STATISTICAL DAMAGE DETECTION IN A STATIONARY ROTOR SYSTEMS THROUGH TIME SERIES ANALYSIS

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***Abstract*— A novel approach to detect damage in stationary rotating systems excited by unbalance and stochastic forces is presented. The methodology is based solely on output time series measured at the bearings stations. The method deals with the application of auto-regressive models and statistical modeling for the linear prediction of damage diagnosis. The results showed that the approach is suitable for practical applications.**

***Keywords*— damage detection, rotor systems, AR-ARX models.**

I. INTRODUCTION

The analysis and monitoring of machines play an important hole in modern industries due to economical, equipment availability, and safety reasons. Farrar *et al.* (2005) stated that in-service failure corresponds to 20-40% of all losses in engineering sector, mainly in petrochemical industry. And rotating components are presented in the majority of the machines usually found in industries as, for example, pumps, compressors, fans, turbines, etc. Hence, the monitoring and maintenance of this kind of equipment – the rotating machines – is a crucial issue in any industrial plant.

Several technical standards adopt the root means square (RMS) values (DIN 45666, for example) and/or the overall vibration value (ISO 10816) as damage-sensitive index in rotor systems (NBR 10082, 1987). Unfortunately, in some cases, these features are contaminated by the unavoidable experimental errors or dynamical effects. To overcome these facts, the spectral analysis can be performed, but the results are highly dependent on the human experience (Mitchell, 1993).

Another more elaborated approach for damage detection is the use of mathematical models, which can be generated numerically, using the finite element methods and/or experimental, obtained through modal analysis. Based on these models, different strategies are described in the literature to identify a damage as, for instance, correlation analysis (Eduardo, 2003); model updating by using optimization methods (Castro *et al.*, 2005); state observers based methods (Melo and Lemos, 2004), etc. It is worth noting, however, that model-based assessment approaches are usually computationally intensive and requires a quite accurate model of the rotor system.

The present paper addresses the damage detection problem of a rotor system. The methodology is based on an AR-ARX model, as described by Sohn and Farrar (2001), who used this procedure for structural application. The main idea is to use the one-step-ahead error prediction as damage-sensitive index. Large prediction error comparing to the actual measurement will occur if the system presents accumulated damage (Silva *et al.*, 2007).

This paradigm has successful applications for gear fault detection comparing with wavelet analysis and resonance demodulation (Wang, 2003). The partitioning of this damage-sensitive feature in healthy or damaged state is made in this paper by using two different statistical modeling. The first one is the ratio between the residual errors (Sohn and Farrar, 2001). The second one is based on limits control constructed by statistical process control (Silva *et al.*, 2005). The performance for both threshold values determination are compared and discussed. Tests are made in a rotor system with different damage patterns. The capability to reach good diagnostic based solely in response measurements is demonstrated.

Additionally, the procedure proposed is not based on the human knowledge and experience, as is the case in the classical spectral and/or RMS analysis.

II. DAMAGE-SENSITIVE FEATURE

Initially, it is considered signals, $z[k]$, measured from the undamaged rotor system (healthy state) in N environmental conditions, always running in stationary condition. In order to obtain all signals with zero sample mean and standard deviation equals to one, these time series must be standardized, as given by the following expression:

$$x[k] = \frac{z[k] - m(z)}{s(z)}, \tag{1}$$

where $x[k]$ is the standardized signal at the k^{th} time instant, $m(z)$ and $s(z)$ are, respectively, the mean value and standard deviation of the $z[k]$ sequence.

The first phase of the methodology is devoted to the construction of an AR model, with order p, for each $x_i[k]$, $i = 1, 2, \ldots, N$. The AR(p) model can be written as:

$$A_{xi}(q)\, x_i[k] = e_{xi}[k], \tag{2}$$

where $e_{xi}[k]$ is the error between the i^{th} measured signal and the output from the prediction model. $A_{xi}(q)$ is the i^{th} polynomial in the delay operator q^{-1}. The coefficients of the AR model can be found by the Yule-Walker equations (Wang, 2003), while the polinomial order, p, can be obtained by using Akaike's information theoretic criterion (AIC).

A new vector of data measured in unknown structural condition (undamaged or damaged), after standardization, is used to obtain another polinomial, $A_y(q)$, also of order p:

$$A_y(q)\,y[k] = e_y[k]\,, \tag{3}$$

where $y[k]$ is this new standardized signal at the k^{th} time instant.

The AR model in Eq. (3) is compared with each model of the signals $x_i[k]$ in the reference database – Eq. (2) – in order to select the signal $x_R[k]$ "closest" to the unknown condition block $y[k]$. To accomplish this, the Euclidean distance,

$$Distance = \sum_{l=1}^{p}(a_{yl} - a_{xil})^2\,, \tag{4}$$

is minimized.

The signal $x_R[k]$ which coefficients satisfy the minimum distance in Eq. (4) is called the reference signal. The idea behind this procedure is that if the normalized vector $y[k]$ is obtained under the same operational condition of one of signal in the reference database and there has been no damage in the system, the AR model (the coefficients of the $A_y(p)$ polinomial) in Eq. (3) should be similar to the model obtained for the $x_R[k]$ signal (Sohn and Farrar, 2001). Otherwise, the coefficients of the $A_y(p)$ polinomial will be different from the coefficients of any of the polinomials $A_{xi}(p)$, indicating a damage or a significant change in the operational condition.

The next stage is the obtention of an ARX (autoregressive with exogenous input) model from the reference signal $x_R[k]$. This model can be written as:

$$A_{xR}(q)\,x_R[k] = B_{xR}(q)\,e_{xR}[k] + \varepsilon_{xR}[k]\,, \tag{5}$$

where $\varepsilon_{xR}[k]$ is the residual error of the ARX(na, nb) model, $e_{xR}[k]$ is the residual error of the AR(p) model given by Eq. (2). The orders na and nb of the polynomials $A_{xR}(q)$ and $B_{xR}(q)$ are set arbitrarily.

Now, the same model associated with Eq. (3) is used to investigate if it is capable to predict the vector of data obtained in any unknown condition:

$$A_{xR}(q)\,y[k] = B_{xR}(q)\,e_y[k] + \varepsilon_y[k]\,. \tag{6}$$

If the ARX model obtained from Eq. (5) is not a good prediction for the unknown signal $y[k]$ and $e_y[k]$, then the residual error $\varepsilon_y[k]$ in Eq. (6) and its probability distribution will change.

A common approach is to monitor the standard deviation of $\varepsilon_y(k)$ and compare it with the standard deviation of the healthy state $\varepsilon_{xR}(k)$. This can be easily done by computing the ratio between the standard deviations of the residual errors from Eqs. (5) and (6) through the following expression:

$$\gamma = \frac{s(\varepsilon_y)}{s(\varepsilon_{xR})}\,. \tag{7}$$

A significant increase in this index indicates that the location where the measurement is made is close to the damaged spot. Another approach used in damage detection is the statistical process control (SPC). This method is based on a control chart which is used for automatic continuous monitoring (Silva *et al.*, 2005).

A control chart is composed by a centerline (CL) located at the mean value of the reference residual error $\varepsilon_{xR}(k)$ and two additional horizontal line corresponding to the upper and lower control limits (UCL & LCL) versus the sample numbers. The CL, UCL, and LCL are given by (Montgomery, 1996)

$$CL = \text{mean}(\overline{X}), \quad \text{and} \quad UCL,LCL = CL \pm Z_{\alpha/2}\frac{S}{\sqrt{n}}\,, \tag{8}$$

where $\overline{X}$ is the sample mean and S is the standard deviation, both with the respect to n observations in each sample. $Z_{\alpha/2}$ is the percentage point of the normal distribution.

In general, when the rotating mechanical system presents some deterioration or fault, a statically significant number of samples outside the control limits, called as outliers, are observed (Silva *et al.*, 2005).

III. RESULTS

To illustrate the methodology, some tests in a vertical rotating system were made. A schematic drawing of the 6 DOF rotor used to run the simulations is shown in Fig. 1. This system was already used by Eduardo (2003) and details about the equations of motion can be found in his work. In the present work, this model was used to obtain the resulting vibration at several points of the rotor when subjected to unbalance and sthocastic forces (white noise). The geometrical and physical properties are shown in Table 1 while Table 2 presents the five damage patterns considered for this system.

Five different scenarios of undamaged state were considered in this work, each one obtained by the variantion of the operational condition (% of noise

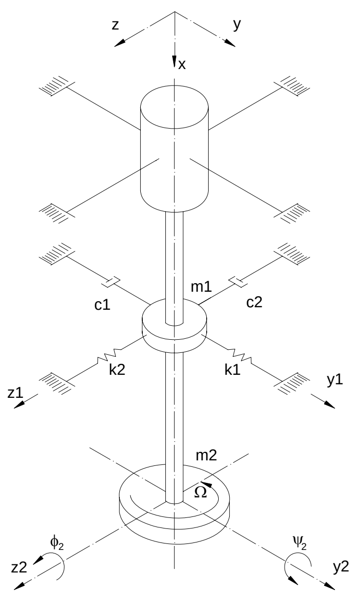

Figure 1. Rotating mechanical system.

Table 1. Properties of rotor sytem.

Property	Value
m_1: bearing mass [kg]	15
m_2: disk mass [kg]	10
I_2: tranversal moment of inertia [kg.m^2]	0.25
I_{2p}: polar moment of inertia [kg.m^2]	0.5
L: length of rotor [m]	0.8
k_1: bearing stiffness in y direction [N/m]	90e3
k_2: bearing stiffness in z direction [N/m]	120e3
c_1: bearing damping in y direction [kg/s]	30e3
c2: bearing damping in z direction [kg/s]	30.75e3
Ω: constant speed rotation [rad/s]	60
e: unbalance eccentricity [m]	1e-5

Table 2. Damage patterns.

Damage	Description
(1)	Reduction of 20% in k_1
(2)	Reduction of 30% in k_1 and c_1
(3)	Reduction of 20% in k_2
(4)	Reduction of 30% in k_1
(5)	Variation of 20% disk unbalance

Table 3. List of studied undamaged scenarios.

	Damage Pattern	Peak Amplitude Force (N)	% RMS noise
Case 1	No damage	5	10
Case 2	No damage	10	10
Case 3	No damage	10	5
Case 4	No damage	15	10
Case 5	No damage	18	10

added and input level). The list of these 5 undamaged scenarios is shown in Table 3. Table 4 describes the 18 "unknown" conditions studied, which can correspond to measurements performed in the healthy or damaged system.

The rotor response was obtained by numerical integration of the equations of motion using a sampling rate of 1000 samples/sec and a total time of 10 sec. The first half of the data (5000 points) was used to obtain the AR(13) model while the second half was used to validated the model. The reference signal was obtained by using Eq. (4). The ARX model for the reference signal was constructed using the second half of the data block. Next, this model was used to predict the signals obtained in unknown conditions.

Some examples of the displacement in the y_1 coordinate are shown in Figs. 2 and 3. Figure 2 shows the error prediction for case 6, where the reference database is case 2 (see Table 4). The prediction error $\varepsilon_y(k)$ is arranged in 5 groups with 1000 samples each. $Z_{\alpha/2}$ chosen was 2.57 and it corresponds to 99% of confidence. Thus, 10 samples (=1% of total 1000 samples) are expected to be outsides the control limits even for the rotor system without any damage. The outliers are marked by "*" in all figures. Therefore, the 5 outliers in fig. 2b do not indicate a clear damage. However, a significant number of outliers (76) appears in Fig. 3b, which corresponds to case 11, indicating the existence of damage.

Figure 4 presents the ratio between the standard deviations of the residual errors given by Eq. (7) for

Table 4. List of studied unknown set (healthy or damaged).

	Damage Pattern	Peak Amplitude Force (N)	% RMS noise
Case 6*	No damage	8	10
Case 7*	No damage	12	15
Case 8*	No damage	20	20
Case 9	Pattern 1	10	5
Case 10	Pattern 1	10	10
Case 11	Pattern 2	10	5
Case 12	Pattern 2	10	10
Case 13	Pattern 3	10	5
Case 14	Pattern 3	10	10
Case 15	Pattern 4	10	5
Case 16	Pattern 4	10	10
Case 17	Pattern 5	10	5
Case 18	Pattern 5	10	10

* This set of data was not used to construct the AR-ARX model. It was considered in unknown condition to test false-positive.

various undamage and damage source. The results of damage diagnosis by using this procedure, as made by Sohn and Farrar (2001), do not appears to be robust, because the ratio γ do not present a significant change to give a clear indication of a system anomaly. However, the SPC present a suitable detection, once a significant number of outliers are expected to be outside the limits. Figure 5 provides the number of outliers of the raw time series $\varepsilon_y(k)$.

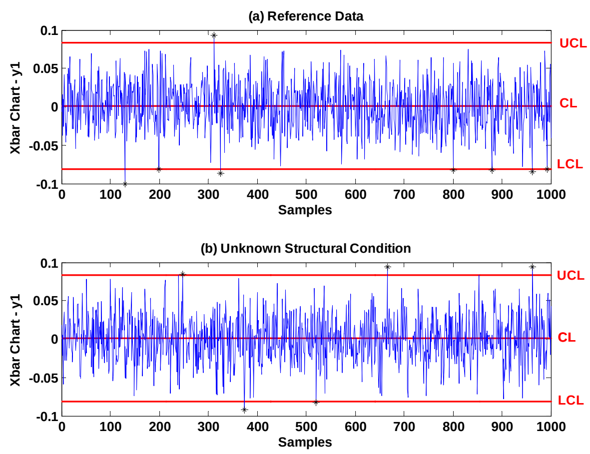

Figure 2. AR-ARX prediction error– (a) reference database (case 2). (b) Unknown condition (case 6). The limits control are constructed by a previous time series from case 2 (reference). The number of outliers in (b) shows normal condition (5 outliers).

III. FINAL REMARKS

The approach demonstrated to be able to detect damage in rotating machines without deep knowledge of the system. Two statistical modeling are exemplified in order to obtain a threshold value with minimum interaction with the user. The SPC was found to be more suitable for a further automated continuous monitoring in a real-world rotor system because of its simplicity and clearness.

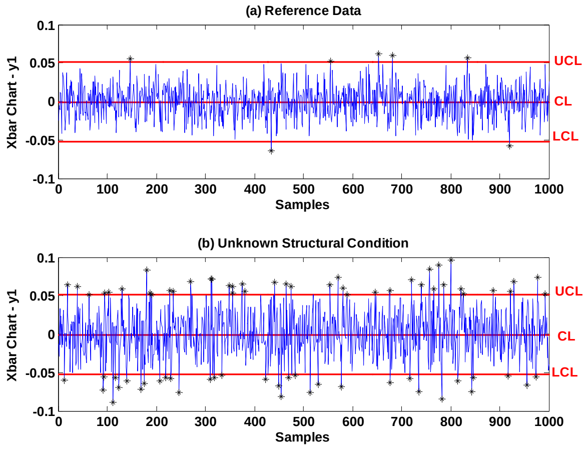

Figure 3. AR-ARX prediction error– (a) reference database (case 1). (b) Unknown condition (case 11). The limits control are constructed by a previous time series from case 1 (reference). The number of outliers in (b) indicates the existence of damage (76 outliers).

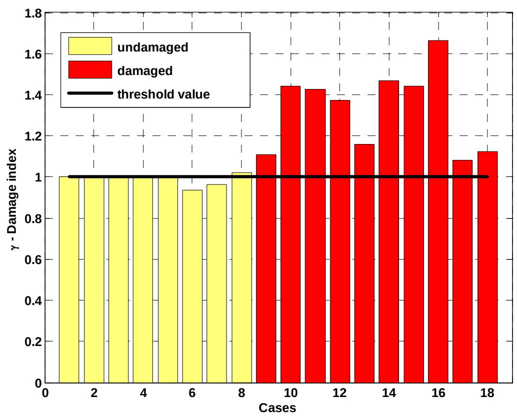

Figure 4. γ ratio framework.

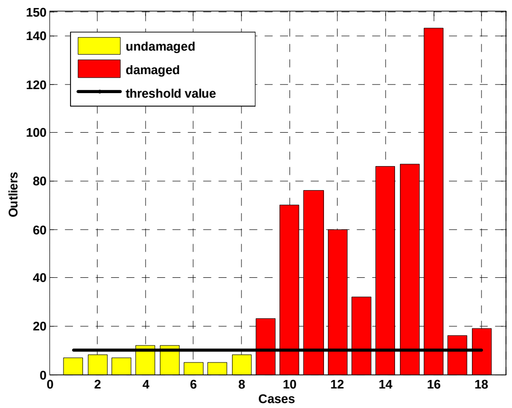

Figure 5. Outliers evolution. For each case is obtained a control limit based on reference data-base and confidence of 99%.

In order to quantify the damage and to obtain the remaining service life without mathematical model, one could use these features, outliers and/or error predictions, considering the system associated with different fault levels. Hence, it is possible to drive a supervised learning, as for example, by using classical neural networking to try to obtain correlations between outliers and damage sources. However, the simple question of whether damage is present or not in the rotor is the most fundamental issue. Unfortunately, this goal is still a daunting problem for some practical applications in the industry. In this sense, the results in this paper encourage the authors and it seems that the methodology can be used successfully in real cases, where other approaches fail.

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