KINEMATIC CONTROL OF WHEELED MOBILE ROBOTS

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***Abstract*— A generic kinematic control, which is directly applicable to any type of wheeled mobile robot, is proposed in this work. Firstly, it is presented the kinematic models of the four common types of wheels (*fixed*, centered *orientable*, *castor* and *Swedish*) a classification of wheeled mobile robots. Afterwards, it is proposed a kinematic control scheme with three nested loops: dynamic, kinematic and planning. In particular, it is studied in depth the kinematic loop through the position control and the inverse kinematics of wheels. It is also studied indirectly the planning loop through the characterization of the references that each type of mobile robot can track with no error. Finally, the kinematic control proposed is applied to an industrial forklift in simulation and in a real situation.**

***Keywords*— Kinematic control, wheeled mobile robots, tracking references, nested loops.**

I. INTRODUCTION

Wheeled mobile robots (WMR) are used in the automation of industrial processes and in other areas like agriculture. They are mainly involved in reference tracking. So, a proper control is essential for that purpose.

Some researches have developed a WMR control based on geometric methods. It consists on applying control signals in such a way that the WMR follows a curve that connects its actual position with a goal position on the reference trajectory. For instance, Ollero *et al.* (1994) used circular arcs as connecting curves, meanwhile Shin (1990) used fifth order polynomials. This control develops a point-to-point trajectory tracking (pure-pursuit) so it is difficult to guarantee stability for a particular trajectory and to optimize some pursuit parameters for a good reference tracking.

Other authors have used tools of the classical control theory for the tracking control. One possibility is to obtain a linear approximation of the WMR model around an equilibrium point and then design a classical linear control. Even, it is possible to obtain the discrete model of the linear approximated system and apply a discrete control. The main drawback is that the linear approximation is valid, especially for WMR, for a small range around the equilibrium point, arising sometimes instability. As an example, O'Connor *et al.* (1996) develop a continuous control for the WMR direction based on a linear approximation. Similar controls are proposed by Canudas *et al.* (1997).

A state feedback linearization has been also used as a previous stage of a linear classical control. It has the drawback that the singularities invalidate the linearization. However, if the linearization is possible and there are no singularities, or they are never achieved, this control is very suitable. Examples of kinematic controls based on this approach are d'Andrea-Novel *et al.* (1995) and Gracia and Tornero (2002), meanwhile Tzafestas and Tzafestas (2001) use the state feedback linearization for a dynamic control. Some other researches (Samson, 1995; Murray and Sastry, 1993) have studied the conditions of WMR controllability to design control lows.

Some approaches use non-linear control lows for which stability is guaranteed through the existence of a *Lyapunov* function that fulfills the *Lyapunov* theorem for stability. For example, Lyshevski and Nazarov (2000) apply a continuous dynamic control for a WMR with the existence of such a function. Similarly, Dixon *et al.* (2000) present other stable control lows.

Other authors have applied an adaptative control. For example, Inoue *et al.* (1997) apply an adaptative discrete kinematic control with the experimental tuning of some parameters of the algorithm to achieve stability. Dixon *et al.* (2001) develop an adaptative control for reference tracking with an uncalibrated vision system.

More recently, most of the works that nowadays deal with WMR control use *fuzzy*, see for example Wong *et al.* (2005), *neural networks*, see for example Antonini *et al.* (2006) and d'Amico *et al.* (2006), or both, see for example Hui *et al.* (2006). Nevertheless, these approaches have two major drawbacks: firstly, the optimization process of the fuzzy functions/rules/ parameters and the learning of the neural networks become complex and they take much time; secondly, most of these approaches are valid just for a specific WMR.

This research proposes a kinematic control scheme (subsection III.B) with three nested loops (dynamic, kinematic, and planning) that is similar to the approaches used for robotic manipulators. It is studied in depth the kinematic loop (subsections III.C and III.D) and indirectly the planning loop, through the characterization of the references that each WMR can track with no error (section IV). The kinematic control proposed is applied to an industrial forklift (section V), which is equivalent to the tricycle WMR, in simulation and in a real situation. Finally, section VI points out the most outstanding contributions of this approach and suggests extensions. In particular, with respect to previous works, the main advantages of the proposed approach for WMR control are: it is valid for *any* type of WMR; it uses all the maneuverability of the WMR; and it takes into account the type of references that can be tracked with no error. Meanwhile, it has the main limitation that the low-level control must be much faster that the medium-level control to guarantee the global stability.

II. KINEMATIC MODELING

Assuming horizontal movement, the position of the WMR body is completely specified by 3 scalar variables (*e.g.* x, y, θ), referred by Campion *et al.* (1996) as *WMR posture*, $\mathbf{p}$ in vector form. Its first-order time derivative $\dot{\mathbf{p}}$ is called (Muir and Neuman, 1987) *WMR velocity vector*, and separately (v_x, v_y, ω) *WMR velocities*. Similarly, for each wheel, *wheel velocity vector* and *wheel velocities* are defined.

A. Wheel Equations with No-Slip

The kinematic modeling of a wheel has been tackled in many researches (Muir and Neuman, 1987; Alexander and Maddocks, 1989; Kim *et al.*, 2004) as a previous stage for modeling the whole WMR. Here it will be considered the approach of Gracia and Tornero (2007) and their wheel equations.

Four types of wheels will be considered: *fixed*, *centered orientable* (hereinafter *orientable*), *off-centered orientable* or *castor* and *Swedish* or *Mecanum* (Fig. 2 (a)). Under the no-slip assumption each wheel introduces two scalar equations given by the no-slip constraint in the contact point between the wheel and the floor. The matrix equation of the *castor* wheel is (1) where it has been used a compact trigonometric notation ($\cos(x) \equiv \mathrm{c}x,\ \sin(x) \equiv \mathrm{s}x$), the parameters of Fig.1, and the variables and constants of Table 1.

$$\begin{pmatrix} \mathrm{c}(\beta_i+\delta_i) & \mathrm{s}(\beta_i+\delta_i) & \mathrm{l}_i\,\mathrm{s}(\beta_i+\delta_i-\alpha_i)-\mathrm{d}_i\mathrm{c}\delta_i & -\mathrm{d}_i\,\mathrm{c}\delta_i & 0 \\ -\mathrm{s}(\beta_i+\delta_i) & \mathrm{c}(\beta_i+\delta_i) & \mathrm{l}_i\,\mathrm{c}(\beta_i+\delta_i-\alpha_i)+\mathrm{d}_i\,\mathrm{s}\delta_i & \mathrm{d}_i\,\mathrm{s}\delta_i & \mathrm{r}_i \end{pmatrix} \begin{pmatrix} {}^{\bar{\mathrm{R}}}\dot{\mathbf{p}} \\ \dot{\beta}_i \\ \dot{\varphi}_i \end{pmatrix} = \mathbf{0} \quad (1)$$

The matrix equation of the *orientable* wheel can be obtained from (1) with $\mathrm{d}_i = \delta_i = 0$:

$$\begin{pmatrix} \mathrm{c}\beta_i & \mathrm{s}\beta_i & \mathrm{l}_i\,\mathrm{s}(\beta_i-\alpha_i) & 0 \\ -\mathrm{s}\beta_i & \mathrm{c}\beta_i & \mathrm{l}_i\,\mathrm{c}(\beta_i-\alpha_i) & \mathrm{r}_i \end{pmatrix} \begin{pmatrix} {}^{\bar{\mathrm{R}}}\dot{\mathbf{p}} \\ \dot{\varphi}_i \end{pmatrix} = \mathbf{0}, \quad (2)$$

which is also valid for *fixed* wheels, where β_i is constant.

The matrix equation of the *Swedish* wheel is:

$$\begin{pmatrix} \mathrm{c}(\beta_i+\gamma_i) & \mathrm{s}(\beta_i+\gamma_i) & \mathrm{l}_i\,\mathrm{s}(\beta_i+\gamma_i-\alpha_i) & \mathrm{r}_i\,\mathrm{s}\gamma_i & 0 \\ -\mathrm{s}(\beta_i+\gamma_i) & \mathrm{c}(\beta_i+\gamma_i) & \mathrm{l}_i\,\mathrm{c}(\beta_i+\gamma_i-\alpha_i) & \mathrm{r}_i\,\mathrm{c}\gamma_i & \mathrm{r}_{ri} \end{pmatrix} \begin{pmatrix} {}^{\bar{\mathrm{R}}}\dot{\mathbf{p}} \\ \dot{\varphi}_i \\ \dot{\varphi}_{ri} \end{pmatrix} = \mathbf{0}, \quad (3)$$

where it has been used the parameters of Fig. 2 (b), and the variables and constants of Table 1. Note that, in the previous wheel equations, the instantaneously coincident frame $\bar{\mathrm{R}}$ avoids dependency on the global stationary frame G (Muir and Neuman 1987).

The WMR velocity vector in coordinate frame G is (4) where $\theta = \int \omega$ is the scalar angle between the axis $\bar{\mathrm{R}}_x$ and the axis G_x in coordinate Z of frame G.

$$\dot{\mathbf{p}} = \begin{pmatrix} \mathrm{c}\theta & -\mathrm{s}\theta & 0 \\ \mathrm{s}\theta & \mathrm{c}\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} {}^{\bar{\mathrm{R}}}\dot{\mathbf{p}}, \quad (4)$$

TABLE 1 – Frames, Variables and Constants

Symbol	Description
G	Global frame attached to the floor with the Z axis perpendicular to the floor surface
R	Frame attached to the robot with the Z axis perpendicular to the floor surface. The origin of this frame will track the reference
$\bar{\mathrm{R}}$	Frame attached to the floor, coincident with the robot frame R, with the Z axis perpendicular to the floor surface
${}^{\bar{\mathrm{R}}}\dot{\mathbf{p}}$	WMR velocity vector in coordinate frame $\bar{\mathrm{R}}$, equivalent to $({}^{\bar{\mathrm{R}}}v_x\ {}^{\bar{\mathrm{R}}}v_y\ \omega)^{\mathrm{T}}$
$\dot{\mathbf{p}}$	WMR velocity vector in coordinate frame G, equivalent to $(v_x\ v_y\ \omega)^{\mathrm{T}}$
$\dot{\beta}_i$	Angular velocity of the steering link with respect to the WMR
$(\mathrm{L}_{xi}, \mathrm{M}_{xi})$	Rotation axle of the wheel and the rollers
$(\dot{\varphi}_i, \dot{\varphi}_{ri})$	Rotation velocity of the wheel and the rollers in coordinate of axles L_{xi} and M_{xi}
$(\mathrm{r}_i, \mathrm{r}_{ri})$	Wheel equivalent radius and roller radius

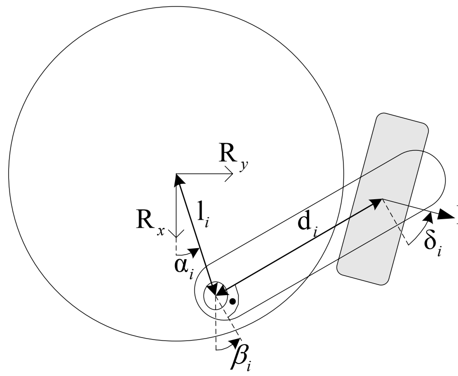
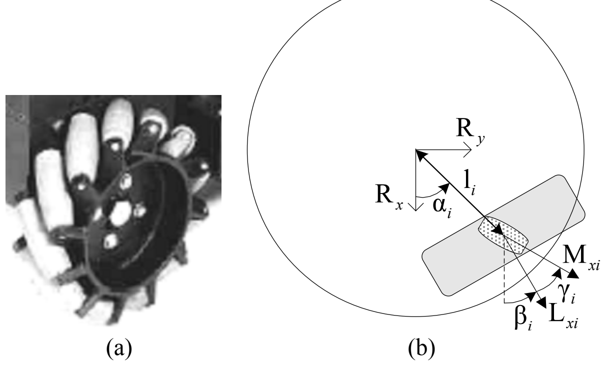


Figure 1. Parameters of the *castor* wheel: $\mathrm{l}_i, \mathrm{d}_i, \alpha_i, \beta_i, \delta_i$



(a) (b)

Figure 2. (a) *Swedish* wheel with rollers at 45º; (b) Parameters of the *Swedish* wheel: $\mathrm{l}_i, \alpha_i, \beta_i, \gamma_i$

From the previous wheel equations, it can be obtained the kinematic models of the WMR with the procedure shown in Gracia and Tornero (2007).

TABLE 2 - Classification of the Five Types of WMR

Type (m,s)	Type 1 (3,0)	Type 2 (2,0)	Type 3 (2,1)	Type 4 (1,1)	Type 5 (1,2)
m	3	2	2	1	1
s	0	0	1	1	2

B. Classification of WMR

Campion *et al.* (1996) classified the WMR into five generic types (see Table 2) through WMR *mobility degree m* (number of WMR velocities that can be assigned instantaneously and independently) and WMR *steeribility degree s* (number of *orientable* wheels that are steered independently).

The WMR of type 1 are constructed with no *fixed* or *orientable* wheels, *i.e.* with either *Swedish* or *castor* wheels, and are called omnidirectional. The WMR of type 2 have one independent *fixed* wheel and other possibly *omnidirectional* wheels (*Swedish* or *castor*). An example of this type is the differential-drive WMR. The WMR of type 3 have one independent *orientable* wheel and other possibly *omnidirectional* wheels. An example is the syncro-drive WMR. The WMR of type 4 have one independent *orientable* wheel and another independent *fixed* wheel. Examples of this type are the tricycle WMR, the bicycle WMR, and the car-like WMR. The WMR of type 5 are characterized by two independent *orientable* wheels.

Note that, the mobility degree represents the number of degrees of freedom that can be *instantaneously* used, without reorientation of the *orientable* wheels. Thus it is also defined *maneuverability degree g* as the sum of the mobility and steeribility degrees, and represents the total degrees of freedom, with reorientation of the *orientables* wheels. Therefore the WMR of types 1, 3, and 5 have full maneuverability (g = 3) and the WMR of types 2 and 4 have restricted maneuverability (g = 2).

III. KINEMATIC CONTROL

A. Introduction

The WMR control can be done from a kinematic or a dynamic perspective. The *kinematic perspective* consists on decoupling the control in two nested loops: kinematic loop and dynamic loop. The *dynamic perspective* considers only one loop with a global dynamic control. It has several drawbacks: the required analysis and computation become very complex; it is very sensitive to the uncertainties in the model parameters; the required inertial sensors may be expensive and inaccurate.

The kinematic perspective is simpler and its global stability is guaranteed if the low-level control (dynamic loop) is much faster than the medium-level control (kinematic loop). This option, more common in the bibliography, will be considered here.

The input to the WMR control is the tracking reference generated by a high-level *planner*. If these references are dynamical generated (*i.e.* they are recomputed online to deal with unexpected situations) it his obtained an outer loop (planning loop). The reference can be two-dimensional, *e.g.* position coordinates (x, y), or three-dimensional, *e.g.* position and orientation coordinates (x, y, θ), and may have associated time values (*e.g.* 2-D trajectory) or not (*e.g.* 2-D path), in which case it is usually assumed a constant motion velocity.

B. General Scheme

Figure 3 shows the proposed control scheme (see Table 3). In order to guarantee the stability of the previous control scheme with three nested loops, the dynamic loop must bee much faster than the kinematic loop and this one much faster than the planning loop.

The *low-level control* (dynamic loop) may include an inner current control loop (not depicted in Fig. 3) and would consider the masses, moments of inertia, torques, and forces to generate the input voltages to the actuators (motors). For this purpose the actuators can be considered decoupled (ideal), *i.e.* the influence between them is neglected, or coupled (real), *i.e.* their interaction is taken into account. The *high-level planner* may consider several aspects for reference generation: the possible references of section IV, collision avoidance, singular configurations, etc.

The *forward kinematics* consists on obtaining the WMR velocity vector $\dot{\mathbf{p}}$ from the sensed wheel velocities and may use kinematic models, slip models, etc. The estimation of the WMR posture $\mathbf{p}$ may be carried out simply through the numerical integration of the WMR velocity vector $\dot{\mathbf{p}}$ or through the *extended Kalman filter*, which is useful when there are other non-odometric sensors (sensor fusion) like in Scheding *et al.* (1999), Lindgren *et al.* (2002), etc.

The posture reference $\mathbf{p}_{\text{ref}}$ and the control velocity vector $\dot{\mathbf{p}}_{\text{control}}$ are complete (three elements) for WMR of full maneuverability (types 1, 3, and 5) and limited to two elements for WMR of restricted maneuverability (types 2 and 4). For this second case it will be assumed, in the next section, that the reference will be given by both linear variables.

The number of actuators for a consistent actuation is given by the *adequate actuation criterion* of Muir and Neuman (1987), and it coincides with the mobility degree m. In addition all the steering of the *orientable* wheels must be actuated. Note that all the non-actuated wheel velocities are auto-adjusted if the friction available between the wheel and the floor is enough.

Next subsections go deeper on the blocks of the kinematic loop (medium-level control) in Fig. 3.

C. Position Control

The position control of Fig. 3 establishes the WMR velocity vector $\dot{\mathbf{p}}$ to be achieved by the low-level control (control velocity vector $\dot{\mathbf{p}}_{\text{control}}$) depending on the WMR posture reference $\mathbf{p}_{\text{ref}}$ given by the high-level planner. It will be considered the following control:

$$\dot{\mathbf{p}}_{\text{control}} = \dot{\mathbf{p}}_{\text{ref}} + \mathbf{A}_{\text{c}}(\mathbf{p}_{\text{ref}} - \mathbf{p}) \tag{5}$$

where $\mathbf{A}_{\text{c}}$ is a diagonal matrix of dimension 3.

The low-level dynamic control gives rise to the convergence of the WMR velocity vector $\dot{\mathbf{p}}$ to the control velocity vector $\dot{\mathbf{p}}_{\text{control}}$. If the convergence time is neglected, *i.e.* the low-level control is considered instantaneous or is much faster than the kinematic control, the dynamics of the kinematic control (5) is given by:

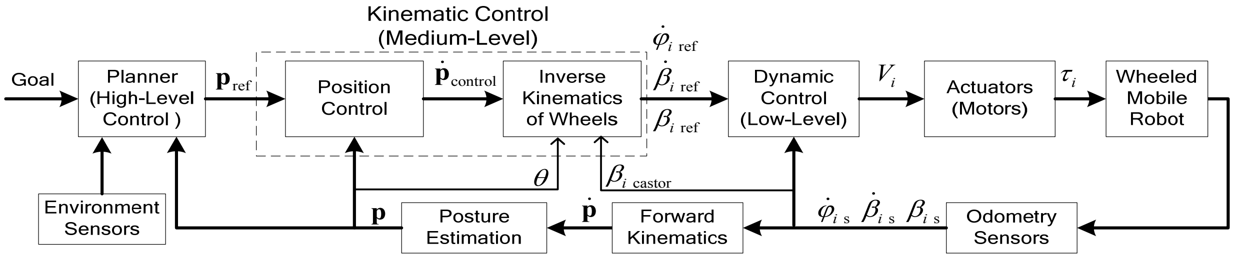


Figure 3. Control scheme of the wheeled mobile robot (with the symbol definitions of Table 3)

TABLE 3 – New Symbols of Figure 3

Symbol	Description
$\mathbf{p}_{\text{ref}}$	Posture reference given by the planner
$\dot{\mathbf{p}}_{\text{control}}$	WMR velocity vector to be achieved by the low-level dynamic control
$\dot{\varphi}_{i\ \text{ref}}$	Rotation velocity to be achieved by the low-level dynamic control
$\dot{\beta}_{i\ \text{ref}}$	Steering velocity to be achieved by the low-level dynamic control
$\beta_{i\ \text{ref}}$	Orientation of the *orientable* wheel *i* to be achieved by the low-level dynamic control
V_i	Voltage applied to the actuator *i* by the dynamic control
τ_i	Torque applied by the actuator *i*
$\dot{\varphi}_{i\ \text{s}}$	Sensed rotation velocity
$\dot{\beta}_{i\ \text{s}}$	Sensed steering velocity
$\beta_{i\ \text{s}}$	Sensed wheel orientation
$\beta_{i\ \text{castor}}$	Orientation of the *castor* wheel *i* with some wheel velocity actuated

$$\mathrm{d}(\mathbf{p}-\mathbf{p}_{\text{ref}})/\mathrm{d}t+\mathbf{A}_{\text{c}}(\mathbf{p}-\mathbf{p}_{\text{ref}})=\mathbf{0} \tag{6}$$

Equation (5) represents a trajectory controls, due to the derivative feedforward of the reference (first term of the second member), and it has no stationary error for any continuous reference. Moreover, it applies a proportional feedback and therefore the error converges asymptotically. In this sense, the poles (error dynamics) are assigned for each coordinate with the diagonal elements a_{c} of $\mathbf{A}_{\text{c}}$. If the kinematic control is implemented discretely the assigned dynamics must be slower than the sample time T to guarantee stability (*Shanon-Nyquist* theorem), *e.g.* $1/a_{\text{c}} \geq 10 \cdot \text{T}$. Finally, if the dead-zone of the actuators is not negligible, it can be necessary to add an integral effect in (5) and the error convergence may result oscillating.

D. Inverse Kinematics

The desired WMR velocity vector $\dot{\mathbf{p}}$ is the input to the inverse kinematics of a wheel, which returns the wheel velocities $\dot{\mathbf{q}}_{\text{w}i}$ and the orientation β_i if it is an *orientable* wheel. For the control scheme of Fig. 3, the desired WMR velocity vector is the control velocity vector $\dot{\mathbf{p}}_{\text{control}}$ obtained from the position control.

The values of wheel velocities $\dot{\mathbf{q}}_{\text{w}i}$ given by the inverse kinematics can be used or not in the low-level dynamic control. If not, they auto-adjust as previously mentioned. Meanwhile the orientation β_i of an *orientable* wheel must be always used by the low-level dynamic control.

Orientable wheel with no fixed wheels

From the first scalar equation of (2), β_i results:

$$\beta_i=\arctan\left((-{}^{\text{R}}v_x+\omega\,\text{l}_i\,\text{s}\alpha_i)({}^{\text{R}}v_y+\omega\,\text{l}_i\,\text{c}\alpha_i)^{-1}\right). \tag{7}$$

The previous expression using (4) is:

$$\beta_i=\arctan\left(\frac{-v_x\,\text{c}\,\theta-v_y\,\text{s}\,\theta+\omega\,\text{l}_i\,\text{s}\alpha_i}{-v_x\,\text{s}\,\theta+v_y\,\text{c}\,\theta+\omega\,\text{l}_i\,\text{c}\alpha_i}\right). \tag{8}$$

From the second element of (2) and taking into account the expression (7), the rotation velocity results:

$$\dot{\varphi}_i=-(1/\text{r}_i)\sqrt{(-{}^{\text{R}}v_x+\omega\,\text{l}_i\,\text{s}\alpha_i)^2+({}^{\text{R}}v_y+\omega\,\text{l}_i\,\text{c}\alpha_i)^2}. \tag{9}$$

The previous expression using (4) is:

$$\dot{\varphi}_i=-\frac{1}{\text{r}_i}\sqrt{\begin{aligned}&(-v_x\,\text{c}\,\theta-v_y\,\text{s}\,\theta+\omega\,\text{l}_i\,\text{s}\alpha_i)^2\\&+(-v_x\,\text{s}\,\theta+v_y\,\text{c}\,\theta+\omega\,\text{l}_i\,\text{c}\alpha_i)^2.\end{aligned}} \tag{10}$$

Therefore the inverse kinematics of a *orientable* wheel is given by (8) and (10), although it is particularized in the next point for when there is also one independent *fixed* wheel.

Note that to compute β_i in (8) the *arctan* is in all four quadrants and $\dot{\varphi}_i$ in (10) is always negative. Alternative it is possible to compute the *arctan* in fourth and first quadrants and multiply the second member of (10) by the sign of the second squared term in the root.

Fixed wheel and particularized orientable wheel

The inverse kinematics of a *fixed* wheel is given by Eq. (10). Although, since β_i is constant, there is a relationship (constraint) between the three elements of the WMR velocity vector, given by the first element of (2).

If the origin of frame R is on the rotation axle of the *fixed* wheels ($\beta_{\text{f}}-\alpha_{\text{f}}=0$), the relationship is exclusively between the linear velocities of $\dot{\mathbf{p}}$:

$$\beta_{\text{f}}-\alpha_{\text{f}}=0\rightarrow{}^{\text{R}}v_x={}^{\text{R}}v_y\tan\beta_i\ \text{ or }\ v_x=v_y\tan(\theta+\beta_i). \tag{11}$$

Otherwise, if it is not on the mentioned rotation axle it is possible to obtain ω depending on linear velocities:

$$\begin{aligned}\omega&=\frac{{}^{\text{R}}v_x\text{c}\beta_i+{}^{\text{R}}v_y\text{s}\beta_i}{-\text{l}_i\,\text{s}(\beta_i-\alpha_i)}=\frac{v_x\text{c}(\theta+\beta_i)+v_y\text{s}(\theta+\beta_i)}{-\text{l}_i\,\text{s}(\beta_i-\alpha_i)}\\ \omega&=f_1(\theta)\,v_x+f_2(\theta)\,v_y,\end{aligned} \tag{12}$$

where $f_i(\theta)$ is a generic function depending on θ.

Replacing (11) or (12), as appropriate, in (8) and (10), it is obtained:

$$\beta_i = \arctan\left(\frac{f_3(\theta)+f_4(\theta)\left(\omega/v_y\right)}{f_5(\theta)+f_6(\theta)\left(\omega/v_y\right)}\right) \quad (13)$$

$$\dot{\varphi}_i = -\sqrt{(f_7(\theta)\, v_y + f_8(\theta)\, \omega)^2 + (f_9(\theta)\, v_y + f_{10}(\theta)\, \omega)^2} \quad (14)$$

$$\beta_i = \arctan\left(\frac{f_{11}(\theta)+f_{12}(\theta)\left(v_y/v_x\right)}{f_{13}(\theta)+f_{14}(\theta)\left(v_y/v_x\right)}\right) \quad (15)$$

$$\dot{\varphi}_i = -\sqrt{(f_{15}(\theta)\, v_x + f_{16}(\theta)\, v_y)^2 + (f_{17}(\theta)\, v_x + f_{18}(\theta)\, v_y)^2} \quad (16)$$

Therefore, the inverse kinematics of a *orientable* wheel for when there is one independent *fixed* wheel is {(13),(14)} if the origin of frame R is on the rotation axle of *fixed* wheels or {(15),(16)} if it is not. Similarly (14) or (16) is the inverse kinematics of *fixed* wheels.

Castor wheel

From the first scalar equation of (1), $\dot{\beta}_i$ is:

$$\dot{\beta}_i = \frac{\left(\mathrm{c}(\beta_i+\delta_i)\ \ \mathrm{s}(\beta_i+\delta_i)\ \ \mathrm{l}_i\,\mathrm{s}(\beta_i+\delta_i-\alpha_i)-\mathrm{d}_i\mathrm{c}\delta_i\right)}{\mathrm{d}_i\,\mathrm{c}\delta_i}\,{}^{\bar{\mathrm{R}}}\dot{\mathbf{p}}$$
$$\dot{\beta}_i = \frac{\left(\mathrm{c}(\theta+\beta_i+\delta_i)\ \ \mathrm{s}(\theta+\beta_i+\delta_i)\ \ \mathrm{l}_i\,\mathrm{s}(\beta_i+\delta_i-\alpha_i)-\mathrm{d}_i\mathrm{c}\delta_i\right)}{\mathrm{d}_i\,\mathrm{c}\delta_i}\,\dot{\mathbf{p}} \quad (17)$$

Premultiplying (1) by $\left(\mathrm{s}\delta_i\ \ \mathrm{c}\delta_i\right)$, $\dot{\varphi}_i$ results:

$$\dot{\varphi}_i = -(\mathrm{r}_i\cdot\mathrm{c}\delta_i)^{-1}\left(-\mathrm{s}\beta_i\ \ \mathrm{c}\beta_i\ \ \mathrm{l}_i\,\mathrm{c}(\beta_i-\alpha_i)\right){}^{\bar{\mathrm{R}}}\dot{\mathbf{p}}$$
$$\dot{\varphi}_i = -(\mathrm{r}_i\cdot\mathrm{c}\delta_i)^{-1}\left(-\mathrm{s}(\theta+\beta_i)\ \ \mathrm{c}(\theta+\beta_i)\ \ \mathrm{l}_i\,\mathrm{c}(\beta_i-\alpha_i)\right)\dot{\mathbf{p}}. \quad (18)$$

Note that, the previous inverse kinematics equations depend on the orientation of the steering link. Therefore, if any of the velocities of this wheel is actuated the steering angle β_i must be sensed (see Fig. 3).

Swedish wheel

From the first scalar equation of (3), $\dot{\varphi}_i$ is:

$$\dot{\varphi}_i = -\frac{\left(\mathrm{c}(\beta_i+\gamma_i)\ \ \mathrm{s}(\beta_i+\gamma_i)\ \ \mathrm{l}_i\,\mathrm{s}(\beta_i+\gamma_i-\alpha_i)\right)}{\mathrm{r}_i\,\mathrm{s}\gamma_i}\,{}^{\bar{\mathrm{R}}}\dot{\mathbf{p}}$$
$$\dot{\varphi}_i = -\frac{\left(\mathrm{c}(\theta+\beta_i+\gamma_i)\ \ \mathrm{s}(\theta+\beta_i+\gamma_i)\ \ \mathrm{l}_i\,\mathrm{s}(\beta_i+\gamma_i-\alpha_i)\right)}{\mathrm{r}_i\,\mathrm{s}\gamma_i}\,\dot{\mathbf{p}} \quad (19)$$

Premultiplying (3) by $\left(\mathrm{c}\gamma_i\ \ -\mathrm{s}\gamma_i\right)$, $\dot{\varphi}_{\mathrm{r}i}$ results:

$$\dot{\varphi}_{\mathrm{r}i} = (\mathrm{r}_i\,\mathrm{s}\gamma_i)^{-1}\left(\mathrm{c}\beta_i\ \ \mathrm{s}\beta_i\ \ \mathrm{l}_i\,\mathrm{s}(\beta_i-\alpha_i)\right){}^{\bar{\mathrm{R}}}\dot{\mathbf{p}}$$
$$\dot{\varphi}_{\mathrm{r}i} = (\mathrm{r}_i\,\mathrm{s}\gamma_i)^{-1}\left(\mathrm{c}(\theta+\beta_i)\ \ \mathrm{s}(\theta+\beta_i)\ \ \mathrm{l}_i\,\mathrm{s}(\beta_i-\alpha_i)\right)\dot{\mathbf{p}}. \quad (20)$$

Nevertheless, the previous roller rotation velocity is not usually sensed nor actuated, since the roller is an auxiliary element of not much importance. That is why this wheel velocity has not bee included as an output of the inverse kinematics block in Fig. 3.

IV. POSSIBLE TRACKING REFERENCES

A. Introduction

Firstly it must be clarified that the adjective *possible* is referred to the fact that the WMR tracks the reference with no error, assuming that the initial conditions are adequate and that the elements of the control scheme are ideal. For example, if it is not respected the continuity condition of one variable (obtained in the analysis of the next subsections) an instantaneous tracking error would arise at the discontinuity points, which would be progressively corrected by the position control. The same happens for non-adequate initial conditions.

Under the *kinematic framework* tackled in this research the low-level dynamic control is obviated or considered instantaneous, and therefore the wheel velocities $\{\dot{\varphi}_i, \dot{\beta}_i, \dot{\varphi}_{ri}\}$ that are actuated may change instantaneously. Obviously, the posture references generated by the high-level planner should logically be continuous, *i.e.* they should not change instantaneously.

We have the following math definitions:

- A function is C^0 if it is continuous (see Fig. 4 (a));
- A function is C^1 if it has a continuous derivative;
- A function is C^n (with n>0) if it has a continuous n-th derivative;

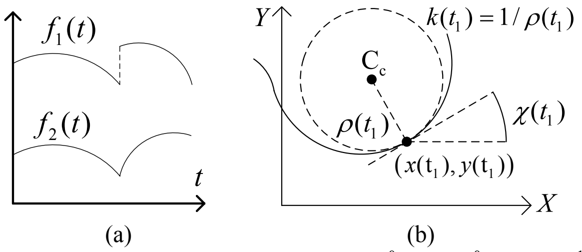


Figure 4.(a)Example functions: f_1 is not C^0; f_2 is C^0 but not C^1 (b)Variables of the path at a continuous tangent angle point

Next, it will be considered the path or curve given by two linear coordinates, $x(t)$ and $y(t)$, through the time, *i.e.* a two-dimensional (2D) trajectory (Fig. 4 (b)).

The angle χ between the tangent vector to the path/curve and the X-axis will be referred to as *tangent angle to the path* or simply *tangent angle*. It is computed from:

$$\chi(t) = \arctan\left(dy(t)/dx(t)\right) = \arctan\left(v_y/v_x\right). \quad (21)$$

Note that a continuous tangent angle χ does not imply that the linear positions $x(t)$ and $y(t)$ are continuous, *e.g.* dashed straight line in the X- or Y-axis direction. Nevertheless, it will be assumed in the subsequent developments that the references of the linear positions generated by the planner are, logically, continuous.

The *forward velocity* v_t and the *turning velocity* ω_t of the 2D trajectory (Fig. 4 (b)) are:

$$v_t(t) = \sqrt{v_x^2 + v_y^2} \quad (22)$$

$$\omega_t(t) = d\left(\chi(t)\right)/dt = (\dot{v}_y v_x - v_y \dot{v}_x)(v_x^2+v_y^2)^{-1}. \quad (23)$$

It is obvious the following coimplication:

$$\omega_t \text{ is } C^0 \Leftrightarrow \chi \text{ is } C^1. \quad (24)$$

The *curvature* κ *of the path* is the inverse of the *radius of curvature* ρ (Fig. 4 (b)) and is obtained with:

$$\kappa(t) = (\rho(t))^{-1} = \omega_t/v_t = (\dot{v}_y v_x - v_y \dot{v}_x)\left(v_x^2+v_y^2\right)^{-3/2}. \quad (25)$$

From (22), (24), and (25), it is obtained:

$$\text{If } \chi \text{ is } C^1 \Rightarrow \kappa \text{ is } C^0. \quad (26)$$

The previous statement is not true in the opposite direction, since v_t and ω_t may change instantaneously and keep their relationship continuous. If the curvature κ is *defined* at one point or through the entire path, the tangent angle χ is continuous at that point or through the entire path:

$$\text{If } \kappa \text{ is defined} \Rightarrow \chi \text{ is } C^0 . \qquad (27)$$

The *center of curvature* C_c *of the path* is given instantaneously by the radius of curvature ρ and the tangent angle to the path χ. Thus, if the curvature κ is continuous (ρ and χ are continuous) the set of the points defined by the center of curvature are continuous.

Note that it is possible to talk about a path with continuous (C^0) tangent-angle/curvature but not about a path with C^1 tangent-angle/curvature. In that case we would have to talk about a 2D trajectory, since it depends on forward velocity.

B. Type 1: Omnidirectional WMR

This WMR has full maneuverability, so the planner will generate references for the three elements of **p**. Under the kinematic framework (the wheel velocities change instantaneously), and taking into account the inverse kinematics of {(17),(18),(19),(20)}, this WMR can track any C^0 posture reference.

C. Type 2: Differential-Drive WMR

This WMR has restricted maneuverability, so the planner will generate references only for the linear coordinates $\{x(t),y(t)\}$ of the posture **p**.

If the origin of frame R (point that tracks the reference) is not on the axle of the *fixed* wheels $(\beta_f - \alpha_f \neq 0)$, the inverse kinematics of the *fixed* wheels is given by (16). In that case the WMR can track any 2D trajectory with C^0 position coordinates (kinematic framework) or with C^1 position coordinates (dynamic framework).

If the origin of R is on the rotation axle of the *fixed* wheels $(\beta_f - \alpha_f = 0)$, the inverse kinematics of the *fixed* wheels is given by (14). Using the constraint (11), given by the *fixed* wheels, it is obtained the relationship:

$$\tan(\theta+\beta_i) = v_x / v_y \rightarrow \theta + \beta_i = \alpha_\chi \rightarrow \dot{\theta} = \dot{\alpha}_\chi \rightarrow \omega = \omega_t \; ,(28)$$

i.e. the WMR angular velocity ω coincides with the turning velocity ω_t of the 2D trajectory.

So the inverse kinematics (14) is rewritten as:

$$\dot{\varphi}_i = -\sqrt{(f_7(\theta)\, v_y + f_8(\theta)\, \omega_t)^2 + (f_9(\theta)\, v_y + f_{10}(\theta)\, \omega_t)^2} \quad (29)$$

Therefore, under the kinematic framework the WMR can track any path with C^0 tangent angle. Although the WMR can track paths with tangent angle discontinuities if stops to reorientate itself at the discontinuity points.

D. Type 4: Tricycle and Bicycle WMR

This WMR has restricted maneuverability, so the planner generates references only for the linear coordinates. If the origin of frame R is not on the axle of the *fixed* wheels $(\beta_f - \alpha_f \neq 0)$, the inverse kinematics is {(15),(16)} for the *orientable* wheel and (16) for the *fixed* wheels. Tacking into account (21), these equations result:

$$\beta_i = \arctan\left(\frac{f_{11}(\theta) + f_{12}(\theta)\, \chi}{f_{13}(\theta) + f_{14}(\theta)\, \chi}\right) \quad (30)$$

$$\dot{\varphi}_i = -v_x \sqrt{(f_{15}(\theta) + f_{16}(\theta)\, \chi)^2 + (f_{17}(\theta) + f_{18}(\theta)\, \chi)^2} \quad (31)$$

In that case, under the kinematic framework the WMR can track any path with C^0 tangent angle. Although the WMR can track paths with tangent angle discontinuities if stops to reorientate the *orientable* wheel. Under the dynamic framework the WMR can track any 2D trajectories with C^1 tangent angle.

If the origin of frame R is on the axle of the *fixed* wheels $(\beta_f - \alpha_f = 0)$, the inverse kinematics is {(13),(14)} for the *orientable* wheel and (14) for the *fixed* wheels. For this case (28) and (29) are valid and (13) is rewritten as:

$$\beta_i = \arctan\left(\frac{f_3(\theta) + f_4(\theta)\left(\omega_t / v_y\right)}{f_5(\theta) + f_6(\theta)\left(\omega_t / v_y\right)}\right). \quad (32)$$

Using the constraint (11), the curvature of (25) is:

$$\kappa = \omega_t (v_x^2 + v_y^2)^{-1/2} = \omega_t (f_{19}(\theta)\, v_y)^{-1} . \quad (33)$$

So (32) and (29) result:

$$\beta_i = \arctan\left(\frac{f_3(\theta) + f_{20}(\theta)\, \kappa}{f_5(\theta) + f_{21}(\theta)\, \kappa}\right) \quad (34)$$

$$\dot{\varphi}_i = -v_y \sqrt{(f_7(\theta) + f_{22}(\theta)\, \kappa)^2 + (f_9(\theta) + f_{23}(\theta)\, \kappa)^2} \; . \quad (35)$$

Then, under the kinematic framework the WMR can track any path with C^0 curvature. Although the WMR can track paths with curvature discontinuities and C^0 tangent angle if stops to reorientate the *orientable* wheel. If there are curvature and tangent angle discontinuities the WMR would have to reorientate itself through a double reorientation of the *orientable* wheel.

E. Type 3/5: WMR with One/Two *Orientable* Wheels

Here, the WMR of types 3 and 5 are tackled simultaneously. They have full maneuverability, since they have no *fixed* wheels, so the planner will generate references for the three elements of **p**.

If there is more that one *orientable* wheel or there is one *orientable* wheel and origin of R is not on the center of the wheel, *i.e.* there is some *orientable* wheel with a non-zero l_i parameter ($\exists\, l_{oi} \neq 0$), the inverse kinematics of the *orientable* wheels with $l_{oi} \neq 0$ is {(8),(10)}. In that case, the WMR can track any C^1/C^2 posture reference under the kinematic/dynamic framework. Although, under the kinematic framework, the WMR can track posture references with derivative discontinuities if stops to reorient the *orientable* wheels.

If there is only one *orientable* wheel and origin of R is on its center ($\nexists\, l_{oi} \neq 0$), the inverse kinematics of {(8),(10)} is particularized:

$$\beta_i = \arctan\left((c\,\theta + \chi\, s\,\theta)(s\,\theta - \chi\, c\,\theta)^{-1}\right) \quad (36)$$

$$\dot{\varphi}_i = -\left(v_x / \mathrm{r}_i\right)\sqrt{(\mathrm{c}\,\theta + \chi\,\mathrm{s}\,\theta)^2 + (-\mathrm{s}\,\theta + \chi\,\mathrm{c}\,\theta)^2}\ . \quad (37)$$

Therefore, under the kinematic framework the WMR can track any posture references with C^0 tangent angle and C^0 angular reference (θ). Although the WMR can track paths with tangent angle discontinuities if stops to reorient the *orientable* wheel.

F. Summary of Possible Tracking References

Table 4 summarizes the possible tracking references depending on the WMR with the following aspects:
- Number of elements of **p** generated by the planner;
- Reference conditions under the kinematic framework;
- Procedure to avoid the tracking error if a particular condition is not satisfied.

V. APPLICATION OF THE KINEMATIC CONTROL TO AN INDUSTRIAL FORKLIFT

A. Particularization of the Kinematic Control

Here it will be particularized the WMR control to the case of the industrial forklift of Fig. 5, which is equivalent to the tricycle WMR of Fig. 6, where the origin of R (point that tracks the reference) has been located at the end of the blades $(\beta_\mathrm{f} - \alpha_\mathrm{f} \neq 0)$.

The parameters of each wheel are:

$$\begin{array}{lll} \mathrm{l}_1 = \sqrt{\mathrm{e}^2 + \mathrm{l}_{12}^2} & \alpha_1 = -\mathrm{atan}(\mathrm{e}/\mathrm{l}_{12}) & \beta_1 = 0 \\ \mathrm{l}_2 = \sqrt{\mathrm{e}^2 + \mathrm{l}_{12}^2} & \alpha_1 = \pi + \mathrm{atan}(\mathrm{e}/\mathrm{l}_{12}) & \beta_2 = 0 \\ \mathrm{l}_3 = \mathrm{l}_3 & \alpha_3 = -\pi/2. & \end{array} \quad (38)$$

The constraint (12) given by the *fixed* wheels is:

$$\omega = (\mathrm{l}_1\,\mathrm{s}(\alpha_1))^{-1}(v_x \mathrm{c}\theta + v_y \mathrm{s}\theta) = -\mathrm{e}^{-1}(v_x \mathrm{c}\theta + v_y \mathrm{s}\theta). \quad (39)$$

The inverse kinematics (16) of the *fixed* wheels is:

$$\begin{aligned} \dot{\varphi}_1 &= -\mathrm{r}^{-1}(-v_x\,\mathrm{s}\,\theta + v_y\,\mathrm{c}\,\theta - \mathrm{l}_{12}\mathrm{e}^{-1}(v_x\,\mathrm{c}\,\theta + v_y\,\mathrm{s}\,\theta)) \\ \dot{\varphi}_2 &= -\mathrm{r}^{-1}(-v_x\,\mathrm{s}\,\theta + v_y\,\mathrm{c}\,\theta + \mathrm{l}_{12}\mathrm{e}^{-1}(v_x\,\mathrm{c}\,\theta + v_y\,\mathrm{s}\,\theta)). \end{aligned} \quad (40)$$

TABLE 4 – POSSIBLE TRACKING REFERENCES

	Type 1	Type 2	
		$\beta_\mathrm{f} - \alpha_\mathrm{f} \neq 0$	$\beta_\mathrm{f} - \alpha_\mathrm{f} = 0$
No. refs.	3	2	2
Reference conditions	$\mathbf{p}_\mathrm{ref}$ is C^0	x_ref is C^0 y_ref is C^0	χ is C^0 *

	Type 4		Types 3 and 5	
	$\beta_\mathrm{f} - \alpha_\mathrm{f} \neq 0$	$\beta_\mathrm{f} - \alpha_\mathrm{f} = 0$	$\exists\, \mathrm{l}_{oi} \neq 0$	$\nexists\, \mathrm{l}_{oi} \neq 0$
No. refs.	2	2	3	3
Reference conditions	χ is C^0 **	κ is C^0 ***	$\mathbf{p}_\mathrm{ref}$ is C^1 ****	θ_ref is C^0 χ is C^0 **

* Stop to reorientate the WMR at

** Stop to reorientate the *orientable* wh

*** Stop to reorientate the *orientable* wheel at the curvatu

**** Stop to reorientate the *orientable* wheels at the derivative discontinuities of the posture reference

Figure 5. Industrial forklift Nichiyu FBT15 series 65

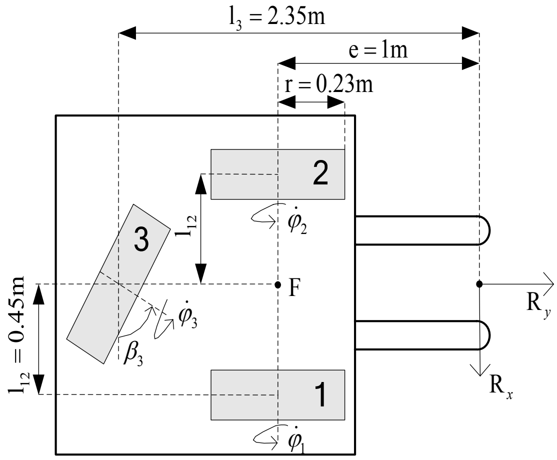


Figure 6. Variables and parameters of the industrial forklift

The inverse kinematics {(15),(16)} of the *orientable* wheel results:

$$\beta_3 = \arctan\left((\mathrm{l}_3 - \mathrm{e})(v_x\,\mathrm{c}\,\theta + v_y\,\mathrm{s}\,\theta)(-v_x\,\mathrm{s}\,\theta + v_y\,\mathrm{c}\,\theta)^{-1}\mathrm{e}^{-1}\right) \quad (41)$$

$$\dot{\varphi}_3 = -\frac{1}{\mathrm{r}}\sqrt{\frac{(\mathrm{l}_3 - \mathrm{e})^2}{\mathrm{e}^2}(v_x\,\mathrm{c}\,\theta + v_y\,\mathrm{s}\,\theta)^2 + (-v_x\,\mathrm{s}\,\theta + v_y\,\mathrm{c}\,\theta)^2} \quad (42)$$

If β_3 is computed in two quadrants (first and fourth quadrants), Eq. (42) is modified:

$$\dot{\varphi}_{3\mathrm{b}} = \mathrm{sign}(-v_x\,\mathrm{s}\,\theta + v_y\,\mathrm{c}\,\theta)\ \dot{\varphi}_3. \quad (43)$$

The position control (5) results:

$$\begin{aligned} v_{x\ \mathrm{control}} &= \dot{x}_{\mathrm{R\ ref}} + a_x(x_{\mathrm{R\ ref}} - x_\mathrm{R}) \\ v_{y\ \mathrm{control}} &= \dot{y}_{\mathrm{R\ ref}} + a_y(y_{\mathrm{R\ ref}} - y_\mathrm{R}) \end{aligned} \quad (44)$$

where $\{x_{\mathrm{R\ ref}}, y_{\mathrm{R\ ref}}\}$ are the reference of the linear coordinates and $\{a_x, a_y\}$ are the poles of the error dynamics in each coordinate.

The inputs to the inverse kinematics {(40),(41),(42)/(43)} are the control velocities $\{v_{x\ \mathrm{control}}, v_{y\ \mathrm{control}}\}$ of (44), and the outputs are $\{\dot{\varphi}_{1\ \mathrm{ref}}, \dot{\varphi}_{2\ \mathrm{ref}}, \beta_{3\ \mathrm{ref}}, \dot{\varphi}_{3\ \mathrm{ref}}\}$, which have to be achieved by the low-level dynamic control of Fig.

3. For a consistent actuation only the orientation angle β_3 and one rotation velocity must be actuated.

B. Simulation Results

Next it is presented an example, in a simulation environment, of the kinematic control of the previous subsection. For this example the control has been applied continuously and the low-level dynamics has been obviated. Also, it has been considered a null initial posture **p** and the poles $a_x = a_y = 2$ seconds^{-1}. It can be seen that the origin of R tracks the reference (Fig. 7 (a)) with the assigned dynamics (Fig. 7 (b)), although the orientation β_3 has two discontinuities.

The first discontinuity of β_3 (step of π) proves that the *arctan* of β_3 must be computed in all four quadrants to avoid it. If the *orientable* wheel has a +/–90º range, as is the case of the industrial forklift, the WMR has to stop to reorientate the wheel. The second discontinuity of β_3 (step of π/2) is produced due to the discontinuity of the tangent angle to the path (Table 4) and would imply a control action of infinite value. In order to track the reference with no error the WMR has to stop to reorientate the wheel.

Note that if it is considered a differential-drive WMR those discontinuities are not a problem. One interesting fact is that there is a maneuver, *i.e.* a change in the direction of the motion, at the beginning of the tracking (Fig. 7 (a)). This denotes that the designed kinematic control does not distinguish between forward or backward tracking. So, the WMR should be initially orientated to the reference to avoid backward tracking, *e.g.* with a previous path connecting the initial WMR posture and the original path.

It is also interesting to note that the track of the mid-distance point between the *fixed* wheels (Fig. 7 (a)) has a continuous tangent angle although the tangent angle of the reference path has discontinuities.

C. Experimental Results

The kinematic control given by {(40),(41),(43),(44)} has been implemented on a programmable logic controller (PLC) that governs the industrial forklift. The industrial forklift has two electric motors, one for the traction of the *fixed* wheels and the other one for the steering of the *orientable* wheel. The low-level dynamic control of the two motors has been designed neglecting their mutual influence (decoupled).The obtained dynamics of the decoupled control is: the steering motor has a setting time of around 1.5 seconds and no overshoot; the traction motor has a setting time of around 3.5 seconds and an overshoot of around 15%.

It has been carried out experiences of two classical paths, the circle and the straight line, both with a forward velocity v_t of around 0.5 m/s. For the circle experience (Fig. 8) it has been considered the pole values $a_x = a_y = 1$ s^{-1}. It can be seen that the tracking is more or less acceptable, although the orientation β_3 of the *orientable* wheel is constantly being corrected.

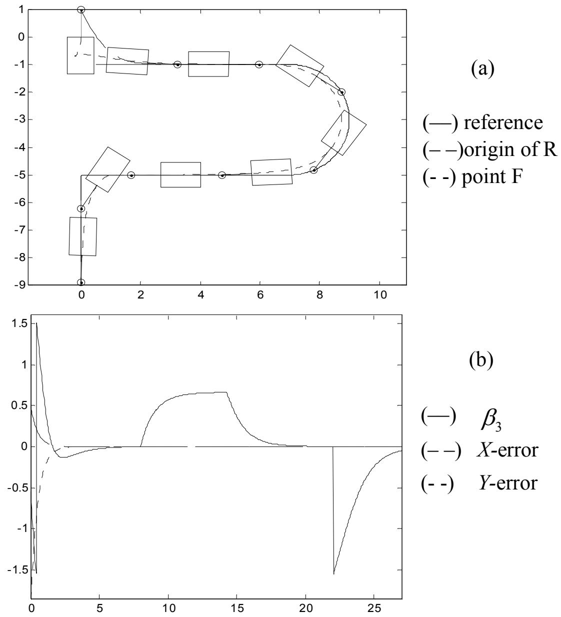


Figure 7. Simulation example

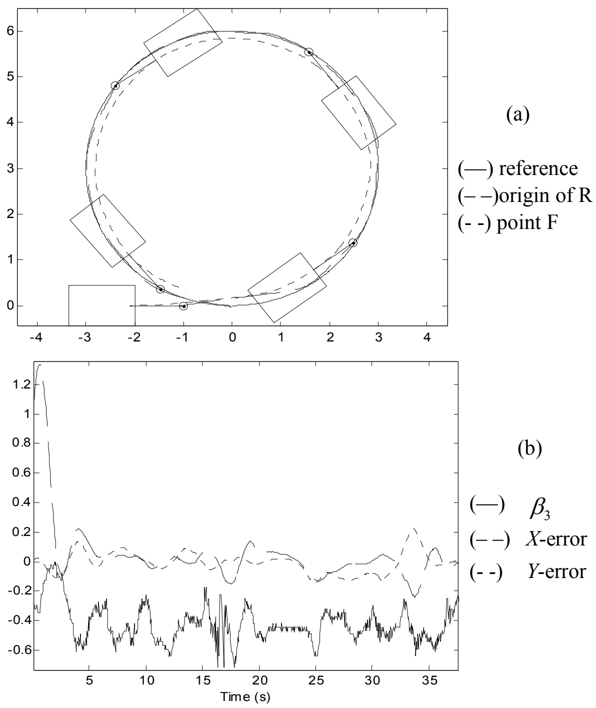


Figure 8. Circle experience

For the straight line experience (Fig. 9) it has been considered two values for the poles $\{a_x, a_y\}$. For the first values ($a_x = a_y = 1$ s^{-1}) the forklift converges to the straight line and the error dynamics is a bit oscillating, meanwhile for the second values ($a_x = a_y = 0.5$ s^{-1}) the convergence is slower and less oscillating. In order to justify the oscillating behavior of the control in Fig. 9 (b), it is shown in Fig. 9 (c) the (non-zero) error of the WMR velocities (*i.e.* the WMR errors in the forward and turning motion), which is given by the (non-zero) difference between $\{\beta_{3\,\mathrm{ref}}, \dot{\varphi}_{3\,\mathrm{ref}}\}$ and $\{\beta_3, \dot{\varphi}_3\}$. Then, it can be conclude that the stability of the kinematic control is conditioned by the error of the WMR velocities, *i.e.* by the dynamics of the low-level control.

In this sense, it has been observed empirically that poles with a bigger value (faster) than the first case produce instability. In fact, the setting time for the first case is of around 5 seconds, which is a bit longer than the setting time of the slower low-level dynamic controller.

In order to analyze the *stability* of the proposed kinematic control, it has been developed a simulation for the straight line tracking. In particular, it has been simulated the kinematic control together with a first order system between $\beta_{3\,\mathrm{ref}}$ and β_3, and a second order system between $\dot{\varphi}_{3\,\mathrm{ref}}$ and $\dot{\varphi}_3$, both with the dynamics indicated at the beginning of this subsection. The simulation results obtained evidence that the errors do not converge to zero for $\{a_x, a_y\} \geq 2.2$ s^{-1}. Note that, this instability condition is achieved earlier for the real WMR (instability arises approximately for $\{a_x, a_y\} \geq 1.2$ s^{-1}) because of modeling errors, non-linearities of the real system, non-null signal-to-noise ratio, coupling of the actuators, etc.

From the previous analysis, it can be established the following practical criterion: the maximum acceptable value for the poles of the kinematic control is such that the setting time of the slower low-level dynamic controller is half the setting time of the kinematic control. For example, in the case of the industrial forklift it would be $\{a_x, a_y\} \leq 5 / (3.5 \cdot 2) = 0.71$ s^{-1}. Obviously, the proposed criterion has been derived empirically. However, the stability could be further analyzed from an analytical point of view, which is not the goal of this research, through the use of the non-linear tools of control engineering.

VI. CONCLUSIONS

This research has presented a generic kinematic control that is valid for any type of WMR. The proposed control uses all the maneuverability of the WMR and takes into account the type of references that can be tracked by each WMR with no error. In fact, the characterization of the possible tracking references for each type of WMR (Table 4) is one of the most important contributions of the research.

Nevertheless the control has two limitations:

- The low-level dynamic control must be much faster that the medium-level kinematic loop to guarantee the global stability. This limitation has been observed in the experimental results with the industrial forklift and has been verified through simulation.
- The designed kinematic control does not distinguish between forward or backward tracking. So, the WMR should be initially orientated to the reference to avoid backward tracking. This limitation has been pointed out by the simulation results.

As further work, it is suggested to design a low-level dynamic control with coupled actuators and/or to develop a global dynamic control without decoupling the WMR kinematics and dynamics. The previous would produce a better performance in the reference tracking of the industrial forklift.

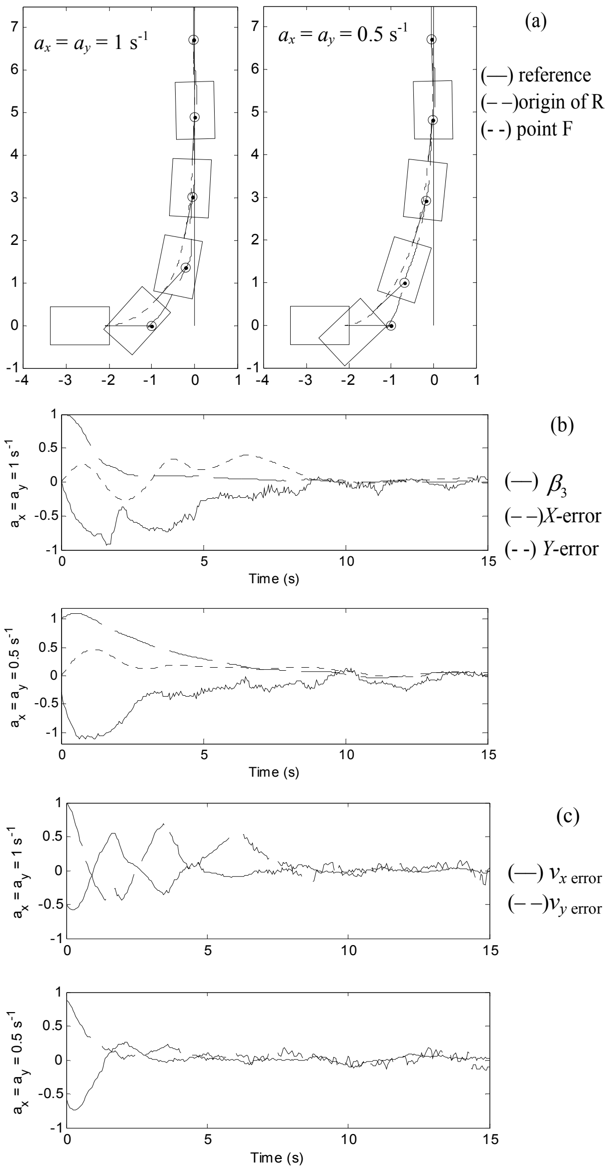


Figure 9. Straight line experiences

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