da$ and $b = b_0 + db$ with $E[a] = a_0$ and $E[b] = b_0$ respectively. The zero mean perturbations da and db are described by the *a priori* known covariance matrices $E[dada^T] = g_a$ and $E[dbdb^T] = g_b$.

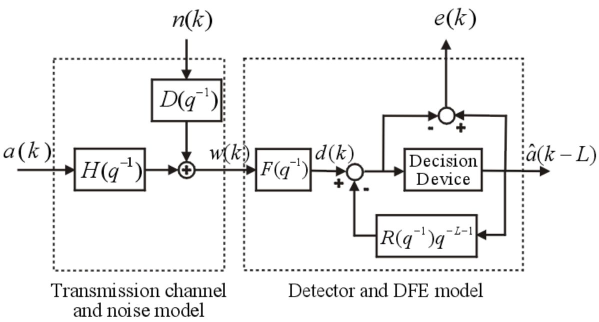

Fig. 1: Transmission channel, noise and DFE models.

The uncertainty on the parameters results in an uncertain channel plus noise system which can be described as having a different realization for each particular value of the parameter vectors a and b. The uncertain system is modeled as

$$H(a_0 + da) = H(a_0) + \Delta H \qquad (1)$$

$$D(b_0 + db) = D(b_0) + \Delta D \qquad (2)$$

The perturbations ΔH and ΔD are approximated by the linear terms of the Taylor series expansion of $H(a)$ and $D(b)$ around the mean values $H(a_0)$ and $D(b_0)$, so $\Delta H \approx (da)^T \left.\frac{\partial H(a)}{\partial a}\right|_{a=a_0}$, and $\Delta D \approx (db)^T \left.\frac{\partial D(b)}{\partial b}\right|_{b=b_0}$ where $\frac{\partial H}{\partial a}$ and $\frac{\partial D}{\partial a}$ are the Jacobian matrices of H and D, respectively. The statistical characterization of the uncertainties is now straightforward:

$E[\Delta H] = 0$;

$$E[\Delta H^*(a)\Delta H(a)] = \left(\frac{\partial H(a)}{\partial a}\right)^* g_a \left(\frac{\partial H(a)}{\partial a}\right) = \Gamma_{\Delta H} \qquad (3)$$

and

$E[\Delta D] = 0$;

$$E[\Delta D^*(b)\Delta D(b)] = \left(\frac{\partial D(b)}{\partial b}\right)^* g_b \left(\frac{\partial D(b)}{\partial b}\right) = \Gamma_{\Delta D} \qquad (4)$$

where the symbol * stands for the joint operations of conjugation and transposition.

III. DFE DESIGN

To present the design procedure, the blocks representing the precursor and poscursor filters of the DFE along with the signals involved are expanded and shown with more detail in Fig. 2.

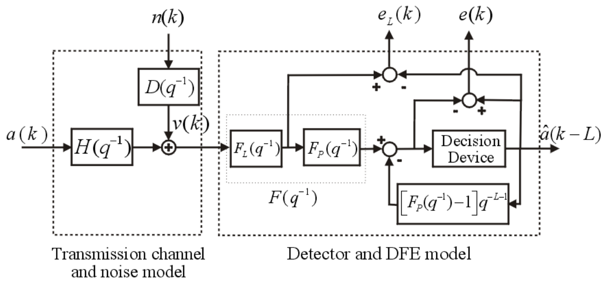

Fig. 2: Expanded view of the DFE

F and *R* are (initially) time invariant linear filters, and will be designed with an infinite number of terms in their impulse responses (IIR). In this case, the optimum design can be performed in two steps (Lee and Messerschmitt, 1994). First, the linear equalizer $F_L(q^{-1})$ is constructed to minimize the *Mean Square Error* (MSE) $E\{e_L^2(k)\}$. Second, as the remaining error sequence $e_L(k)$ will generally be correlated, and thus its variance (power) can be further reduced, the addition of a prediction error filter is considered.

Assuming the decisions of the detector are correct, $F_P(q^{-1})$ and the block $F_P(q^{-1}) - 1$ in the feedback loop of the detector constitute a practical prediction error filter in the sense it is strictly causal ($F_P(q^{-1})$ is monic) and is so realizable (Lee and Messerschmitt, 1994). Its output will depend only of past values of the input, i.e., it depends only of past symbols. When the delay *L* is different from zero, the filters that constitute the equalizer are actually performing smoothing on the received signal $w(k)$.

The problem to solve is to design the DFE following the two steps mentioned above and additionally give the design an orthogonal structure that can easily be adjusted in an adaptive manner. For this purpose the orthogonal family of functions considered were introduced by Ninnes and Gustafsson (1997):

$$L_n(q^{-1}, l) = q^d n_n \frac{q^{-1}}{1 - q^{-1} l_n} \prod_{k=0}^{n-1} \frac{q^{-1} - l_k^*}{1 - q^{-1} l_k} \qquad (5)$$

where $d = 0$ or 1 and $n_n = \sqrt{1 - |l_n|^2}$ is a normalization constant. The basis of (5) has the property of allowing the inclusion of a variety of modes (different poles) and provides a unifying formulation for almost all known system identification techniques supported by orthogonal basis, such as FIR, Laguerre and Kautz models.

A. Precursor equalizer design

The problem of designing the precursor portion of the DFE when the system (channel plus noise) has an uncertain type of model description can be treated as a deconvolution problem (Doñate *et al.*, 2000a, b). From Fig. 2, the expression for the error $e_L(k)$ is obtained when $R=0$ or equivalently when $F_P=1$:

$$e_L(k)=[a(k)\left(q^{-L}-H(a_0)F\right)-n(k)\left(D(b_0)F\right)] \\ -[a(k)\Delta H+n(k)\Delta D]F \tag{6}$$

where q^{-L} takes into account the propagation delay considered for the error evaluation. Assuming $a(k)$ and $n(k)$ are independent of the models uncertainties and using (3) and (4), the *mean square error over the system uncertainties* can be evaluated as

$$\begin{aligned} x &= E_\Delta\{e_L^*(k)e_L(k)\} = \\ &[a(k)\left(q^{-L}-H(a_0)F\right)-n(k)\left(D(b_0)F\right)]^* \\ &[a(k)\left(q^{-L}-H(a_0)F\right)-n(k)\left(D(b_0)F\right)]+ \\ &a^*(k)F^*\Gamma_{\Delta H}Fa(k)+n^*(k)F^*\Gamma_{\Delta D}Fn(k) \end{aligned} \tag{7}$$

where $E_\Delta\{\cdot\}$ is the expectation operator over the models uncertainties. The performance index to be used is the mean value of x over time:

$$\begin{aligned} J=E_k\{x\} &= s_a^2-s_a^2(q^{-L})^*HF-s_a^2F^*H^*q^{-L}+ \\ &s_a^2F^*(H^*H+\Gamma_{\Delta H})F \ \ +s_n^2F^*(D^*D+\Gamma_{\Delta D})F \end{aligned} \tag{8}$$

We see from (8) that the uncertainties in the parameters of the system are contemplated in the objective function J by means of the rational functions $\Gamma_{\Delta H}$ and $\Gamma_{\Delta D}$. Finding the optimal F_{LO} that minimizes J will yield a filter that is slightly more conservative than the classical nominal design but that will generally have a better overall performance in terms of the MSE if the system parameters depart from their nominal values.

To minimize J, (8) is first written in the transform domain with the aid of Parseval´s theorem. Then, a perturbation is added to the optimal deconvolution processor $F=F_{LO}+kz(z^{-1})$, where k is a small, real, positive number and $z(z^{-1})$ is an arbitrary, rational in z and realizable function, analytic in $|z|\geq 1$. To accomplish the necessary and sufficient conditions for a minimum in J, *i.e.* $\left.\frac{\partial J}{\partial k}\right|_{k=0}=0$ and $\left.\frac{\partial^2 J}{\partial k^2}\right|_{k=0}>0$ (Leitman, 1981), the following integrals must be zero,

$$\frac{1}{2p\,j}\oint_{|z|=1}\left(s_a^2\left[Hz(z^{-L}-HF_{LO})^*+H^*z^*(z^{-L}-HF_{LO})\right]\right)\frac{dz}{z}+ \\ \frac{1}{2p\,j}\oint_{|z|=1}\left(s_n^2\left[z^*D^*DF_{LO}+F_{LO}^*D^*Dz\right]\right)\frac{dz}{z}=0 \tag{9}$$

Introducing the spectral factorization,

$$y^*y=s_a^2(H^*H+\Gamma_{\Delta H})+s_n^2(D^*D+\Gamma_{\Delta D}) \tag{10}$$

replacing in (9) and rearranging, the condition to be satisfied is now

$$\frac{1}{2p\,j}\oint_{|z|=1}A\frac{dz}{z}+\frac{1}{2p\,j}\oint_{|z|=1}A^*\frac{dz}{z}=0 \tag{11}$$

where

$$A=y^*z^*\left[F_{LO}y-\{(y^*)^{-1}H^*s_a^2z^{-L}\}_+-\{(y^*)^{-1}H^*s_a^2(z^{-L})\}_-\right]$$

The operator $\{\cdot\}_+$ (resp., $\{\cdot\}_-$) takes the analytic part of the argument outside (resp., inside) the unit circle. Applying Cauchy's theorem to (11), the following part of the integrand must be zero, since it is the only one which may have poles inside the unit circle:

$$F_{LO}y-\{(y^*)^{-1}H^*s_d^2z^{-L}\}_+=0 \tag{12}$$

From (12), J is minimized by

$$F_{LO}=\{(y^*)^{-1}H^*s_d^2z^{-L}\}_+y^{-1} \tag{13}$$

Three comments on this result follow. First, a realizable F_{LO} can only eliminate the parts of the integrand that involve $\{\cdot\}_+$ terms (must be analytic outside the unit circle and as so, causal). Second, the $\{\cdot\}_-$ term must be a rational function starting with a free z to cancel the pole at the origin of the integrand. Third, for symmetry reasons, if one of the integrals in (11) is zero so will be the other. The readers may like to compare this derivation with the one in Appendix A of Ahlén and Sternad (1991) and also the approach of Chen and Lin (1996).

Assuming there are uncertain parameters in the denominators of both *H* and *D*, from (3), (4), and (10), the maximum degree for the polynomials of y is $2(N+S)$. Therefore, the order of y^{-1}, the IIR part of the optimal precursor equalizer, if the prediction error filter is included, will generally be $M\geq 2(N+S)$. From (13), noting that H^* and y^* have all the roots outside the unit circle, the $\{\}_+$ operation reduces to a constant if $L=0$ or a FIR type of filter, of length *L*, if $L>0$. So, the resulting structure for the precursor portion of the DFE including F_P, may generally (when the symbol sequence is white noise) be considered as a cascade of a FIR filter and an IIR filter as shown in Fig. 3.

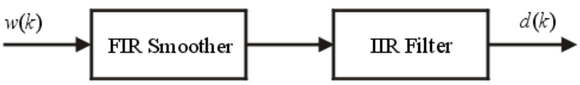

Fig. 3: General structure for the precursor portion of the DFE

B. Poscursor equalizer design

As a second step, we want to evaluate the part F_P of the equalizer that minimizes the prediction error. The spectral density of the error over the detector when

$F_P = 1$ is:

$$\begin{aligned} P_{e_L} &= (HF_{LO} - 1)^* (HF_{L0} - 1) s_a^2 + F_{LO}^* D^* D F_{LO} s_n^2 \\ &= s_e^2 E^* E \end{aligned} \quad (14)$$

where $s_e^2 E^* E$ is the spectral factorization of P_{e_L}, s_e^2 is the MSE and E is a monic, rational, minimum phase function that is also analytic in $|z| \geq 1$. The optimal prediction error filter is then

$$F_P(q^{-1}) = E^{-1}(q^{-1}) \quad (15)$$

In addition, the existence of the poscursor filter is guaranteed, even for the case when the delay $L = 0$, because $R(q^{-1}) = F_P(q^{-1}) - 1$ is a strictly causal filter.

IV. ORTHOGONAL REALIZATION

Each of the filters that conform the robust DFE is to be realized using a linear combination of the orthogonal basis of (5) as:

$$F_i = \sum_{n=0}^{Q} q_n L_n(q^{-1}, l) \quad (16)$$

where the number of terms $Q+1$ of Eqn. (13) must be greater or equal to the order of the functions to be represented. The parameter vector l is chosen to include all of the poles of F and R in each case, to achieve an exact representation. The parameters that linearly combine the basis elements are calculated by

$$q_n = \frac{1}{2p\, j} \oint L_n(l, z^{-1})^* F_i(z^{-1}) z^{-1} dz \quad (17)$$

where the integral can be evaluated using Cauchy's residue theorem.

Assuming M is the order of the robust precursor filter, the vector Θ_F groups the coefficients associated with this filter:

$$\Theta_F = [q_0, ..., q_L, q_{L+1}, ..., q_M]^T \quad (18)$$

and Θ_R groups the coefficients of the orthogonal realization of the poscursor filter *R*:

$$\Theta_R = [q_{M+1}, ..., q_{M+P}]^T \quad (19)$$

P in (18) is the largest degree of the polynomials of *R*. If the degree of F_{LO} is $N_{F_{L0}} \leq 2(N+S)+L$, then from (14), the polynomials of *E* (and *R*) will have a degree $P \leq N_{F_{L0}}$.

From (5) is simple to establish the relation between two successive members of the basis:

$$L_{n+1} = h_{n+1} L_n \frac{\overline{C}(l_n)}{C(l_{n+1})} \quad (20)$$

where $h_{n+1} = \frac{n_{n+1}}{n_n}$, $C(l_n) \equiv 1 - q^{-1} l_n$ and $\overline{C}(l_n) \equiv q^{-1} - l_n$. This relation will be useful for the practical implementation of the DFE.

Up to this point in the design procedure, we have a DFE that has an orthogonal structure for each of its filters and is, by design, robust to small parametric perturbations in the system model.

V. UPDATING OF THE BASIS COEFFICIENTS

To improve the performance of the equalizer and to incorporate the ability of tracking larger changes in the system dynamics, *i. e.* changes that exceed g_a and g_b, the updating of the coefficients that linearly combine the basis elements is next considered. The coefficients Θ are now treated as time varying and noted:

$$\Theta = \Theta(k) = [\Theta_F^T(k), \Theta_R^T(k)]^T \quad (21)$$

The error functional for the adaptive algorithm, assuming the decisions of the detector are correct and as a function of the coefficient set is

$$\begin{aligned} z(\Theta, k) &= E[(a(k-L) - \hat{a}(k-L))^2] = \\ &\quad E[(a(k-L) - \Theta^T(k) X(k))^2] \end{aligned} \quad (22)$$

where

$$X(k) = [X_F^T(k), X_R^T(k)]^T \quad (23)$$

is conformed by

$$\begin{aligned} X_F(k) = [&L_0(q^{-1}, 0), L_1(q^{-1}, 0), ..., L_l(q^{-1}, 0), \\ &L_{l+1}(q^{-1}, z_1), ..., L_M(q^{-1}, z_M)]^T w(k) \end{aligned} \quad (24)$$

which is the generalized precursor regressor build of delayed and basis-filtered versions of the received signal $w(k)$ and

$$\begin{aligned} X_R(k) = [&L_0(q^{-1}, r_0),\ L_1(q^{-1}, r_1),\ ...\ , \\ &L_P(q^{-1}, r_P)]^T a(k-L) \end{aligned} \quad (25)$$

that is the generalized poscursor regressor composed of basis-filtered versions of the detected symbols $a(k)$.

Expanding (21)

$$z(\Theta, k) = E[a^2(k-L)] - 2\Theta^T(k) p + \Theta^T(k) R_I \Theta(k) \quad (26)$$

with $p = E[a(k-L) X(k)]$ and $R_I = E[X(k) X^T(k)]$, and considering the update equation

$$\Theta(k+1) = \Theta(k) - m G(k) \quad (27)$$

where

$$G(k) = \frac{\partial z}{\partial \Theta} = 2(R_I \Theta(k) - p) \quad (28)$$

is the gradient of the error functional in the parameter space, a gradient-based family of adaptive algorithms can be generated.

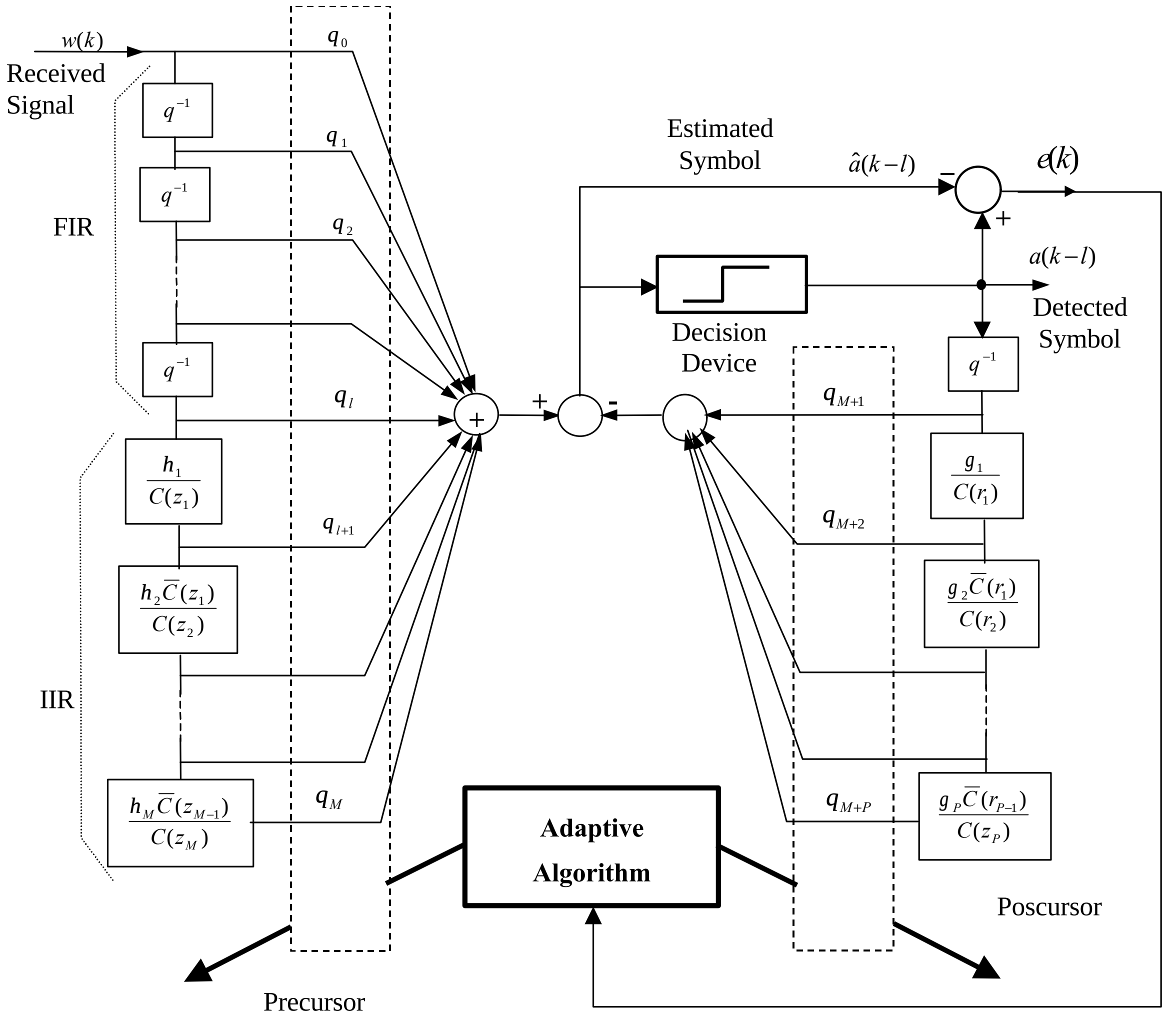

Fig. 4: Structure of the Robust Orthogonal Adaptive DFE. The blocks illustrate the case when the basis parameters are real.

Different estimates of $G(k)$ result in different algorithms. The approach considered for the DFE in this paper uses the instantaneous value of p and R_I as estimates of their mean so, $\hat{p} = a(k-L)X(k)$, $\hat{R}_I = X(k)X^T(k)$ and the estimated gradient is

$$\hat{G}(k) = -2X(k)e(k) \qquad (29)$$

The updating equation finally results

$$\Theta(k+1) = \Theta(k) + 2me(k)X(k) \qquad (30)$$

and the algorithm may be classified as a transform domain *Least Mean Square* (Diniz, 1997).

Figure 4 shows the complete structure for the implementation of the Robust Orthogonal Adaptive DFE. This structure follows from (20) and is illustrated for the particular case of a DFE with real poles in both filters *F* and *R*. The inclusion of complex modes in the basis has to be done in conjugate pairs. In this case, the new basis functions are formed as linear combinations of the basis generated by (5), (see Ninnes and Gustafsson, 1997, for the details of this construction). This procedure assures that the impulse response of the basis functions with complex modes is real.

VI. EXAMPLE

A first order low pass transmission channel model corrupted by colored noise will be used to illustrate the performance of the proposed design. The system is

$$H(z^{-1}) = \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1}} = \frac{1 + 0.9z^{-1}}{1 - 0.9z^{-1}}$$

and

$$D(b, z^{-1}) = \frac{d_0 + d_1 z^{-1}}{1 + c_1 z^{-1}} = \frac{1 + 0.5z^{-1}}{1 - 0.7z^{-1}}.$$

The gain of these filters are adjusted so that the signal to noise ratio in the input of the precursor filter is 30 dB. The design is performed assuming uncertainty in the parameter b_1 of *H*, with $g_{b_1} = 0.0003$ and $L = 0$.

Figure 5 shows the variation of MSE achieved by the

DFE when the parameter b_1 departs from its nominal value. The curve in solid line is the nominal fixed recursive design. Both filters of the DFE are IIR. This design gives the best performance when the parameters remain at the nominal design values. When b_1 changes the MSE raises and the performance degrades quickly.

The curve in dashed line is the fixed robust design. The performance around the nominal value of the parameter is slightly worse than the nominal design. When b_1 is perturbed, the robust DFE equals and outperform the nominal design for positive (negative) variations of more that 4% (-2%) in the parameter.

The robust design has the effect of extending the range of "good" performance of the equalizer for changes in b_1 at the expense of loosing performance around the nominal value of this parameter.

Figure 6 shows the performance of the robust orthogonal adaptive DFE. In this case the initial value of the coefficients Θ are those of the robust design and the updating Eqn. (30) is used to adjust these coefficients as b_1 departs from its nominal value.

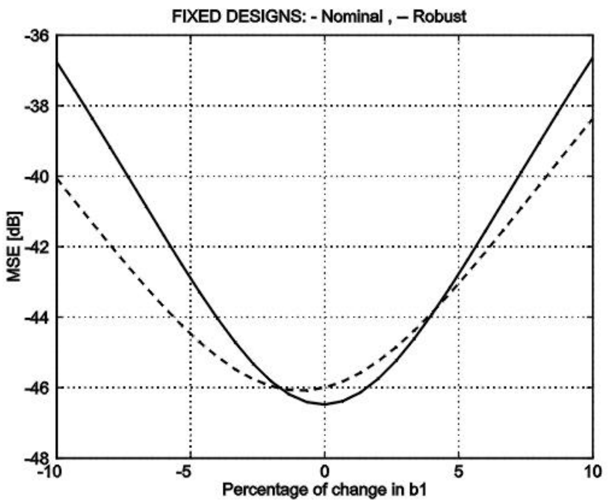

Fig. 5: MSE vs. Percentage of change in parameter b_1 for fixed recursive DFE designs. Solid line: Nominal design. Dashed line: Robust design.

The MSE attainable when the algorithm converges is plotted vs. the percentage of variation of b_1 (dashed line curve). Also included in this figure is the performance of the fixed nominal design (solid line) and the MSE attainable by an adaptive FIR DFE (dash-dot line). The FIR DFE requires 20 taps to achieve a constant MSE that equals the nominal design. The robust orthogonal adaptive DFE exhibits a performance that is less than 1 dB within the minimum MSE for 6% of variation in b_1. This performance is achieved using only 5 adaptive coefficients.

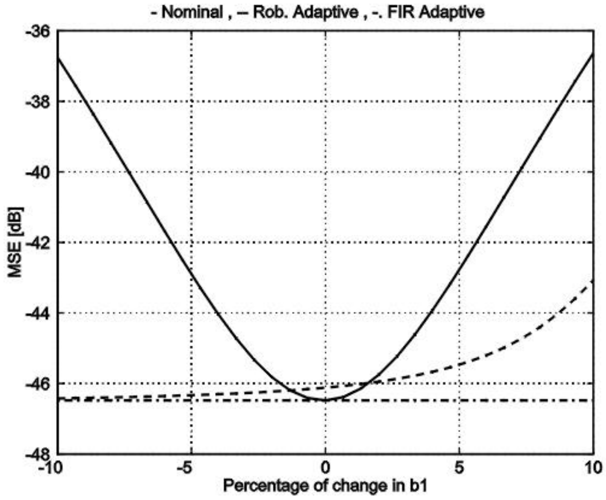

Fig. 6: MSE vs. Percentage of change in parameter b_1 for adaptive DFE designs. Solid line: Nominal fixed design included for reference. Dashed line: Robust orthogonal adaptive design (5 adaptive coefficients). Dash-dot line: FIR adaptive design, 10 taps precursor filter and 10 taps poscursor filter.

VII. CONCLUSIONS

The paper has presented the theoretical aspects for the design of robust adaptive DFE with an orthogonal implementation structure. The DFE designed by this methodology presents robustness to small parametric uncertainties in the channel and noise models, a very modular construction by the use of orthogonal bases, and adaptation capability with the use of a simple algorithm. A numerical example was presented that shows the good performance of the robust orthogonal adaptive DFE in terms of MSE. The example also shows the reduced computational cost when compared with adaptive FIR designs. All these properties make this design a good candidate for practical DSP implementations.

Further research is currently being performed to apply this robust DFE to xDSL systems.

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